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Conducting Plate's Eddy-Current Losses

L. Andriušienė, A. Degutis, P. Kostrauskas,
Department of Electric Power Systems, Kaunas University of Technology
Studentų srt. 48, LT-51367 Kaunas, Lithuania, Tel.: +370 37 351318, e-mail: algirdas.degutis@ktu.lt

D. Mikalajūnas
Lietuvos Energija AB
Žvejų srt. 14, 09310, Vilnius, Lithuania, Tel.: +370 5 2782434

Introduction

Technological air-gaps are inbuilt in the laminated corners of the transformers' magnetic systems. Though attempts are made to reduce the air-gaps, it is not possible to eliminate them completely. The additional magnetisation losses occurring due to the existence of technological air-gaps may be classified in the following way:

1) losses conditioned by increased magnetic flux density in the surroundings of technological air-gaps;

2) eddy-current losses in the plates of electrical sheet-steel.

The losses of the first group drop with the decrease of technological air-gaps. The losses of the second group may be reduced using discretization for the steel plates in the surroundings of the magnetic system corners [1]. The purpose of this proceeding is to offer a technique for the definition of eddy-current losses together with the skin-effect assessment for electrical sheet-steel.

Mathematical model

The formulae are derived for the determination of eddy-current losses [1], presuming that magnetic flux density is constant in all points of the plate in the corner vicinity of magnetic system. However, the plate of electrical sheet-steel is a one-piece conductive structure, conditioning the skin effect of the electromagnetic field, i.e. uneven distribution of magnetic flux density in different points of the steel plate.

Assume now that electromagnetic quantities vary in a harmonic way and the permeability of electrical steel is constant in all points of the plate.

An electrical sheet-steel plate having the shape of a rectangle with dimensions *a* and *b*, as shown in Fig. 1, is crossed perpendicularly by alternating magnetic flux density *b(t,x)*. Taking into account that eddy-currents flow in closed ellipse-shaped contours, Fig. 1 shows the elementary tube of eddy-currents having the width *dx* according to *x*-axis and $\frac{b}{a}dx$ according to *y*-axis. The electromotive force induced in the inner contour $4\overset{\cup}{MN}$ of the elementary tube is

$$L(x)\,e(t,x); \tag{1}$$

where $L(x)=\pi x\left[1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}\right]$ - length of ellipse arc $4\overset{\cup}{MN}$ [2];

$e(t,x)$ - electromotive force induced in one length unit.

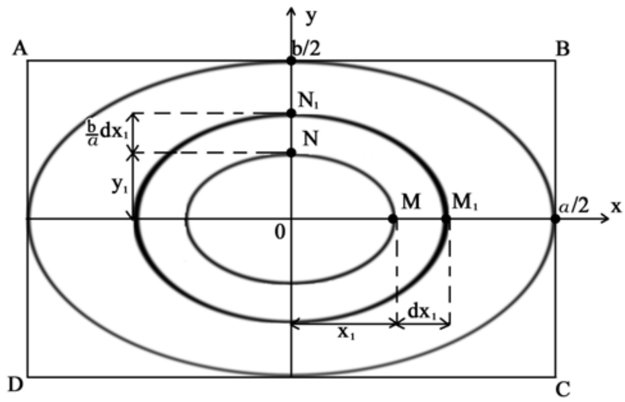

Fig. 1. Rectangle with dimensions *a* and b of electrical steel plate, the centre of which is superposed with the initial point 0 of Cartesian coordinates system

The electromotive force induced in the outer contour $4\overset{\cup}{M_1N_1}$ of the elementary tube is

$$L(x+dx)\left[e(t,x)+de(t,x)\right]; \tag{2}$$

where $L(x+dx)=\pi(x+dx)\left[1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}\right]$.

Obviously, it is possible to write

$$L(x+dx)[e(t,x)+de(t,x)]-L(x)e(t,x)=-\frac{d}{dt}d\Phi(t,x_1); \quad (3)$$

where $d\Phi(t,x_1)$ is the alternating magnetic flow, crossing the ring formed by the following ellipses:

$$\frac{x^2}{x_1^2}+\frac{y^2}{y_1^2}=1, \quad \frac{x^2}{(x_1+dx)^2}+\frac{y^2}{(y_1+dy)^2}=1;$$

x_1 – the abscissa of the variable point of the ellipse's maximal semi-axis;

$$y_1=\frac{b}{a}x_1.$$

In order to determine the skin-effect of magnetic flux it is essential to know the minimal and maximal values of magnetic flux density as well as the variation character of the density between the above mentioned values. The maximal value of magnetic flux density $B_m\cos\omega t$ equals the medium density of the magnetic system's magnetic flux

$$B_m=(B_c+B_y)/2; \quad (4)$$

Where B_c, B_y are the densities of the magnetic fluxes in the magnetic system's core and yoke correspondingly.

The minimal value of magnetic flux density $B_0\cos\omega t$ is defined in the ellipse's centre of magnetic system's corner. On the assumption that the magnetic flux density varies according to the law of the elliptic paraboloid between the minimal and maximal values, it follows that

$$b(t,x,y)=\left[B_0+(B_m-B_0)\left(\frac{x^2}{(a/2)^2}+\frac{y^2}{(b/2)^2}\right)\right]\cos\omega t \quad (5)$$

If the ellipse $\frac{x^2}{x_1^2}+\frac{y^2}{y_1^2}=1$ is used to determine the magnetic flux it is possible to describe it as

$$\Phi(t,x_1,y_1)=\iint_D b(t,x,y)d\sigma=4\int_0^{x_1}dx\int_0^{y_1\sqrt{1-(x/x_1)^2}}\left[B_0+(B_m-\right.$$

$$\left.-B_0)\left(\frac{x^2}{(a/2)^2}+\frac{y^2}{(b/2)^2}\right)\right]\cos\omega t dy=$$

$$=\pi x_1y_1\left[B_0+(B_m-B_0)\left(\frac{x_1^2}{(a/2)^2}+\frac{y_1^2}{(b/2)^2}\right)\right]\cos\omega t. \quad (6)$$

The medium value of the magnetic flux density becomes

$$b_v(t,x_1)=\frac{\Phi(t,x_1,y_1)}{\pi x_1y_1}=\left[B_0+\frac{1}{4}(B_m-B_0)\left(\frac{x_1^2}{(a/2)^2}+\right.\right.$$

$$\left.\left.+\frac{y_1^2}{(b/2)^2}\right)\right]\cos\omega t=\left[B_0+\frac{1}{2}(B_m-B_0)\frac{x_1^2}{(a/2)^2}\right]\cos\omega t; \quad (7)$$

where the dependence $y_1=\frac{b}{a}x_1$ is used.

For the elementary tube, in which eddy-currents flow, the area's differential can be written as follows

$$dS(x_1)=\pi(x_1+dx_1)(y_1+dy_1)-\pi x_1y_1=2\pi\frac{b}{a}x_1dx_1. \quad (8)$$

The differential of the magnetic flux created in the elementary tube is

$$d\Phi(t,x_1)=b_v(t,x_1)dS(x_1)=2\pi\frac{b}{a}\left[B_0x_1+\frac{1}{2}(B_m-\right.$$

$$\left.-B_0)\frac{x_1^3}{(a/2)^2}\right]\cos\omega t dx_1. \quad (9)$$

By transferring (9) to the differential equation (3), we receive

$$\pi\left[1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}\right][e(t,x_1)dx_1+x_1de(t,x_1)]=$$

$$=2\pi\omega\frac{b}{a}\left[B_0x_1+\frac{1}{2}(B_m-B_0)\frac{x_1^3}{(a/2)^2}\right]\sin\omega t dx. \quad (10)$$

The above relation can be rewritten as

$$\frac{de(t,x_1)}{dx_1}+\frac{e(t,x_1)}{x_1}=\left(\alpha+\beta x_1^2\right)\sin\omega t; \quad (11)$$

where

$$\alpha=\frac{2\omega\frac{b}{a}B_0}{1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}}; \quad \beta=\frac{\omega\frac{b}{a}\frac{B_m-B_0}{(a/2)^2}}{1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}}.$$

The latter differential equation is linear and its general solution available from [2] is

$$e(t,x)=\left(\frac{\alpha}{2}x+\frac{\beta}{4}x^3+\frac{C}{x}\right)\sin\omega t. \quad (12)$$

If the boundary condition $e(t,x)=0$ when $x=0$ is used, the integration constant is

$$C=0. \quad (13)$$

The partial solution for electromotive force, induced in one length unit, is

$$e(t,x)=\left(\frac{\alpha}{2}x+\frac{\beta}{4}x^3\right)\sin\omega t. \quad (14)$$

Consequently, the power differential of the eddy-currents is

$$dp(x)=\frac{[L(x)e(t,x)]^2}{2r(x)}=$$

$$=\frac{\left\{\pi x\left[1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}\right]\right\}^2\left(\frac{\alpha}{2}x+\frac{\beta}{4}x^3\right)^2}{2r(x)}dx; \quad (15)$$

where [1]

$$r(x)=\frac{\pi\rho x\left[1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}\right]^2}{2\frac{b}{a}\Delta dx};$$

Δ - thickness of electrical sheet-steel plate; ρ - resistivity of electrical sheet-steel.

Total losses of eddy-currents

$$P = \int_0^{a/2} dp(x) =$$

$$= \frac{\pi \frac{b}{a} a^4 \Delta \left[\frac{\alpha^2}{2} + \frac{\alpha\beta}{3}(a/2)^2 + \frac{\beta^2}{12}(a/2)^4 \right]}{96\rho \left[1{,}5\left(1 + \frac{b}{a}\right) - \sqrt{\frac{b}{a}} \right]^2}. \qquad (16)$$

Using α and β expressions the following relation is received

$$P = \frac{\pi^3 f^2 \left(\frac{b}{a}\right)^3 a^4 \Delta B_m^2}{16\rho \left[1{,}5\left(1 + \frac{b}{a}\right) - \sqrt{\frac{b}{a}} \right]^2} \left[\left(\frac{B_0}{B_m}\right)^2 + \right.$$

$$\left. + \frac{1}{3}\frac{B_0}{B_m}\left(1 - \frac{B_0}{B_m}\right) + \frac{1}{24}\left(1 - \frac{B_0}{B_m}\right)^2 \right]. \qquad (17)$$

The eddy-current losses are described by the formula from [1] if the skin-effect $(B_m = B_0)$ is not taken into account:

$$P_1 = \frac{\pi^3 f^2 \left(\frac{b}{a}\right)^3 a^4 \Delta B_m^2}{16\rho \left[1{,}5\left(1 + \frac{b}{a}\right) - \sqrt{\frac{b}{a}} \right]^2}. \qquad (18)$$

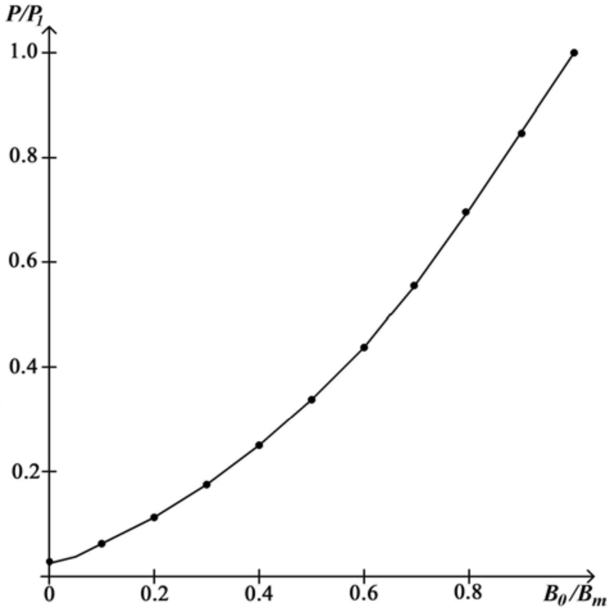

Fig. 2. Dependence of eddy-currents' specific losses on the ratio of magnetic flux densities

Keeping in mind formula (18), equation (17) can be transformed into

$$P = P_1 \left[\left(\frac{B_0}{B_m}\right)^2 + \frac{B_0}{3B_m}\left(1 - \frac{B_0}{B_m}\right) + \frac{1}{24}\left(1 - \frac{B_0}{B_m}\right)^2 \right]. \qquad (19)$$

It follows from the latter, that function P/P_1 is the rising function of B_0 / B_m. The character of the function's variation is shown in Fig. 2.

Fig. 2 above shows that the decrease in specific losses and B_0 is directly proportional. This effect may be reached by diminishing the permeability in the centre of the rectangle of the magnetic system's corner. It could be implemented technically by decreasing the permeability in the central zone of the magnetic system's corners.

Applying $b = a/n$ to formula (17) we find that

$$P_n = \frac{\pi^3 (fB_m)^2 \left(\frac{1}{n}\right)^3 a^4 \Delta}{16\rho \left[1{,}5\left(1 + \frac{1}{n}\right) - \sqrt{\frac{1}{n}} \right]^2} \left[\left(\frac{B_0}{B_m}\right)^2 + \right.$$

$$\left. + \frac{1}{3}\frac{B_0}{B_m}\left(1 - \frac{B_0}{B_m}\right) + \frac{1}{24}\left(1 - \frac{B_0}{B_m}\right)^2 \right]. \qquad (20)$$

The latter formula defines the active losses of the eddy-currents in the rectangular electrical steel plate whose dimensions are a and $b = a/n$. It is obvious that the square plate with side a contains n rectangular plates with the dimensions of sides a and $b = a/n$. Hence, the total eddy-current losses of all the n rectangular electrical steel plates with dimensions a and $b = a/n$ are expressed as

$$P_\Sigma = nP_n = \frac{\pi^3 (fB_m)^2 \left(\frac{1}{n}\right)^2 a^4 \Delta}{16\rho \left[1{,}5\left(1 + \frac{1}{n}\right) - \sqrt{\frac{1}{n}} \right]^2} \left[\left(\frac{B_0}{B_m}\right)^2 + \right.$$

$$\left. + \frac{1}{3}\frac{B_0}{B_m}\left(1 - \frac{B_0}{B_m}\right) + \frac{1}{24}\left(1 - \frac{B_0}{B_m}\right)^2 \right]. \qquad (21)$$

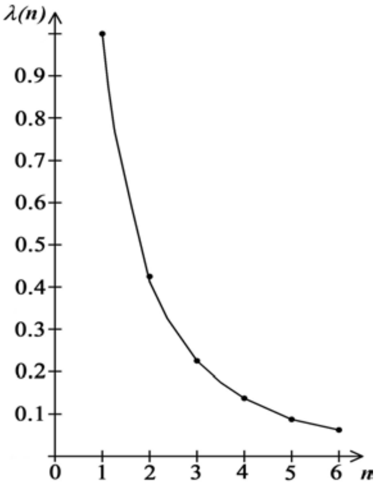

Fig. 3. Dependence of active losses ratio $\lambda(n)$ on n

Using the ratio of active losses (21) and (17) it is possible to define how many times the active losses decreased after the discretization of the rectangular plate ABCD (Fig. 1) into n rectangles:

$$\lambda(n) = \frac{P_\Sigma}{P} = \frac{\left(\frac{1}{n}\right)^2 \left[1{,}5\left(1+\frac{b}{a}\right)-\sqrt{\frac{b}{a}}\right]^2}{\left(\frac{b}{a}\right)^3 \left[1{,}5\left(1+\frac{1}{n}\right)-\sqrt{\frac{1}{n}}\right]^2}. \qquad (22)$$

When the magnetic system corner is square $(a = b)$, we get

$$\lambda(n) = \frac{4}{n^2\left[1{,}5\left(1+\frac{1}{n}\right)-\sqrt{\frac{1}{n}}\right]^2}. \qquad (23)$$

The curve of the function (23) is plotted in Fig. 3.

Conclusions

1. Reasons are defined for the origination of the skin-effect of electromagnetic processes in the laminated corners of the transformer's magnetic system.

2. Differential equations are obtained and solved for the magnetic field distribution in the electrical sheet-steel plate of the transformer's magnetic system.

3. The dependence of the electrical steel plate eddy-currents' active losses on the ratio of the magnetic flux density in the centre and periphery of the corner is determined. Recommendations are offered for the reduction of the losses.

4. It is proved that the discretization of the laminated corners zone in the transformer's magnetic system determines the decrease in the active losses.

References

1. **Andriušienė L., Kostrauskas P., Mikalajūnas D.** Galios transformatorių tuščiosios veikos nuostolių minimizavimas // Elektronika ir elektrotechnika.– Kaunas: Technologija, 2004.– Nr. 1(50).– P. 38-41.
2. **Бронштейн И. Н., Семендяев К. А.** Справочник по математике (перевод с немецкого).– Москва: Наука, 1981.– 718 с.

Pateikta spaudai 2003 11 27

L. Andriušienė, A. Degutis, P. Kostrauskas, D. Mikalajūnas. Sūkurinių srovių nuostoliai elektrai laidžioje plokštelėje // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2004. – Nr. 5 (54). – P. 66-69.

Tariama, kad visi elektromagnetiniai procesai transformatoriaus magnetinės sistemos užleistinės konstrukcijos kampe kinta harmoningai. Įrodyta, kad galios transformatorių magnetinės sistemos užleistinių konstrukcijų kampuose dėl technologinių oro tarpų susidaro papildomos sūkurinės srovės, kurios dėl paviršiaus efekto pasiskirsto netolygiai. Teigiama, kad sūkurinių srovių kontūrai yra elipsės formos. Tariant, kad yra žinomi magnetinių srautų tankiai magnetinės sistemos kampo centre ir periferijoje, sudarytos magnetinio lauko pasiskirstymo diferencialinės lygtys. Išsprendus diferencialines lygtis apskaičiuoti sūkurinių srovių aktyvieji nuostoliai, priklausantys nuo magnetinio srauto tankių kampo centre ir periferijoje kvadrato, lakšto storio, kampo matmenų, savitosios varžos ir dažnio kvadrato. Pateikti būdai sūkurinių srovių aktyviesiems nuostoliams minimizuoti, mažinant magnetinės sistemos kampo centrinės srities magnetinę skvarbą arba diskretizuojant magnetinės sistemos kampo sritį elementariosiomis sritimis. Il. 3, bibl. 2 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

L. Andriušienė, A. Degutis, P. Kostrauskas, D. Mikalajūnas. Conducting Plate's Eddy-Current Losses // Electronics and Electrical Engineering. – Kaunas: Technologija, 2004. – No. 5 (54). – P. 66-69.

It is assumed that all electromagnetic processes vary in a harmonic way in the laminated corner of the transformer's magnetic system.It is proved that due to technological air-gaps existing in the laminated corners of power transformers' magnetic systems, additional eddy-currents originate which are distributed unevenly because of the skin-effect. Ellipticity of eddy-currents contours is stated. Presuming that the magnetic flux densities both in the centre and periphery of the magnetic system's corner are known, differential equations for the distribution of magnetic field are obtained. On the basis of the solved differential equations the active losses of eddy-currents are calculated, which depend on the square of magnetic flux densities in the centre and periphery of the corner, the thickness of plate, dimensions of the corner, specific resistance and the square of frequency. Methods are proposed for the minimization of eddy-currents active losses by reducing the permeability in the central zone of the magnetic system's corner or by using the elementary zones for the discretization of the magnetic system's corner zone. Ill. 3, bibl. 2 (in English; summaries in Lithuanian, English, Russian).

Л. Андрюшене, А. Дегутис, П. Костраускас, Д. Микалаюнас. Потери вихревых токов в электропроводной пластине // Электроника и электротехника. – Каунас: Технология, 2004. – № 5 (54). – С. 66-69.

Предполагается, что все электромагнитные процессы в шихтованных углах магнитной системы силового трансформатора протекают по гармоническому закону. Доказано, что благодаря технологическим зазорам в шихтованных углах магнитной системы трансформатора возникают дополнительные вихревые токи в листах магнитопровода; из-за поверхностного эффекта интенсивность вихревых токов возрастает от центра угла магнитной системы к его периферии. По заданной магнитной индукции в центре и на периферии угла магнитной системы составлено дифференциальное уравнение для определения распределения магнитной индукции в зоне угла магнитной системы. На основании решения дифференциального уравнения распределения магнитной индукции вычислены активные потери, обусловленные вихревыми токами в углах магнитной системы трансформатора. Показано, что активные потери от вихревых токов зависят от квадратов магнитных индукций в центре и на периферии угла магнитопровода, толщины листа, размеров угла, удельного сопротивления и квадрата частоты перемагничивания. Предложены способы по уменьшению активных потерь от вихревых токов: снижением магнитной проницаемости в области центра угла магнитопровода или дискретизацией области угла на элементарные составляющие. Ил. 3, библ. 2 (на английском языке; рефераты на литовском, английском и русском яз.).