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Influence of multilayer beams cross-section shape on stiffness under bending

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1. Introduction

A majority of modern construction materials are made of composites. Each component of composites has its concrete destination in the product. Recently the employment of multilayer structural elements (MSE) is increasing because these elements allow to produce constructions of the required characteristics [1, 2]. Mechanical characteristics of the materials employed, as well as the arrangement of these in the construction and geometrical parameters of the structural element determine strength, stiffness and other characteristics of MSE [3 - 6]. A proper choice and a rational arrangement of materials provide to the structure, and multilayer beams (MB) among these, optimum characteristics. One of the most important parameters of beams is stiffness under bending. In [4 - 6] the influence of various factors, like elasticity modules, a number and position of layers in MSE, symmetry with respect to one axis on strength of multilayer beams and bars is revealed. Some aspects of geometric and stiffness centers, directions of neutral layers and varying stiffness under bending in case of asymmetry in the both, geometrical and stiffness sense, of structures are concerned in [7, 8].

The aim of the present study is to offer a new methodology of stiffness under bending in any direction calculation of the MB subjected to deformation within elasticity limits, to determine extreme values of elasticity and to examine regularities of variations depending on the beam cross-section shape and formation trajectories.

2. Mathematical model of a multilayer beam

Let us assume that MB is formed of n layers (Fig. 1), the elasticity modules of the layers are $E_1, E_2, ..., E_n$, and the cross-sections occupy the simply connected domains K_i such that

$$K \subseteq K_{\square} = [0,1]\times[0,1],\ K = \bigcup_{i=1}^{n} K_i,\ K_i \cap K_j = \varnothing,\ i \neq j \quad (1)$$

then coordinates of stiffness centre of MB, directions of neutral layers and extreme stiffness under bending values can be expressed by inertia tensor and its characteristic directions and values. A density of axial stiffness of MB in this case can be defined by the function

$$E(x, y) = \sum_{i=1}^{n} E_i Ind_i(x, y) \quad (2)$$

where $Ind_i(x,y) = \begin{cases} 0, & (x,y) \notin K_i \\ 1, & (x,y) \in K_i \end{cases}$ is the indication function of the set.

Let us assume $\boldsymbol{E} = (E_1, E_2, ..., E_n)$, $\boldsymbol{1} = (\underbrace{1, 1, ..., 1}_{n})$, then, with respect to (2)

$$m_{pq}(\boldsymbol{E}) = \iint_K x^p y^q E(x, y) dxdy \quad (3)$$

by which all the moments of MB cross-sections, coordinates of stiffness centre, as well as axial stiffness and stiffness under bending will be expressed. Below, the following vectors

$$\boldsymbol{S}(\boldsymbol{E}) = \begin{pmatrix} m_{01} \\ m_{10} \end{pmatrix},\ \boldsymbol{g}(\boldsymbol{E}) = \frac{S(\boldsymbol{E})}{m_{00}(\boldsymbol{E})} \quad (4)$$

and matrices

$$P(\boldsymbol{E}) = \begin{bmatrix} m_{01}^2 & m_{01}m_{10} \\ m_{01}m_{10} & m_{10}^2 \end{bmatrix},\ I(\boldsymbol{E}) = \begin{bmatrix} m_{02} & -m_{11} \\ -m_{11} & m_{20} \end{bmatrix} \quad (5)$$

will be used. Then, a cross-section area of MB is A and axial stiffness is $B(\boldsymbol{E})$

$$A = m_{00}(\boldsymbol{1}),\ B(\boldsymbol{E}) = m_{00}(\boldsymbol{E})$$

and geometrical and stiffness centers radius vectors in a global system of coordinates $\{x, y\}$ will be equal correspondingly to

$$\boldsymbol{C} = \boldsymbol{g}(\boldsymbol{1}),\ \boldsymbol{C}_E = \boldsymbol{g}(\boldsymbol{E}) \quad (6)$$

When employing designations (5) inertia tensor of MB cross-section in the global system $\{x, y\}$ equals to

$$\hat{I}_E(\boldsymbol{E}) = I(\boldsymbol{E}) \quad (7)$$

In stiffness system of coordinates $\{x_E, y_E\}$ (Fig. 1), orientation of the system corresponds to that of the global one, and the origin is defined by radius vector $\boldsymbol{C}_E$, inertia tensor of MB cross-section (following a parallel axis theorem) equals to

$$\hat{J}_E(\boldsymbol{E}) = I(\boldsymbol{E}) - P(\boldsymbol{E}) / m_{00}(\boldsymbol{E}) \quad (8)$$

Now a cross-section inertia moment of MB with respect to any axis crossing stiffness centre and forming the angle φ with a positive abscissa axis of the global coordinate system $\{x, y\}$ can be expressed via tensor $\hat{J}_E(\boldsymbol{E})$

$$M(\varphi) = \boldsymbol{\tau}(\varphi)\,\hat{J}_E(\boldsymbol{E})\,\boldsymbol{\tau}(\varphi)^{\mathrm{T}} \tag{9}$$

where $\boldsymbol{\tau}(\varphi) = \left(cos\,\varphi \quad sin\,\varphi\right)$ is a unit vector of φ direction.

It is note-worthy that tensor (6) is symmetric and elliptic, therefore its eigenvalues λ_1 and λ_2 are real and positive and the corresponding eigenvectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are orthogonal. Namely the latters define the directions of MB neutral layers. Thus, when a bending moment acts in the plane perpendicular to MB cross-section plane crossing the stiffness centre and making angle θ with the positive abscissa axis direction (Fig. 1) of the coordinate system $\{x_E, y_E\}$, stiffness at bending in the direction θ equals to

$$D(\theta) = \boldsymbol{v}(\theta)\,\hat{\boldsymbol{J}}(\boldsymbol{E})\,\boldsymbol{v}(\theta)^{\mathrm{T}} \tag{10}$$

here $\boldsymbol{v}(\theta) = \left(-sin\,\theta \quad cos\,\theta\right)$ is a unit vector of the direction orthogonal to φ direction.

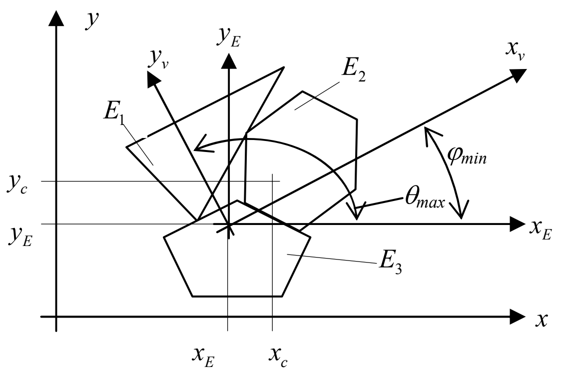


Fig. 1 Geometry of a multilayer structural element, global, stiffness and principal coordinate systems

Now we can define the principal coordinate system $\{x_v, y_v\}$, the origin of which is defined by radius vector $\boldsymbol{C}_E$, and abscissa axis is collinear to the eigenvector of tensor (8) corresponding to a maximum eigenvalue and forming the angle φ_{max} ($-\pi/2 \le \varphi \le \pi/2$) with the positive direction of abscissa axis of the stiffness coordinate system $\{x_E, y_E\}$. Moreover, the ordinate axis is directed so that the principal coordinate system $\{x_v, y_v\}$ is a right-handed one. It must be noted that this choice is always possible and is actually unambiguous, and the system $\{x_v, y_v\}$ satisfies the below conditions:

1) origin of the system is in the stiffness centre (x_E, y_E);

2) inertia moment with respect to abscissa axis is maximal;

3) inertia moment with respect to ordinate axis is minimal;

4) the system $\{x_v, y_v\}$ is right-handed orientated and the orts of it are normal eigenvectors of inertia tensor (8);

5) in the system $\{x_v, y_v\}$ tensor (8) is of a diagonal shape

$$\hat{\boldsymbol{J}}'(\boldsymbol{E}) = \begin{bmatrix} max\{\lambda_1, \lambda_2\} & 0 \\ 0 & min\{\lambda_1, \lambda_2\} \end{bmatrix} \tag{11}$$

3. Method of a multilayer beam geometric and stiffness parameters calculation

Now let us investigate the case when areas K_i are polygons (it is without any loss of generality, because the sides of polygons can be as small as required) not necessarily prominent but simply connected. Provided $P_i^{(j)}$ are the ordered (counter-clockwise) vertex sequences of polygon contour ∂K_i, then the contour of any polygon is

$$\partial K_i = \bigcup_{k=1}^{j_i} \left\{ \overline{P_i^{(j)} P_i^{(j+1)}} \right\}$$

Now let us prove that all the moments $m_{pq}(\boldsymbol{E})$ are expressible in algebraic form. Each side

$$P_i^{(j)}(x_{ij}, y_{ij}) P_i^{(j+1)}(x_{i(j+1)}, y_{i(j+1)}),\ i = 1,2,...n,\ \ j = 1,2,...j_i$$

of each polygon parametric equations in the global coordinate system $\{x, y\}$

$$\begin{cases} x_{ij}(t) = x_{ij} + \overline{x}_{ij}\, t \\ y_{ij}(t) = y_{ij} + \overline{y}_{ij}\, t \end{cases},\ (0 \le t \le 1)$$

here $\overline{x}_{ij} = x_{i(j+1)} - x_{ij}$, $\overline{y}_{ij} = y_{i(j+1)} - y_{ij}$. Therefore the moments $m_{pq}(\boldsymbol{E})$ are equal

$$m_{pq}(\boldsymbol{E}) = \iint_K E(x,y)\, x^p y^q dxdy = \frac{1}{q+1} \oint_{\partial K} E(x,y) x^p y^{q+1} dy =$$
$$= \frac{1}{q+1} \sum_{i=1}^{n} E_i \oint_{\partial K_i} x^p y^{q+1} dy = \frac{1}{q+1} \sum_{i=1}^{n} E_i \sum_{j=1}^{j_i} \int_{\left(P_i^{(j)} P_i^{(j+1)}\right)} x^p y^{q+1} dy =$$
$$= \frac{1}{q+1} \sum_{i=1}^{n} E_i \sum_{j=1}^{j_i} \overline{y}_{ij} \int_0^1 \left(x_{ij}(t)\right)^p \left(y_{ij}(t)\right)^{q+1} dt$$

besides, the definite integrals

$$s_{pq}^{(ij)} = \int_0^1 \left(x_{ij}(t)\right)^p \left(y_{ij}(t)\right)^{q+1} dt$$

at $p \ge 0, q \ge 0,\ p + q \le 2$, can be expressed in a finite form

$$s_{00}^{(ij)} = \frac{1}{2}\overline{y}_{ij} + y_{ij}$$

$$s_{10}^{(ij)} = \frac{1}{3}\overline{x}_{ij}\overline{y}_{ij} + \frac{1}{2}(x_{ij}\overline{y}_{ij} + \overline{x}_{ij} y_{ij}) + x_{ij} y_{ij}$$

$$s_{01}^{(ij)} = \frac{1}{3}\overline{y}_{ij}^2 + y_{ij}(y_{ij} + \overline{y}_{ij})$$

$$s_{20}^{(ij)} = \frac{1}{4}\overline{x}_{ij}^2\overline{y}_{ij} + \frac{1}{3}\overline{x}_{ij}(2x_{ij}\overline{y}_{ij} + \overline{x}_{ij} y_{ij}) +$$

$$+\frac{1}{2}x_{ij}^2\overline{y}_{ij}+x_{ij}y_{ij}(\overline{x}_{ij}+x_{ij})$$

$$s_{02}^{(ij)}=\frac{1}{4}y_{ij}^3+\frac{1}{3}\left(\frac{1}{2}y_{ij}^2\right)+y_{ij}(y_{ij}^2+\overline{y}_{ij}^2)$$

$$s_{11}^{(ij)}=\frac{1}{4}\overline{x}_{ij}\overline{y}_{ij}^2+\frac{1}{3}\overline{y}_{ij}(x_{ij}\overline{y}_{ij}+2y_{ij}\overline{x}_{ij})+\frac{1}{2}y_{ij}^2\overline{x}_{ij}+x_{ij}y_{ij}^2$$

therefore the moments $m_{pq}(\boldsymbol{E})$ can be calculated without employing integration procedure

$$m_{pq}(\boldsymbol{E})=\frac{1}{q+1}\sum_{i=1}^{n}E_i\sum_{j=1}^{j_i}\overline{y}_{ij}s_{pq}^{(ij)} \tag{12}$$

Now, let us investigate the important in practice case when $K\subset K_{\square}$, then the complement to domain K is equal to $K_c=K_{\square}\setminus K\neq\varnothing$ (complement K_c usually is of a more complicated geometry than areas K_i and K_c and is not connected in a majority of cases). Provided the complement K_c is filled with the material of the elasticity module $E_c\neq 0$, then the moments of the entire square (the beam is formed from $n+1$ layer) are equal to

$$m_{pq}^{\square}(\boldsymbol{E}_c)=E_c(m_{pq}^{\square}(\boldsymbol{1}^{\square})-m_{pq}(\boldsymbol{1}))+m_{pq}(\boldsymbol{E})=$$
$$=E_c((1+p+pq+q)^{-1}-m_{pq}(\boldsymbol{1}))+m_{pq}(\boldsymbol{E})$$

here $m_{pq}^{\square}(\boldsymbol{1})$ are moments of the entire square $K_{\square}$, and

$$\boldsymbol{E}_c=(E_1,\,E_2,\ldots,E_n,\,E_c),\ \boldsymbol{1}^{\square}=(\underbrace{1,\,1,\ldots,1}_{n+1})$$

Thus, we introduced a new mathematical model of a multilayer beam allowing to determine stiffness at bending in any direction as well as extreme values of stiffness and the corresponding directions of neutral layers and geometric and stiffness centers coordinates.

4. Object of study

The MB subjected to bending are often formed of rectangular shape cross-section layers, the dimensions generally are not uniform and the cross-section of MB does not possess a single inverse axis of symmetry. Moreover, for the formation of layers the materials of different elasticity modules E_i are employed, therefore the structure can be asymmetric not only in a geometric sense but also in the sense of stiffness, and the stiffness centre generally can not coincide with the geometric one. Such a structure is a two layer ($E_1\neq E_2$) composite formed from two rectangles possessing a mutual share of the contour (Fig. 2, straight-line *AB*). In [8] the dynamics of values variations of a two-layer beam geometric and stiffness centers and that of neutral layers directions and variations of extreme stiffness values under bending when the structural element was formed at point *B* movement along a diagonal of the square (Fig. 2, straight-line *3*) was investigated. In this study, the investigation results obtained at structural element formation at point *B* moving along curves *1-5* of a unit square 1x1 m (Fig. 2), are defined by function $f(t)=t^m$. Thus, the object under study – a two-layer angle iron - satisfies the condition (1) and

$$n=2,\ K_1=[0,1]\times[0,t^m],\ K_2=[0,t^m]\times[t^m,1],\ t\in[0,1]$$
$$P_1^{(1)}=(0,0),\ P_1^{(2)}=(1,0)$$
$$P_1^{(3)}=(1,\,t^m),\ P_1^{(4)}=(0,\,t^m),\ j_1=4$$
$$P_2^{(1)}=(0,\,t^m),\ P_2^{(2)}=(t^m,\,t^m)$$
$$P_2^{(3)}=(t^m,1),\ P_2^{(4)}=(0,1),\ j_2=4$$

here P_i^j are vertexes of rectangular layers. A trajectory of MB cross-section shape formation depends on index *m* therefore *m* further is called a shape index. A part of investigation was performed at m = 0.2; 0.5; 1.0; 2.0; 5.0.

Besides, the results of MB stiffness under bending at different cross-section shapes (Fig. 3) are reviewed. In [8] the first stiffness maximum was determined at $m=1.0$ and $E_1=30$ MPa, $E_2=1500$ MPa. At parameter t = 0.3, a cross-section area of the layer *1* is $A_1=0.3$ m^2, and that of layer *2* – $A_2=0.2$ m^2. The dimensions of the layers of cross-sections given in Fig. 3 were calculated adhering to the designated cross-section areas and elasticity modules of materials to the designated cross-section areas and elasticity modules of materials.

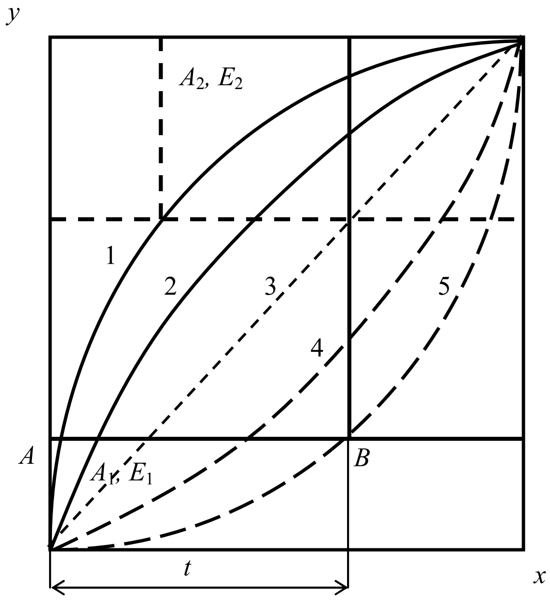


Fig. 2 Methodology of two-layer beam cross-section formation by the use of function $f(t)=t^m$, at *m* equal to: 0.2 – (*1*); 0.5 – (*2*); 1.0 – (*3*); 2.0 – (*4*); 5.0 - (*5*)

A study of geometric and stiffness centers coordinates x_c, y_c; x_E, y_E and the stiffness centre crossing the principal axes (directions of neutral layers as well) was performed by use of the new mathematical model of MB. Though in the global coordinate system $\{x,y\}$ dimensions of the layers varied in relative coordinates from zero to one, it was without any loss of generality of investigations because a proper choice of MB parameters allows to obtain the beams of all the possible cross-section shapes (Fig. 3).

5. Investigation results

In [8] it was proved that at increase of asymmetric MB layers elasticity modules ratio, the stiffness centre moves away from the geometric one and this distance is very important. In Fig. 4 are given the regularities of geometric and stiffness centers variations at constant MB layers (ratio of elasticity modules $E_2/E_1 = 50$), and at varying shape index m of a beam cross-section formation trajectory function $f(t) = t^m$. When formation of the beam occurs along diagonal (m=1) of a unit square, the geometric centre moves by a straight-line *1* and the stiffness centre by a curve *4* (Fig. 4,a and Fig. 4,b).

From the position of the curves it is evident that a distance between the centers does not vary until the pa-

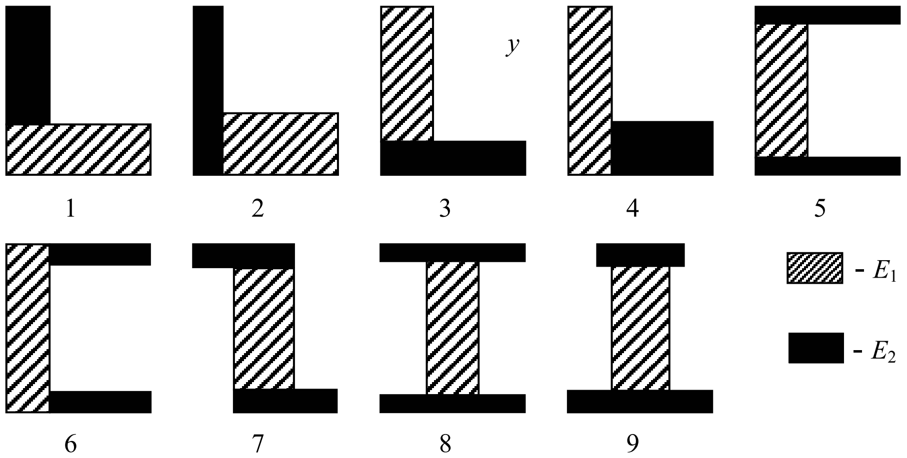
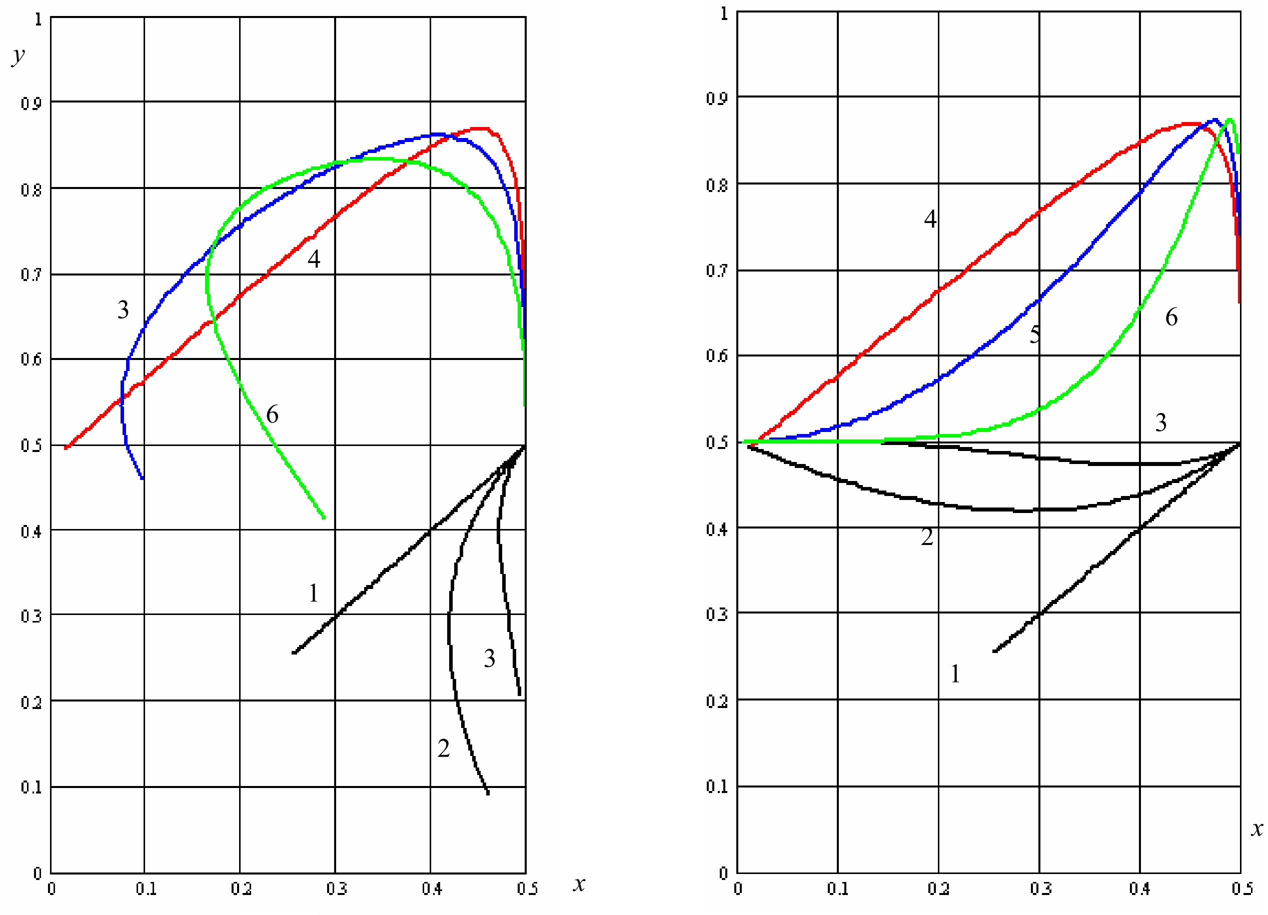


Fig. 3 Investigated cross-sections shapes of multilayer beams



a b

Fig. 4 Variation of the dependences of geometric (*1–3*) and stiffness (*4–6*) centres on MB cross-section geometric shape index *m,* at elasticity modules $E_2/E_1 = 50$, and *m* equal to: a) 0.2 – (*3; 6*); 0.5 – (*2; 5*); 1.0 – (*1; 4*); b) 1.0 – (*1; 4*); 2.0 – (*2; 5*); 5.0 – (3*; 6*) rameter t varies from zero to 0.855. At further t value increase, the distance between the centers decreases to zero because the structure from a two-layer turns into a one-layer. In case formation of the beam cross-section occurs at index m of function $f(t) = t^m$ less than one, i.e. the formation occurs along the trajectory of prominent curves *1, 2* (Fig. 2) then the variation of geometric centre is depicted by curves *2, 3* (Fig. 4, a), and stiffness – by curves *5, 6* (Fig. 4, a). At a shape index m decrease, trajectory of stiffness centre variation is closer to prominent curve *6* and the distance from the geometric centre increases.

At shape index m increase and when $m > 1$, the trajectory of geometric centre approaches to the horizontal axis of symmetry of a unit square (curves *2, 3* Fig. 4, b), and the trajectory of stiffness centre (curves *5, 6* Fig. 4, b) approaches to the geometric one. Moreover, all the curves *2, 3* and *5, 6* become prominent downwards, i.e. the distance between the centers decreases at shape index m increase.

Table

Geometric and stiffness centers, neutral layers directions angles θ_{max} and a maximum stiffness under bending D_{max} at various beam cross-section shapes

Cross-section shapes (Fig. 3)	Coordinates of geometric centers, m		Coordinates of stiffness centers, m		Angle θ_{max}, deg	Stiffness under bending 10^9,Nm^2	
No.	x_c	y_c	x_E	y_E	θ_{max}	D_{min}	D_{max}
1	0.357	0.350	0.153	0.635	98.21	3.6804	14.7269
2	0.400	0.313	0.115	0.491	93.51	3.5812	26.0421
3	0.313	0.400	0.491	0.115	176.5	3.5812	26.0421
4	0.350	0.357	0.635	0.153	171.8	3.6804	14.7269
5	0.313	0.500	0.490	0.500	90.02	25.9588	61.4799
6	0.350	0.500	0.636	0.500	90.02	14.5143	56.4005
7	0.499	0.499	0.499	0.499	112.7	11.1025	63.6425
8	0.500	0.500	0.500	0.500	90.02	25.1055	61.4799
9	0.500	0.456	0.500	0.394	90.02	19.1200	54.7500

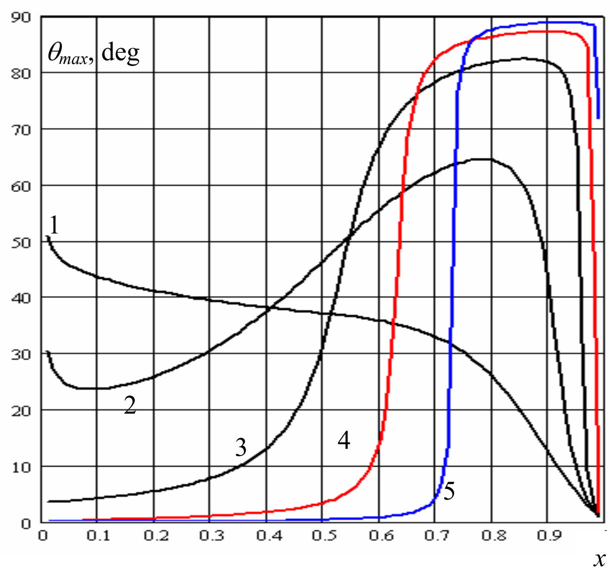


Fig. 5 Variation of angle θ_{max} at elasticity modules ratio $E_2/E_1 = 50$, when m equals to: 0.2 – (*1*); 0.5 – (*2*); 1.0 – (*3*); 2.0 – (*4*); 5.0 – (*5*)

The data in Table depict the influence of various MB cross-section shapes on the values of geometric and stiffness centers coordinates. At the same consumption of materials, the biggest distances (28% of the biggest dimensions size) between the centres were determined at a shape of the beam asymmetry with respect to the both axes (*1–4* Fig. 3). Depending on the position of the layer with the greatest stiffness, this maximum difference is obtained along axes x and y. In case a cross-section shape has at least one axis of symmetry, the distance between geometric and stiffness centers decreases and equals to zero of a double-T beam (Fig. 3, shape *8*).

In [8] it was determined that directions of the composite structural element neutral layers depend on geometric parameters and elasticity modules of the materials employed. Complicated relationships of direction angles show the importance of a precise determination of neutral layers position. Presented in the study variation of neutral layers direction angle θ_{max}, determining the maximum stiffness under bending (in case the moment direction is θ_{max}), depends on the beam cross-section shape index m (Fig. 5) and on shapes of cross-sections (Table 1). The obtained data (Fig. 5) show that the angle of neutral layers direction to a great extent depends on a cross-section shape index m.

At index $m > 0.5$, the curves of angles θ_{max} have well pronounced maximums and at m increase the speed of angle θ_{max} variation increases (Fig. 5, curves *2-5*). It is resulted by the fact that at bigger t values ($t > 0.7$ –curve *5*, Fig. 5) the influence of the layer with the greater stiffness reveals. At shape index $m = 0.2$, the angle θ_{max} varies along the curve *1* (Fig. 5). This curve does not posses maximum and until t=0.7, and angle θ_{max} variation of the values is insignificant because the inertia moments of the second layer with the greater stiffness, in this case, vary also insignificantly. Three pronounced zones of angle θ_{max} variation speed can be determined in curves 2-5 (Fig. 5). The biggest variation speeds of angle θ_{max} are determined at parameter t=0.0 - 0.9. It shows that the employment of the layer of less thickness but with a bigger module of elasticity in the composite element provides a significant variation of neutral layers direction angles.

The data in Table depict the influence of various MB cross-section shapes on neutral layers direction θ_{max} angles values. From the data it is evident that at the same consumption of materials, of angle θ_{max} depends also on the symmetry of a cross-section. When a structure is symmetric at least with respect to axis *x*, then angle θ_{max}=90° (Fig. 3, shapes *5, 6, 8, 9*). The biggest values of angle θ_{max} are in case when a layer with the greatest stiffness is horizontal (Fig. 3, shapes *3, 4*).

The variation of the studied neutral layer angles has a direct influence on MB stiffness when bending (Fig. 6). Maximum variations of stiffness curves are obtained depending on parameter *t* value, at the latter variation from zero to 1 at different values of index *m*. At little *m* values (*m*=0.2-0.5), stiffness curves D_{max} have only one slightly pronounced maximum (Fig. 6, curves *1, 2*). It can be explained by the variation continuous neutral layers direction angle θ_{max}. Moreover, at the mentioned values of shape index *m,* a cross-section area of the first layer ($E_1 = 30$) in the course of parameter *t* variation makes more than 90% of the total beam cross-section area, i.e. the influence of the latter is decisive.

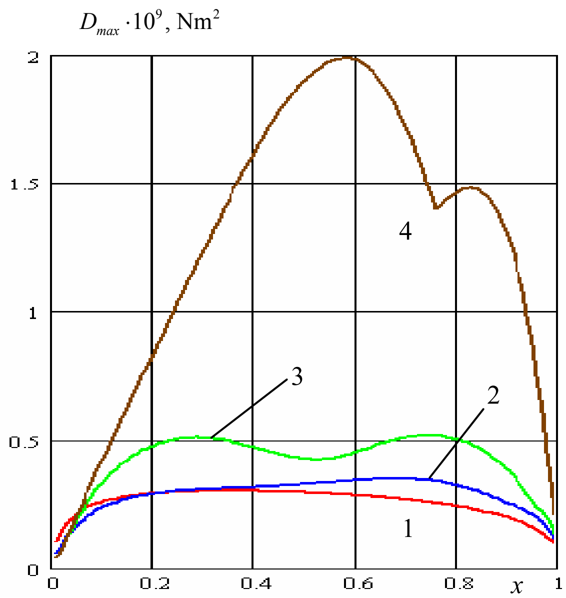


Fig. 6 Maximum stiffness at bending D_{max} variation dependence on parameter *t* at $E_2/E_1 = 50$ and shape index values *m*: 0.2 – (*1*); 0.5 – (*2*); 1.0 – (*3*); 5.0 – (*4*)

At the increase of shape index *m* values, stiffness at bending D_{max} increases and two maximums appear in curves (*3, 4*). At *m*=1.0 the values of the both extremes of stiffness D_{max} are equal, while at *m* increase (*m* > 1.0) the biggest D_{max} value is obtained at the first extreme point, which is moved nearer towards bigger values of parameter *t.* It is related to a prevalent influence of the second layer with the bigger elasticity module. The influence of MB cross-section shapes on stiffness at bending D_{max} is clearly evident from the data in Table. At equal amounts of materials consumption (cross-section areas of all the beams are equal) the biggest stiffness at bending is of shape Z cross-section (shape *7* Fig. 3) beam. Beams of cross-section shapes *5* and *8* (Fig. 3) are of the similar stiffness. It is note-worthy that the dimensions of layers of the biggest stiffness of shapes *6* and *7* (Fig. 3) are equal and are at the same distance from *x,* nevertheless, a stiffness of Z shape cross-section is 12.5 % bigger than that of shape *6*. Values of θ_{max} angle are different as well. It is a proof that a cross-section shape which is hardly evaluated without calculations is a very important factor in beam stiffness. The least and equal stiffness is of *1* and *4* shapes angles, though the values of angle θ_{max} differ by 73.6°, but linear dimensions are equal and only the arrangement of these differs (Fig. 3). The comparison of *1* and *2* cross-section shape (Fig. 3) beams which differ only by linear dimensions of the layers revealed that stiffness of MB is dependent on linear dimensions as well.

The study revealed that stiffness under bending of asymmetric MB depends on linear dimensions of the layers (in case of equal cross-section areas), on position of the layers and on the cross-section shape of the beam.

6. Conclusions

1. The suggested new mathematical model of a multilayer beam with polygon shape layers (without loss of generality as the edges of polygons can be as small as required) allows the determination of beam stiffness under bending in any direction extreme bending stiffness values and the corresponding direction of neutral layers as well as the coordinates of geometric and stiffness centers.

2. The influence of shape index *m* of the beam cross-section formation trajectory function $f(t) = t^m$ as well as the influence of various cross-sections shapes and linear dimensions of layers for geometric and stiffness centers, neutral layers direction angle θ_{max} and a maximum stiffness at bending D_{max} variations regularities is determined.

3. It is revealed that at increase of shape index *m* values, stiffness under bending D_{max} increases and two maximums appear in variation curves. At *m*=1.0, the both extremes of stiffness D_{max} values are equal, meanwhile at *m* > 1.0 the biggest D_{max} value is in the first extreme the position of which is nearer towards the bigger values of *t* parameter. At equal consumption of materials, the biggest stiffness at bending obtained is of asymmetric Z shape beam, and it is 12 % bigger than that of a double – T beam.

4. The study revealed that neutral layers angles in the structures investigated depend on geometric parameters and cross-section shapes of the beams. Complicated relationships of direction angles dependences prove the importance of precise determination of neutral layers positions.

5. It is determined that the variation of MB layers geometry and cross-section shape results in the variation of geometric and stiffness centers trajectories and the distances between the centers. Shape index *m* decrease, the variation of stiffness curve occurs along the prominent curve and the distance from geometric centre increases. At equal consumption of materials, the least distances between the two centers are of the beams the cross-sections of which possess at least one axis of symmetry.

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DAUGIASLUOKSNIŲ SIJŲ SKERSPJŪVIO FORMOS ĮTAKA JŲ LENKIAMAJAM STANDŽIUI

R e z i u m ė

Straipsnyje pasiūlytas naujas daugiasluoksnės sijos matematinis modelis, leidžiantis nustatyti sijos lenkiamąjį standį bet kuria kryptimi, taip pat ekstremalias standžio vertes ir jas atitinkančias neutraliųjų sluoksnių kryptis bei geometrinio ir standžio centrų koordinates. Nustatyta sijos skerspjūvio formavimo trajektorijos funkcijos $f(t)=t^m$ formos rodiklio m bei įvairių skerspjūvių formų įtaka sijos geometrinio ir standžio centrų, neutraliųjų sluoksnių krypčių kampo θ_{max} bei maksimalaus lenkiamojo standžio D_{max} kitimo dėsningumams.

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INFLUENCE OF MULTILAYER BEAMS CROSS-SECTION SHAPE ON STIFFNESS UNDER BENDING

S u m m a r y

The study offers a new mathematical model of a multilayer beam stiffness under bending in any direction determination as well as the evaluation of extreme values and the corresponding directions of neutral layers and coordinates of geometric and stiffness centers. The influence of beam cross-section formation trajectory function $f(t)=t^m$ shape index m and a variety of different shapes of cross-sections geometric and stiffness centers, neutral layers direction angle θ_{max} and a maximum stiffness under bending D_{max} on variation regularities is determined.

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ВЛИЯНИЕ ФОРМЫ ПОПЕРЕЧНОГО СЕЧЕНИЯ МНОГОСЛОЙНОЙ БАЛКИ НА ЕЕ ЖЕСТКОСТЬ ПРИ ИЗГИБЕ

Р е з ю м е

В статье предложена математическая модель многослойной балки, позволяющая определить жесткость при изгибе в любом направлении, а также ее экстремальные значения и им соответствующие направления нейтральных слоев, координаты жесткостных и геометрических центров. Установлено влияние степенного показателя формы m функции формирования траектории поперечного сечения и ему соответствующих форм на закономерности изменения координат геометрического и жесткостного центров, угла θ_{max}, направлений нейтраьных слоев, а также максимальных значений жесткости при изгибе.

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