T 121 SIGNALŲ TECHNOLOGIJA

A Method for Measuring an Electric Conductivity Tensor of Plane Media

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Introduction

Thin layer structures are wide used in the contemporary microelectronic technique devices, therefore investigations of the later by solid body physical methods acquire ever more important significance [1]. Thin layers of ordered structure (first of all monocrystalline) are of particular importance, since many physical effects in such layers distinguish themselves by a good repeatability. Thin films of ordered structure, obtained by epitaxy methods (by growing a layer on a monocrystalline substrate or inducing crystallite orientation by an electric field acting in the plane of the substrate), most frequently are anisotropic relative to electric conductivity, therefore their specific electric conductivity is completely defined by a tensor.

The specific conductivity tensor

In order to determine electric conductivity of an isotropic substance, it is customary to apply the following method: a rectangle-shaped sample of a, b dimensions is prepared with electrodes κ_1, κ_2 (Fig. 1). fixed on its opposite sides.
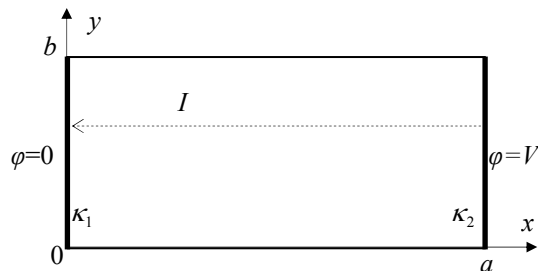

Fig. 1. The sample to investigate the specific conductivity of an isotropic substance

Having applied voltage V between the electrodes, we measure the current intensity I, and then the specific conductivity of the substance is calculated by the formula

$$\sigma = \frac{a}{b\,h}\frac{I}{V}$$

where h is thickness of the sample.

If the sample substance has anisotropic electric conductivity, then the latter is defined by a symmetrical tensor of specific conductivity

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}, \tag{1}$$

The components σ_{ij} of which are functions of the angle θ, made by the sides of rectangle with the principal axes of tensor σ:

$$\sigma = \begin{pmatrix} \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta & (\sigma_1 - \sigma_2)\sin\theta\cos\theta \\ (\sigma_1 - \sigma_2)\sin\theta\cos\theta & \sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta \end{pmatrix} \tag{2}$$

here σ_1, σ_2 – are specific conductivities along the principal axes, i.e., if the angle $\theta = 0$ or $\theta = \pi/2$ then

$$\sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \text{ arba } \sigma = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_1 \end{pmatrix},$$

besides, if $\sigma_1 = \sigma_2$, then the conductivity in all directions is the same and it is defined by one number σ_1 (the case of isotropic conductivity).

In general case (2) ($\sigma_1 \neq \sigma_2$) only the tensor trace $\mathrm{tr}\,\sigma$ and determinant $\det\sigma = \delta^2$ (invariants of tensor σ) do not depend on the angle θ

$$\mathrm{tr}\,\sigma = \sigma_{11} + \sigma_{22} = \sigma_1 + \sigma_2,$$
$$\det\sigma = \sigma_{11}\sigma_{22} - \sigma_{12}^2 = \sigma_1\sigma_2.$$

The problem of potential distribution

Let as consider the distribution of electric field potential $\varphi = \varphi(x, y)$ in a rectangle-shaped sample. If the medium is isotropic or anisotropic and the angle

$$\theta = k\frac{\pi}{2},\ k = 0,\pm 1,\pm 2,\dots,$$

then the potential isolines make up a beam of parallel lines, and in the opposite case, the isolines become curves. If the

lengths of the sides on which the electrodes are arranged, are short in comparison with that of other sides ($b \gg a$), then the shape of the central part isolines of the plate is approximate to straight lines.

With a view to establish their arrangement at least approximately, we formulate a boundary problem [2], the solution of which describes the distribution of the potential in the sample G with anisotropic conductivity (Fig. 1)

$$\begin{cases} \operatorname{div}\sigma\operatorname{grad}\varphi = 0, \ (x,y)\in G, \\ \varphi|_{\kappa}=const, \\ (\sigma\operatorname{grad}\varphi)_{\mathbf{n}}|_{\Gamma\setminus\kappa}=0, \end{cases} \qquad (3)$$

here Γ is in the contour of domain G, κ is the part of contour in which the contacts are arranged, and $\boldsymbol{n}$ is a unit vector of the contour normal. Note that the first differential operator of boundary problem (3) includes only the second order derivatives, i.e.,

$$\operatorname{div}\sigma\operatorname{grad}\varphi = \sigma_{11}\varphi_{xx} + 2\sigma_{12}\varphi_{xy} + \sigma_{22}\varphi_{yy},$$

Therefore each linear function $\varphi(x,y) = c_1 x + c_2 y + c_0$ is the solution to the $\operatorname{div}\sigma\operatorname{grad}\varphi = 0$. By choosing the numbers c_i ($i = 0,1,2$) so that the boundary conditions

$$(\sigma\operatorname{grad}\varphi)_{\mathbf{n}}|_{\Gamma\setminus\kappa}=\sigma_{12}\varphi_x + \sigma_{22}\varphi_y|_{\Gamma\setminus\kappa}=0$$

be satisfied and symmetry of the solution

$$\varphi(a/2+x, b/2+y) = V - \varphi(a/2-x, b/2-y)$$

be retained, we have an approximate solution of problem (3)

$$\varphi(x,y) = \frac{V}{a}\left(x - \left(y - \frac{b}{2}\right)\frac{\sigma_{12}}{\sigma_{22}}\right).$$

Based on this expression of the potential, the current intensity in the sample is

$$I_0 = \int_0^b (\sigma\operatorname{grad}\varphi)_{\mathbf{i}}\,dy = \int_0^b \sigma_{11}\varphi_x + \sigma_{12}\varphi_y\,dy =$$
$$= \frac{V}{a}\int_0^b \sigma_{11} - \sigma_{12}\frac{\sigma_{12}}{\sigma_{22}}\,dy = V\frac{b}{a}\frac{\delta^2}{\sigma_{22}}.$$

here i is the ort vector of the axis of abscissas of the coordinate system $\{x,y\}$.

Now let as go back to the main problem, i.e., to finding the components of the tensor σ (by taking physical measurements). If during the experiment the current intensity I_0 is measured as well as the appearing $\Delta\varphi_0$ (Fig. 2), then, in view of solution (4) we have that the components of the tensor are connected:

$$\begin{cases} \Delta\varphi_0 = \varphi(a/2,0) - \varphi(a/2,b) = Vb\sigma_{12}/a\,\sigma_{22}, \\ I_0 = Vb\delta^2/a\,\sigma_{22}, \\ \sigma_{11}\sigma_{22} - \sigma_{12}^2 = \delta^2. \end{cases} \qquad (4)$$

Indeed, if the determinant δ^2 of the tensor were known, we could find all the components $\sigma_{11}, \sigma_{22}, \sigma_{12}$ single-valued from the equations (4).
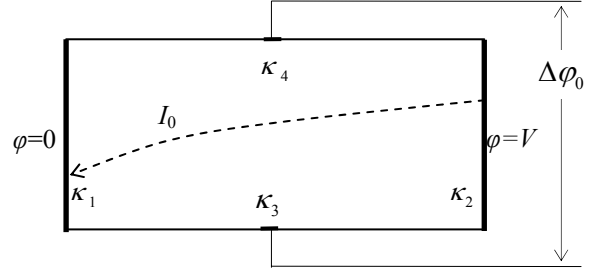

Fig. 2. The sample to determinate the conductivity of an anisotropic substance (longitudinal current)

Calculation of the determinant of the specific conductivity tensor

One can approximately calculate the determinant of the tensor after carrying out such an experiment: in the presence o voltage between the contacts κ_1 and κ_2, to measure the intensity I_1 of current and the appearing difference $\Delta\varphi_1$ of potentials between κ_3 and κ_4 (Fig. 3a). Afterwards, the experiment is repeated by using other pairs of contacts, i.e., by measuring the quantities I_2 and $\Delta\varphi_2$ (Fig. 3b), respectively. Having found the values of the quantities mentioned, the unknown quantity $\delta = \sqrt{\det\sigma}$ is obtained as the solution of the equation [3]:

$$\exp\left(-\pi h\delta\frac{|\Delta\varphi_1|}{I_1}\right) + \exp\left(-\pi h\delta\frac{|\Delta\varphi_2|}{I_2}\right) = 1. \qquad (5)$$

This equation always has the unique solution that can be found by applying the rapidly an unconditionally converging Newton method, choosing he initial value $\delta^{(0)}=0$.

Equation (5) is well known in the Hall effect theory and with sufficiently short lengths of electrodes it defines the conductivity of an isotropic medium

$$\sigma = \delta = \sqrt{\det\sigma}.$$

When investigating the case of an anisotropic substance, the latter is reduced to an equivalent isotropic one, after transforming the variables in boundary problem (3)

$$\xi = x\sigma_{11} - y\sigma_{12}, \ \eta = y\delta. \qquad (6)$$

Then the rectangle-shaped domain G becomes a parallelogram H. In addition, if we define the conductivity in the domain G by tensor (1), and in the domain H by the

number $\delta = \sqrt{\det \sigma}$, then all the electrical characteristics (potential of the electrical field and density of the current) becomes equal at the respective points of domains G and H (taking in to account transformation (6)), therefore equation (5) can be applied in the calculation of anisotropic conductivity parameters (components of the conductivity tensor).
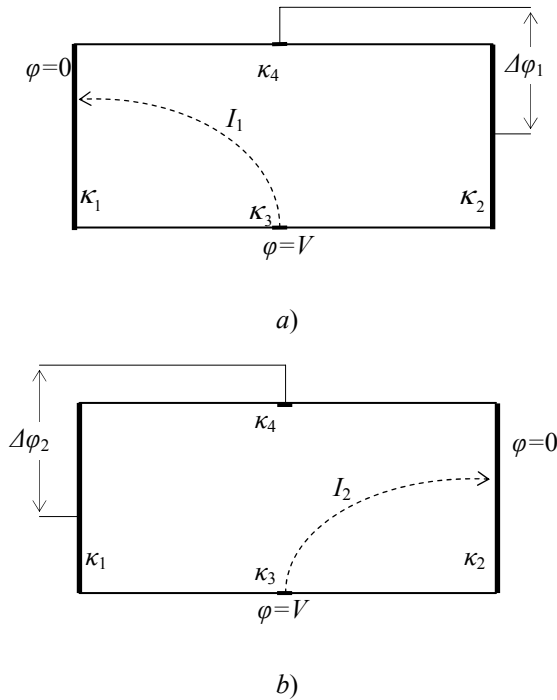

a)

b)

Fig. 3. The sample to determinate the conductivity of an anisotropic substance (transversal current)

Calculation of components of the conductivity tensor

Having measures the differences $\Delta\varphi_0, \Delta\varphi_1, \Delta\varphi_2$ of potentials and current intensities I_0, I_1, I_2 in the above experiment, one can find the solution of equation (5), i.e., $\sqrt{\det \sigma}$, and by (4) all the components of the tensor σ :

$$\sigma_{22} = V\frac{b}{a}\frac{\delta^2}{I_0}, \quad \sigma_{12} = \delta^2\frac{\Delta\varphi_0}{I_0}, \quad \sigma_{11} = \frac{\delta^2 - \sigma_{12}^2}{\sigma_{22}}. \quad (7)$$
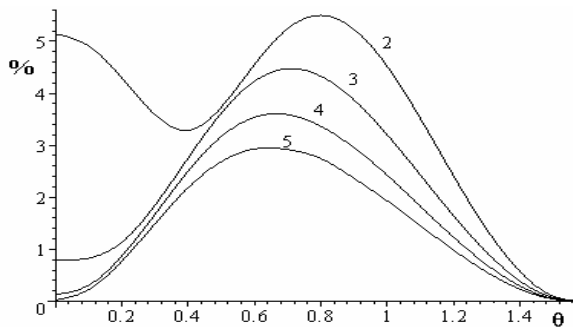

Fig. 4. Relative errors (8) of the tensor components. The numbers in the graph show ratio *a/b* between the side lengths of rectangle.

Evaluation of error of the method

Making use of formulas (7) we can approximately calculate the components of the specific conductivity tensor. The main reasons for emergence of errors of such method are an assumption on the linear potential distribution in the central part of the sample and the fact that nonzero length electrodes are used in the experiment.
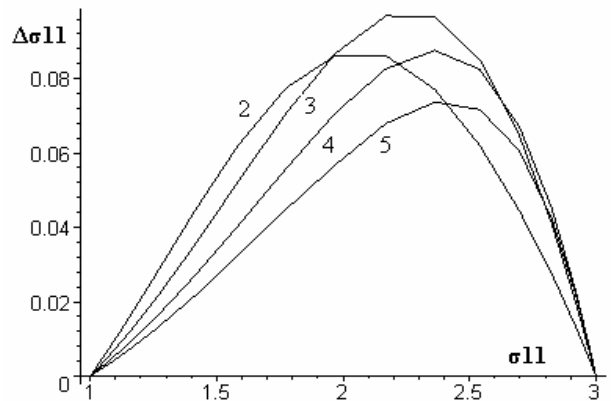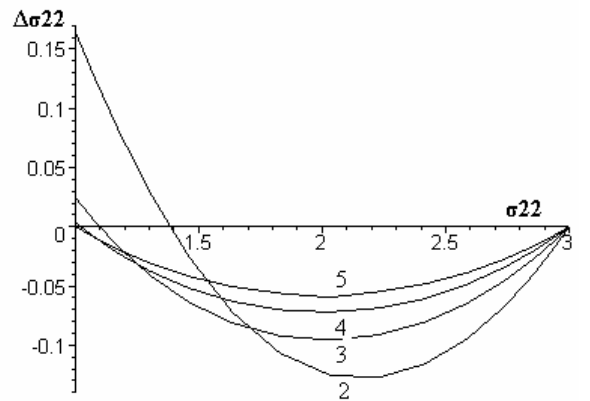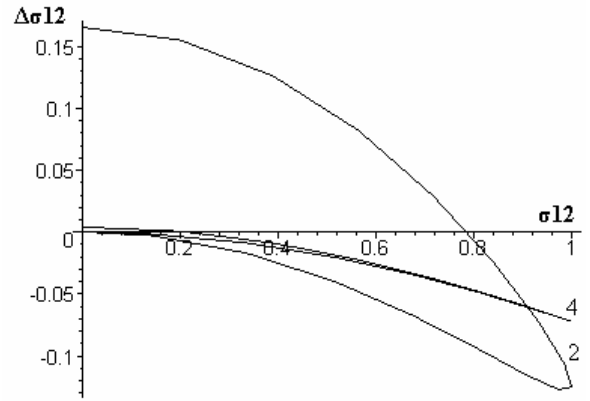

Fig. 5. Errors $\tilde{\sigma}_{ij} - \sigma_{ij}$ of the tensor components. The numbers in the graph show ratio *a/b* between the side lengths of rectangle.

The values of these errors are estimated after making imitative models of experiments and after calculating the differences $\Delta\varphi_0, \Delta\varphi_1, \Delta\varphi_2$ of potentials as well as current intensities I_0, I_1, I_2 . If $\sigma_{11}, \sigma_{12}, \sigma_{22}$ are exact values of the tensor σ components, and $\tilde{\sigma}_{11}, \tilde{\sigma}_{12}, \tilde{\sigma}_{22}$ are the

calculated ones according to (7), then, having defined a relative error

$$\frac{\sqrt{\left(\sigma_{11}-\tilde{\sigma}_{11}\right)^2+\left(\sigma_{12}-\tilde{\sigma}_{12}\right)^2+\left(\sigma_{22}-\tilde{\sigma}_{22}\right)^2}}{\sqrt{\tilde{\sigma}_{11}^2+\tilde{\sigma}_{12}^2+\tilde{\sigma}_{22}^2}}\cdot 100\% \quad (8)$$

One can observe its variation dependent on the angle θ and on the values (2) of the main conductivities σ_1, σ_2.

The relative errors mentioned are illustrated in Fig. 4, where principal values of the specific conductivity tensor are $\sigma_1 = 3$, $\sigma_2 = 1$, and the ratios a/b between the sides of a rectangle shaped domain are equal to 2, 3, 4, 5, respectively, while the lengths of central electrodes κ_3, κ_4 do not exceed $b/20$. Fig. 5 demonstrates the dependence of the error $\sigma_{ij} - \tilde{\sigma}_{ij}$ of each component on its value σ_{ij}.

In the general case, based on the described error emergence reasons and formulas (6) of reduction to the isotropic case, we can assert that the relative error (8) I determined by the following parameters:

1. Extensibility of sample, i.e., the ratio a/b
2. Measure of conductivity anisotropy:

$$\max\left(\sigma_1/\sigma_2, \sigma_2/\sigma_1\right),$$

i.e., the error will be lower under the higher ratio a/b and the lower value of the anisotropy measure.

Conclusions

1. The method for calculating the specific conductivity tensor of an anisotropically conductive medium, proposed in this paper, distinguishes itself by the simplicity of physical measurements: it suffices to make an equally thick rectangle-shaped sample with four electrodes fixed on its sides and to take various measurements of current intensity and differences of potentials.

2. The necessary mathematical calculations can be promptly performed, even without using a complex computing technique.

3. The accuracy of the results obtained depends on the dimensions a and b of the sample and on the ratios of the conductivity tensor components σ_1 and σ_2. For example, if $a/b \geq 3$, then the relative error does not exceed 5% for every substance, the anisotropy measure of which is $\max\left(\sigma_1/\sigma_2, \sigma_2/\sigma_1\right) \leq 3$.

4. By applying this method, it is possible to calculate the second invariant of the specific conductivity tensor in the investigation of the sample of any shape, because equation has been obtained independent of geometry of the sample.

References

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J. Kleiza, V. Kleiza. Plokščios terpės elektrinio laidumo tenzoriaus matavimo metodas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 3(59) – P. 51 –54.

Pasiūlytas anizotropiškai laidžios terpės savitojo laidumo tenzoriaus matavimo metodas pasižymintis fizikinių matavimų paprastumu: pakanka pagaminti vienodo storio stačiakampio formos bandinį su pritvirtintais jos kraštinėse keturiais elektrodais ir atlikti įvairius srovės stiprio ir potencialų skirtumo matavimus. Be to, būtinus skaičiavimus galima atlikti greitai, nesinaudojant sudėtinga skaičiavimo technika. Gaunamų rezultatų tikslumas priklauso tik nuo bandinio matmenų ir tenzoriaus komponenčių santykių. Il.5, bibl.3 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

J. Kleiza, V. Kleiza. A Method of Measuring Electric Conductivity Tensor of a Plane Media // Electronics end Electrical Engineering. – Kaunas: Technologija, 2005. – No. 3(59) – P. 51 –54.

The method for calculating the specific conductivity tensor of an anisotropically conductive medium, proposed in this paper, distinguishes itself by the simplicity of physical measurements: it suffices to make an equally thick rectangle-shaped sample with four electrodes fixed on its sides and to take various measurements of current intensity and differences of potentials. The necessary mathematical calculations can be promptly performed, even without using a complex computing technique. The accuracy of the results obtained depends on the dimensions of the sample and on the ratios of the conductivity tensor components. Ill. 5, bibl. 3 (in English; summaries in Lithuanian, English and Russian).

Й. Клейза, В. Клейза. Метод измерения тензора электрической проводимости плоской среды // Электроника и электротехника. – Каунас: Технология, 2005. – № 3(59) – С. 51 –54.

Предложен метод измерения тензора удельной электрической проводимости плоской анизотропной среды обладающий простотой физических измерений: достаточно изготовить образец постоянной толщины и прямоугольной формы с четырьмя электродами прикрепленными по его сторонам и произвести несколько измерений тока и разности потенциалов. Кроме этого, необходимые расчеты достаточно просты и их можно произвести пользуясь несложной вычислительной техникой. Точность результатов зависит от отношения размеров образца и отношения компонент тензора. Ил.5, библ. 3 (на английском языке; резюме на литовском, английском и русском яз.).