# Modelling Light Transmission in a Fiber-Optical Reflection System 

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#### Abstract

Light transmission in the reflection fiber system, located in external optical media, has been investigated for application as sensors. The system was simulated by different models, including external cavity parameters such as the distance between light emitting and receiving fibers and mirror positioning distance. The sensitivity to a linear displacement of the sensors was studied as a function of the distance between the tips of the light emitting fiber and the center of the pair reflected light collecting fibers, by positioning a mirror. Physical fundamentals and operating principles of the advanced fiber optical sensors were revealed.


Keywords: fiber-optical sensor, displacement sensor, refractometer.

## 1 Introduction

Investigation of the reflection fiber system arrangement and influence of physical and optical parameters of a cavity on light transmission promises the creation of a lot of devices for measuring physical properties of the external optical media and cavity dimension changes.

In recent years W.H.Ko et al [1] have proposed a new innovative, external influence compensated method - a fiber optical reflective displacement micrometer, which consisted of the light transmitting fiber and a pair of light
receiving fibers on a plane. The tips of the pair of the receiving fibers detect the light reflected from a mirror and form an angle with the light emitting fiber. The experimental dependence of the output signal of the micrometer after fixing not commented parameters of the fibers and construction of the micrometer have been obtained.

The aim of this paper is to investigate the influence of the arrangement of the reflection fiber system on light transmission: (i) the influence of physical properties of a cavity, (ii), to show the sensitivity of the fiber optical system dependence on the fiber tips distribution distance b1. We demonstrate a possibility: (i) to use fibers with a defined angle between the tips of fibers at the mirror in a measuring head and the same light source as a sensor for increased sensitivity measuring of the linear displacement, (ii) to show that the light source degradation and temperature fluctuations can be compensated.

## 2 Principle of operation (main equations)

The arrangement of the fiber tips in Fig. 1 was applied to explain the main physical principle of operation of the reflection fiber system. If the reflecting surface is a perfect mirror, the reflected light is equivalent to that transmitted with the source $L$, at the mirrored position $L^{\prime}$ a distance $z$ away. According to [2], the intensity distribution function of the source $I(r, z)$ at the position $Q(r, z)$ on the plane $z(h)$ away from the source and $r$ away from the light beam axis is [2]

$$
\begin{equation*}
I(r, z)=I_{0} K_{0} \frac{\exp \left\{r^{2} / R^{2}(z)\right\}}{R^{2}(z)} \tag{1}
\end{equation*}
$$

$I_{0}$ is the intensity of the light source, $K_{0}$ is the loss of the intensity of the input fiber; $n=1 . R(z)$ is the effective radius of the output optical field, defined as

$$
\begin{equation*}
R(z)=a_{0}+k z \tan \left(\theta_{c}\right) \tag{2}
\end{equation*}
$$

$2 a_{0}$ is the radius of the fiber core, $\theta_{c}$ is the aperture angle of the fiber, $k$ is a constant of the light source. Contrary to [1, 2], we consider $R(z)$ to be a linear function, because of the well-known linear spreading of light.

If the reflecting surface is not perfect and has the reflection coefficient $R_{M}$, the reflected light will be reduced by the factor $r$.


Fig. 1. Fiber tips arrangement of the reflection fiber system [1]. $A, B$ are light receiving fibers; $L$ is a light emitting fiber; $\theta$ is an angle; $2 a_{0}$ is the fiber core diameter; $2 a$ is the fiber diameter with cladding; $2 d$ is the distance between light receiving fibers; $b_{1}$ and $b_{1}^{\prime}$ are the distance between the centers of light emitting and a pair of the receiving fibers with $h=h_{0}$ and $h_{10}$ $\left(h_{10}>h_{0}\right) ; h$ is a distance; $I_{0}$ is the intensity of a light source; $I_{A}, I_{B}$ are the intensity of received light; $R$ is the reflection coefficient of the mirror; $L_{h_{0}}^{\prime}$ is the mirrored position of the fiber $L$ at $h_{0} ; h_{0}$ is the distance $h$ when signals $(A-B)=0 ; L_{ \pm h^{\prime}}^{\prime}$ is the mirrored position at $h=h_{0} \pm h^{\prime} ; \pm X$ is a displacement of mirrored position for $\pm h^{\prime} ; Z_{0}$ is the distance of the mirrored position at $h_{0} ; Z$ is the distance of $L_{+h^{\prime}}^{\prime}$ at the mirror position $h=h_{0}+h^{\prime}$.

The transmitted light intensity at the end of the receiving fiber is [3]:

$$
\begin{equation*}
I(r, z) \approx \iint_{S} \frac{K_{0} R_{M} K I_{0} \exp \left(-r^{2} / R^{2}(z)\right)}{\pi R^{2}(z)} d s \tag{3}
\end{equation*}
$$

$K$ is the loss in the intensity of the receiving fiber, and $S$ is the receiving fiber core area.

If the intensity at the receiving fiber center is used to represent the average intensity received and the reflection coefficient of the mirror is included, then the intensity received by the $A$ and $B$ fibers can be simplified as follows
(Fig. 1):

$$
\begin{align*}
& A=I_{A}=S_{1} \frac{K_{0} R_{M} K_{1} I_{0} \exp \left\{-(x+d)^{2} / R^{2}(z)\right\}}{\pi R^{2}(z)}  \tag{4}\\
& B=I_{B}=S_{2} \frac{K_{0} R_{M} K_{2} I_{0} \exp \left\{-(x-d)^{2} / R^{2}(z)\right\}}{\pi R^{2}(z)} \tag{5}
\end{align*}
$$

where $2 d$ is the distance between the centers of the light receiving fibers, $2 a$ is the diameter of the fiber with cladding. In particular case $2 d$ can be equal to $2 a$.

When the two receiving fibers are identical, $K_{1}=K_{2}, S_{1}=S_{2}$ and

$$
\begin{equation*}
\frac{(A-B)}{(A+B)}=\frac{\exp \left\{-(x+d)^{2} / R^{2}(z)\right\}-\exp \left\{-(x-d)^{2} / R^{2}(z)\right\}}{\exp \left\{-(x+d)^{2} / R^{2}(z)\right\}+\exp \left\{-(x-d)^{2} / R^{2}(z)\right\}} . \tag{6}
\end{equation*}
$$

According to Fig. 1, $X=x=2 h^{\prime} \sin \theta$. Equation (6) is a function not only of $x$, but also of $z$.

$$
\begin{equation*}
z=\left(z_{0} \pm \Delta z\right) n \tag{7}
\end{equation*}
$$

where

$$
z_{0}=\frac{b_{1}}{\sin \theta}, \quad \Delta z=\frac{2 h^{\prime}}{\cos \theta}
$$

Then

$$
\begin{equation*}
z=\left(\frac{b_{1}}{\sin \theta} \pm \frac{2 h^{\prime}}{\cos \theta}\right) n \tag{8}
\end{equation*}
$$

Here $z_{0}$ and $h_{0}$ are the initial reference values when $A=B$, $\left(U_{A}=U_{B}\right)$, $U_{A}-U_{B}=0$ is the crossover point of the signals $A(h)$, and $B(h)$ is the main principle of the system operation. Refractive index $n=1$.

$$
\begin{equation*}
h=h_{0} \pm h^{\prime} . \tag{9}
\end{equation*}
$$

Function (6) can be expressed only as a function $h, n, \theta, \theta_{c}, b_{1}$ (Fig. 1). $(A-B) /(A+B)$ does not depend on $S_{1}, K_{1}, R_{M}, K_{0}, I_{0}$, in contrast to $(A-B)$ that depends on them.

The compensation mechanism, signal $(A-B) /(A+B)$, does not depend on the reflection coefficient of mirror, intensity of the light source, and loss in the fibers. Equations (4), (5) and (6) were used as functions $h$ for simulating the output signals $U_{\text {out }}$ of sensors in order to compare the sensitivity $S=$ $d U_{\text {out }} / d h$ of our fiber sensor constructions with those known in literature.

## 3 Results

The signal characteristics of the system are

$$
\begin{aligned}
& A(h), B(h), C(h)=A(h)-B(h), \quad V(h)=C(h) /[A(h)+B(h)], \\
& S_{A}=G(h)=\frac{d A}{d h}, \quad S_{B}=F(h)=\frac{d B}{d h}, \quad S_{\text {sub }}=\frac{d C}{d h}, \quad S_{d i v}=\frac{d V}{d h} .
\end{aligned}
$$

For calculating applied such parameter $b_{1}$ values $1,2,3,4 \mathrm{~mm}$. The last value is as in [3] for comparison of the $S_{s u b}$ and $S_{\text {div }}$ results. The diameter of the fiber core is $a_{0}=0.2 \mathrm{~mm}$, with cladding $a=0.4 \mathrm{~mm}$, the fiber aperture angle $\theta_{c}=19.5^{\circ}$ and the angle $\theta=25^{\circ}$. The distance between fibers $A$ and $B$ is $d=0.3 \mathrm{~mm}$. The output signal for all pictures is in the same arbitrary units.

The output signals $A(h)$ and $B(h)$ in Fig. 2 rise from zero to the maximum value and then drop off. The portion in affinity of the points $A_{1}, A_{2}$ and $B_{1}$,


Fig. 2. Dependence of signals $A$ and $B$ on the mirror position $h . \theta=30^{\circ}$, $b_{1}=1 \mathrm{~mm}$.
$B_{2}$ for these characteristics is linear. The maximum amplitude $A_{\text {max }}$ of signal $A(h)$ is always higher than $B_{\text {max }}$ for signal $B(h)$. Sensitivity $G(h)$ and $F(h)$ (Fig. 3) on the near side of this characteristics is higher than that as in the far side. Moreover the sensitivity of the curve $B(h)$ is lower than that of $A(h)$ on both sides. The characteristics $A(h)$ and $B(h)$ naturally cross each other at an appropriate value of $h_{0}$.

The calculation results of the characteristics $A(h)$ and $B(h)$ with different values of the system parameter $b_{1}$ are represented in Fig. 4.


Fig. 3. Sensitivity dependence to the signals $A$ and $B$ on the mirror position $h: G(h)=d[A(h)] / d h, F(h)=d[B(h)] / d h$.


Fig. 4. Dependence of signals $A$ and $B$ on the mirror position $h$ as $b_{1}=1,2,3 \mathrm{~mm}$.

The maximum values of the characteristics $A(h)$ and $B(h)$ decrease as $b_{1}$ increases. The maximum points of all characteristics and their crossing points are at the higher values of $h$ with increasing $b_{1}$. As theoretical equations (4) and (5) show the decrease of $A_{\max }, B_{\max }$ will be exponential.

The characteristics $C(h)=A(h)-B(h)$ Fig. 5 rise from zero to the positive maximum value, then drop to zero, and rise to the negative maximum value and afterwards decrease to zero. For application in practical measurements, the linear part of the characteristic at the value $h=h_{0}$ is more important. Measuring values $C(h)$ everyone can determine the value of the mirror or body
displacement from position $h_{0} \pm h^{\prime}$. The amplitude of signal $C(h)$ decreases as $b_{1}$ increases, the distance $h_{0}$ increases as well. The characteristics $S_{\text {sub }}(h)$ in Fig. 6 reveal the sensitivity to displacement to be very low as $b_{1}=4 \mathrm{~mm}$.


Fig. 5. Dependence of signals $C(h)=A(h)-B(h)$ on the mirror position $h$ as $b_{1}=1,2,3,4 \mathrm{~mm}$.


Fig. 6. Dependence of signals $S_{\text {sub }}(h)$ on the mirror position $h$ as $b_{1}=1,2,3,4 \mathrm{~mm}$.

This is the main reason why the values of $S_{\text {sub }}(h)$ are so low as the distance $b_{1}$ is not properly chosen for the system. Moreover, the greater the signal $C(h)$ the better the resolution of the system is.

Very important for compensated measurements is system signal (Fig. 7)

$$
V=(A-B) /(A+B) .
$$

This figure shows the signal to be $\pm 1$ and zero at definite $h=h_{0}$ values.


Fig. 7. Dependence of signals $V(h)=A(h)-B(h) / A(h)+B(h)$ on the mirror position $h$ as $b_{1}=1,2,3,4 \mathrm{~mm}$.

Those values of $h$ increase as $b_{1}$ increase. The characteristics $V(h)$ have linear parts in the affinity of zero. Therefore each of them is proper for measurement not only the mirror displacement $h^{\prime}$, but also its direction. $S_{d i v}$ decreases exponentially as $b_{1}$ increases (Fig. 8).


Fig. 8. Dependence of signals $S_{\text {div }}(h)$ on the mirror position $h$ as $b_{1}=1,2,3,4 \mathrm{~mm}$.

## 4 Conclusion

Light transmission modelling in the reflection fiber-optical system revealed that the sensitivity could increase as the parameter of the system $b_{1}$ is de-
creased as much as possible. All the modelling results are coincidental with the experimental data [3].

## References

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