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Investigation of the Electric Field of the Plane Capacitor with Round Hole

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Introduction

The plane capacitor with round hole can be used as a valve controlled by electric field, if the hole is filled with electrorheological fluid (ERF). The expression of distribution of electric field in the round hole of the air plane capacitor is presented in [1]. It is obtained using conformal mapping technique. But the capacitor with dielectric material between the plates is used usually. The permittivity of ERF is different than permittivity of air. The conformal mapping can not be used in this case. Other theoretical methods are complicated to use because of the open space and inconvenient boundary conditions. We select the numerical method of finite elements for calculation electric field in the hole of capacitor with non-homogeneous medium.

The program pocket COSMOS/M was used. It has the special sub-program ESTAR for the investigation of two- and three-dimensional electromagnetic fields. This program pocket has the bound-elements, which permit to model the open space problems with necessary precision.

The modelling of 2D problem

The distribution of electric field is the same in any plane θ=const. Therefore it is sufficient to investigate the two-dimensional problem. Let the axes r and z of cylindrical coordinate system coincide with axes x and y, correspondingly. Let the axis of hole coincides with axis y and let the origin of coordinate system be in the middle point between the planes of capacitor plates. The equations $E(x,y)=E(-x,y)=E(x,-y)=E(-x,-y)$ are right by symmetry in this case, and we can model one quarter of hole and capacitor. The charges, situated on the external side of plate, create some part of field in the hole. Therefore the model must include sufficient great part of area over the plate of capacitor. The area of modelling is shown in fig. 1. It coincides with the first quarter of Cartesian coordinate system. All area was divided into four parts: I - hole, II – interior of capacitor, III – exterior of hole and IV – exterior of capacitor plate. The width of parts II and IV was accepted $l_{II}=l_{IV}=5r$ (r- the radius of hole). The height of parts III and IV was accepted $m_{IV}=m_{III}=5h$ (h – ½ of distance between the capacitor plates). The errors, which appear because this limit of height, are smaller than 0,5%. Distribution of field in the part I is interesting for us.

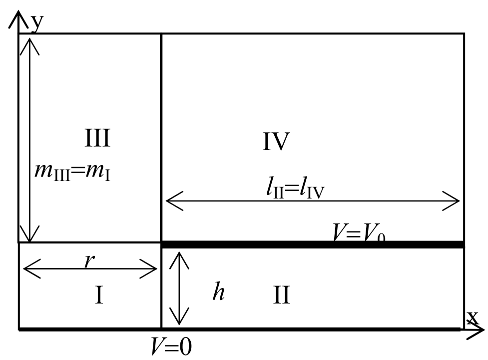

Fig 1. The dividing of modelling area

The investigation of air capacitor

The two-dimensional model of air capacitor with r=h i.e., r_s=r/h=1, was investigated primarily. The mesh of finite elements was chosen uniform with the step 0,1h in both directions. The modelling results were compared with the theoretical results, calculated by the expressions obtained in [1]. The theoretic and modelling values of electric field strength and the coordinates are used relative in this paper:

$$E^s = E/E_0,\quad E_0 = V_0/h,\quad x_s = x/r,\quad y_s = y/h\,. \quad (1)$$

There is no difference between the field in radial plane of the section of plane capacitor with the round hole, containing the axis of hole, and the field of the section of two the same planar capacitors without the holes with distance between them equal to diameter of hole. In the 2D model we obtain the plane parallel field, i.e., field E_{12}

between two plane capacitors. The relative values of field $E_{12}^{s}(0,y_s)$ on the axis x=0, where $y_s = y/h$, were calculated using formula $E_{12}^{s}(0,y_s) = 2K_{12}(r_s)E_{vy}^{s}(r_s,y_s)$ for $r_s = 1$. In this case $K_{12}(1) = 0{,}834$. The results of calculation of field $E_{vy}^{s}(1,y_s)$ near the alone plane capacitor were taken from the paper [1]. The theoretical E_{12}^{sT} and modelling E_{12}^{M} results are compared in the table 1. The δ is the relative difference $\delta = \frac{E_{12}^{sT} - E_{12}^{sM}}{E_{12}^{sT}} \cdot 100\%$ between these results.

Table 1. The comparison of theoretical and modelling results in the two-dimensional case

y_s	0	0,1	0,2	0,3	0,4	0,5
E_{12}^{sT}	0,6305	0,6281	0,6209	0,6090	0,5924	0,5709
E_{12}^{sM}	0,6353	0,6328	0,6251	0,6122	0,5940	0,5705
δ,%	0,76	0,74	0,52	0,27	0,07	-0,56
y_s	0,6	0,7	0,8	0,9	1,0	
E_{12}^{sT}	0,5447	0,5137	0,4780	0,4374	0,3922	
E_{12}^{sM}	0,5417	0,5081	0,4702	0,4290	0,3858	
δ,%	-0,56	-1,10	-1,66	-1,96	-1,66	

The difference between theoretical and modelling results does not exceed 2%. Therefore the theoretical model, obtained in the [1], can be used for calculation of electric field on the axis of hole.

Modelling of the 3D problem

The three-dimensional model is necessary for verification of theoretical expression of axisymmetric field calculation using the results of 2D model and expression of field calculation in the points distant at the axis of hole. These expressions are obtained in [1].

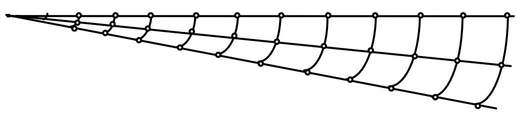

Fig 2. The view of mesh in the plane y=y_0

For the axisymmetric object it is sufficient to model the segment of object, situated between the planes $\theta = 0$ and $\theta = \theta_0$ in the cylindrical coordinate system, if the COSMOS/M is used. The angle θ is obtained turning the plane xy about the axis y. The mesh of the three-dimensional model was selected just the same as the two-dimensional model in the x and y directions (see fig. 1). It was used the segment situated between the planes $\theta = 0$ and $\theta_0 = 10^0$, which was divided into two segments: between $\theta = 0$ and $\theta = 5^0$ and between $\theta = 5^0$ and $\theta_0 = 10^0$. The mesh looks like the fig. 2 in any plane y=y$_0$.

The theoretical E_a^{sT} and modelling E_a^{sM} results of the electrical field strength on the axis of hole are presented in the table 2. Using these results we can verify the accuracy of derived in [1] factor $K_P = K_a K_M = \frac{\pi}{2}\frac{1-2m(1)}{1-m(1)} = 1{,}2581$. Multiplying the results of plane parallel field obtained in the 2D case by this factor we obtain the values of axisymmetric field on the axis. The δ_a is the relative difference between the theoretical E_a^{sT} and modelling E_a^{sM} findings: $\delta_a = \frac{E_a^{sT} - E_a^{sM}}{E_a^{sT}} \cdot 100\%$. The δ_p is the relative difference between the results of three-dimensional modelling and the results of two-dimensional modelling, multiplied by factor K_P: $\delta_p = \frac{E_a^{sM} - K_P E_{12}^{sM}}{E_a^{sM}} \cdot 100\%$. In the three-dimensional case theoretical and modelling results are not different more than 2,1%. The modelling results confirm the accuracy of theoretical value of factor K_P.

Table 2. The comparison of theoretical and modelling results in the three-dimensional case

y_s	0	0,1	0,2	0,3	0,4	0,5
E_a^T	0,7932	0,7902	0,7812	0,7662	0,7452	0,7182
E_a^M	0,8102	0,8071	0,7975	0,7813	0,7582	0,7276
δ_a,%	2,10	2,08	2,04	1,94	1,70	1,30
δ_p,%	1,38	1,38	1,41	1,44	1,41	1,38
y_s	0,6	0,7	0,8	0,9	1,0	
E_a^T	0,6853	0,6462	0,6013	0,5503	0,4934	
E_a^M	0,6895	0,6442	0,5923	0,5455	0,4934	
δ_a,%	0,61	-0,30	-1,52	-0,88	0,0	
δ_p,%	1,38	1,18	1,28	1,07	1,66	

The theoretical values of electric field strength in any point of hole were calculated using expressions (see [1]):

$$E^s = E/E_0 = \sqrt{(E_y^s)^2 + (E_x^s)^2},$$

$$E_y^s = 0{,}7932(1 - 0{,}378 y_s^2 + 0{,}189 x_s^2),$$

$$E_x^s = 0{,}3 x_s y_s,$$

$$E_0 = V_0/h.$$

Some theoretical and modelling values of electric field strength are presented in the fig 3 (Theor and Model curves, correspondingly). Theoretical expressions are not exact, when the values of x_s and y_s approach to 1. The mean value of relative difference between theoretical and modelling values is less than 2%.

The dielectric between the plates of capacitor

We investigate two cases: 1) the dielectric with relative permittivity ε_r>1 is in the II part of area (see fig. 1), 2) the dielectrics are in the I and II parts of area.

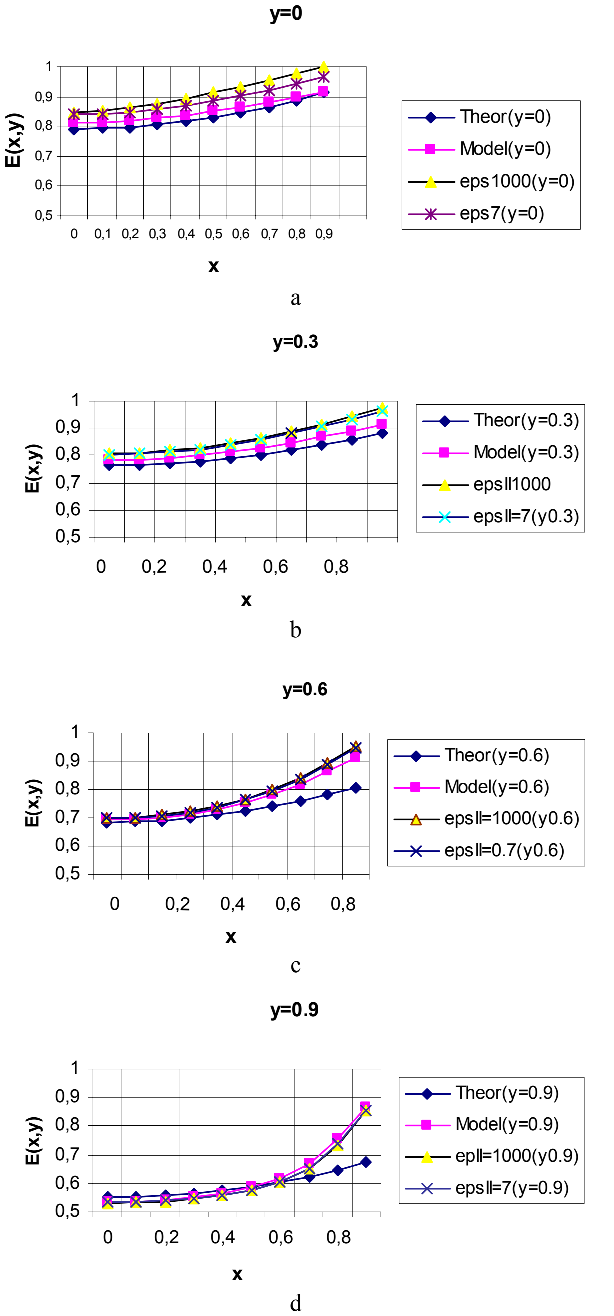

Fig. 3. The results of theoretical calculation (Theor) and modelling for $\varepsilon_{II}=1$ (Model), $\varepsilon_{II}=7$ and $\varepsilon_{II}=1000$, a) in the middle plane of capacitor ($y_s=0$), b) for $y_s=0{,}3$, c) for $y_s=0{,}6$, d) for $y_s=0{,}9$

We can suppose, that the potential V_A of any point A of hole is formed by charges, which are situated on the edge of plate, on the all interior side of plate, excluding edge, and on the all exterior side of plate, excluding edge (see fig. 4, a). The capacitances C_4, C_3 and C_e are partial capacitances of fourth, third and first parts of modelling area through which the charge of the exterior of capacitor plate create potential in the point A. The capacitances C_2 and C_i – partial capacitances of second and first parts of area through which the charge of interior of capacitor plate create potential in the point A. The capacitances C_0 and C_A are the partial capacitances between the point A and edge of plate and plane with zero potential, correspondingly. We can clear up the influence of permittivity of different parts of area to the field in the hole using the equivalent capacitive circuit, shown in fig. 4, b.

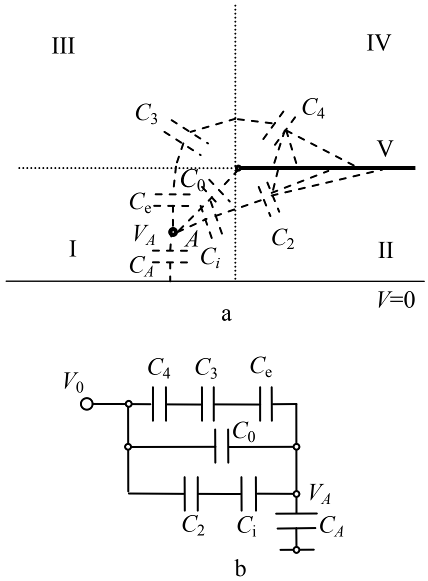

Fig. 4. The formation of potential in any point of hole: a) situation of partial capacitances, b) capacitive circuit

These capacitances are proportional to permittivity of corresponding part of area. The transfer coefficient K_V of this circuit can be expressed this way:

$$K_V = \frac{V_{1k}}{V_0} = \frac{1/C_A}{1/C_A + 1/C_R} = \frac{1}{1 + C_A/C_R}, \qquad (2)$$

where C_R – resultant capacitance of capacitances C_0, C_3, C_4, C_i, C_2 and C_e. It can be expressed:

$$C_A = C_0 + C_{34e} + C_{2i}, \qquad (3)$$

$$C_{34e} = \frac{1}{1/C_3 + 1/C_4 + 1/C_e}, \qquad (4)$$

$$C_{2i} = \frac{1}{1/C_2 + 1/C_i}. \qquad (5)$$

1) *Dielectric between capacitor plates.* The partial capacitance C_2 and resultant capacitances C_{2i} and C_R increase, if the material with permittivity $\varepsilon_{II}>1$ is placed between the capacitor plates. The coefficient K_V and potential V_A increase, correspondingly with (1), too. This increase is not significant and depends on situation of point. The capacitance C_2 has the greatest influence for the points, in which the charge, distant at the edge creates potential. There are the points, situated in the middle plane between the capacitor plates (y=0) near the wall of hole. The modelling results show that the maximal variation in comparison with air capacitor was obtained for the point y=0, x_s=0,9. This variation is 5,7% for dielectric with permittivity ε_r=7 and 9,4% for dielectric with permittivity ε_r=1000. In other points the variation is lesser.

The results of modelling for permittivities ε_r=7 and ε_r=1000 and for the values of y_s: y_s=0, y_s=0,3, y_s=0,6 and

y_s=0,9 are showed in fig. 3. We can see, that for y_s=0,6 the mean value of variation, when the x_s varies between 0 and 0,9, is less then 2%. The dielectric has not influence to field in the hole if $y_s \geq 0{,}9$.

2) *The dielectric between the capacitor plates and the dielectric fluid in the hole*. The capacitance C_A increases proportionally to permittivity of fluid in this case. If other capacitances are not varied, the coefficient K_V decreases (see (1)). However the resultant capacitance C_R varies, because the capacitances C_i and C_e vary, too. The greatest variation of potential and field is in the points of hole, in which the significant influence has the charge of exterior side of plate and the capacitance C_e is not significant, i.e., in the upper part of hole near the axis. The influence of permittivity of fluid is small in the point near the wall of hole. In this case the value of C_R practically is equal to C_e. The dependence of electric field strength upon the x for the values of permittivity ε_r=1, ε_r=2,4, ε_r=3,8, ε_r=7 and relative distance at middle plane of capacitor y_s=0, y_s=0,5, y_s=0,7 and y_s=0,9 are showed in fig. 5.

Analytical approximation of electric field strength in the hole

We consider the capacitor with dielectric permittivity $\varepsilon_{II} \approx 7$. Let be $\varepsilon_{III}=\varepsilon_{IV}=1$. The permittivity of ERF ε_I can vary in large limits, but it is not more than 7. The modelling results of values E of electric field strength, obtained for indicated values ε_{II}, ε_{III}, ε_{IV} and for ε_I=1÷7, can be approximated by this expression:

$$E = E^S \cdot E_0 : \qquad (6)$$

where

$$E^S = K_{1\varepsilon}[1 - K_{2\varepsilon} y_s^2] \cdot [1 + K_{3\varepsilon} x_s^2] \cdot [1 + K_{4\varepsilon} x_s^2 y_s^4], \qquad (7)$$

$$K_{1\varepsilon} = K_1(\varepsilon_I) = \frac{\varepsilon_I}{0{,}027\varepsilon_I^2 + 1{,}29\varepsilon_I - 0{,}12}, \qquad (8)$$

$$K_{2\varepsilon} = K_2(\varepsilon_I) = \frac{0{,}893\varepsilon_I - 0{,}04}{0{,}912\varepsilon_I + 1}, \qquad (9)$$

$$K_{3\varepsilon} = K_3(\varepsilon_I) = \frac{\varepsilon_I}{2{,}96\varepsilon_I + 1{,}77}, \qquad (10)$$

$$K_{4\varepsilon} = K_4(\varepsilon_I) = \frac{2{,}25\varepsilon_I - 1}{0{,}247\varepsilon_I + 1{,}61}, \qquad (11)$$

E_0 is expressed by (1).

For calculation the direction of vector $\boldsymbol{E}$ of electric field strength we can calculate the x component E_x of this vector, using the expression:

$$E_x = [K_{x\varepsilon}(\varepsilon_I) \cdot (x_s + 0{,}1) y + y_s^4 (x_s - 0{,}3)] E_0, \qquad (12)$$

where

$$K_{x\varepsilon}(\varepsilon_I) = 0{,}25 + \frac{1 - 0{,}058\varepsilon_I}{3{,}3\varepsilon_I + 9{,}3}(\varepsilon_I - 1). \qquad (13)$$

The angle α between the vector $\boldsymbol{E}$ and axis x can be calculated in this way:

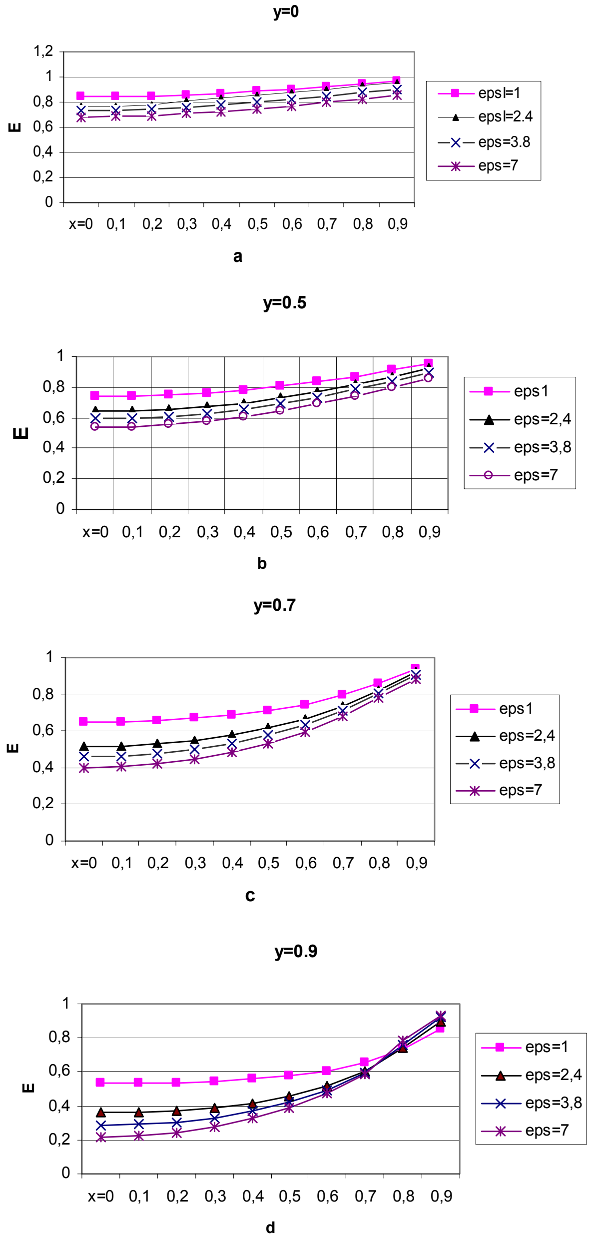

Fig. 5. The distribution of electric field strength in the hole for four values of permittivity (eps) in different levels of hole a) y_s=0, b) y_s=0,5, c) y_s=0,7, d) y_s=0,9

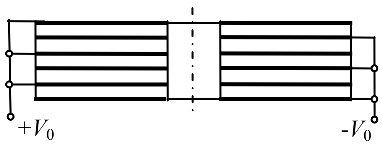

Fig. 6.The multiplayer capacitor with the opposite fields

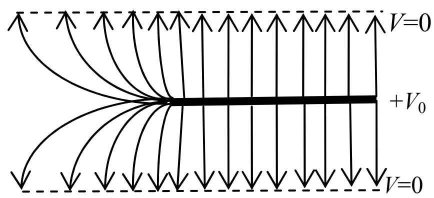

Fig. 7. The distribution of field near the plate of multilayer capacitor

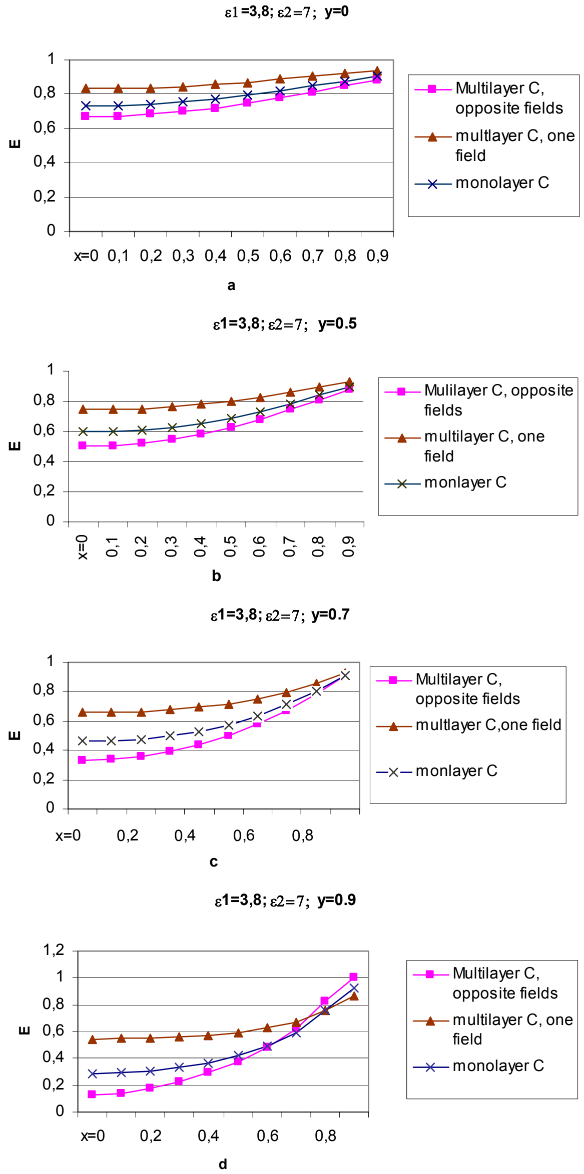

Fig. 8. The field in the multiplayer capacitor with opposite fields and with one field in comparison with the field in the monolayer capacitor: for some planes of hole: a)y_s=0, b) y_s=0,5, c) y_s=0,7, d) y_s=0,9

$$\alpha = \arccos(E_x / E) \qquad (14)$$

Using the expressions (6)-(14) we can calculate the value and direction of vector of electric field strength in any point of plane capacitor hole, filled by ERF, with relative error less than 5%.

The electric field in the hole of multilayer capacitor

The probable construction of valve with the ERF is shown in the fig. 6. The capacitor has some the same layers. In the adjacent layers the electric fields are opposite. It is evident, that in any layer the field will be weaker than in the monolayer capacitor with the same difference of potentials between the plates. The view of field near of any plate with positive charge is presented in fig. 7. The field in any layer is created by the charge accumulated on the interior of the plate only.

The field can be intensified in one layer of multilayer capacitor, if the difference of potentials is created between two adjacent plates only. The field of the part of hole, which is in this layer, will be greater than in the hole of monolayer capacitor. In this case the relative permittivities of part III and IV will be more than 1 (see fig. 1) and the capacitances C_3, C_4 and C_{34e} in the circuit fig.4, a, increase. Therefore the resultant capacitance C_R and coefficient K_V (see (5)) increase, too. The modelling results are presented in fig. 8 for the case of ERF relative permittivity ε_l=3,8 and for the values y_s=0, y_s=0,5, y_s=0,7 and y_s=0,9.

Conclusions and results

1. It is convenient to investigate the electric field in the round hole of capacitor using the finite element method.
2. The results of modelling confirm theoretical expression for calculation the electric field in the hole, obtained in [1].
3. The dielectric material between the plates of capacitor fractionally amplifies the field in the inner layers of hole in comparison with air capacitor.
4. The field in the hole decreases proportional to permittivity of electrorheological fluid which fills the hole.
5. The field in the multilayer capacitor with opposite fields in adjacent layers decreases in comparison with monolayer capacitor.
6. The field in the hole of multilayer capacitor can be increased if the difference of potentials is connected up to two adjacent plates only.

References

Bansevicius R., Virbalis J.A. The electric field in the round hole of the air plain capacitor // Electronics and Electrical Engineering.- Kaunas: Technologija, 2004.- No. 2(51).- P.24-28.

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R. Bansevičius, J.A. Virbalis. Plokščiojo kondensatoriaus kiaurymės elektrinio lauko tyrimas // Elektronika ir elektrotechnika.- Kaunas: Technologija, 2004.- Nr.5(54). - P.9-14.

Elektroreologinio skysčio ERS papildyta plokščiojo kondensatoriaus kiaurymė gali būti naudojama elektrinio lauko valdomiems vožtuvams konstruoti. Todėl svarbu žinoti, kaip pasiskirsto elektrinis laukas, prijungus įtampą prie kondensatoriaus plokščių. Šis laukas buvo ištirtas baigtinių elementų metodu. Buvo naudojamas dvimatis ir trimatis modeliai. Modeliavimo rezultatai patvirtino anksčiau gautų analizinių išraiškų laukui orinio kondensatoriaus kiaurymės ašyje ir bet kuriame kiaurymės taške apskaičiuoti teisingumą. Sudaryta talpinė grandinė, padedanti įvertinti kondensatoriaus dielektriko ir ERS, užpildančio kiaurymę, dielektrinių konstantų įtaką lauko stipriui. Nustatyta, kad dielektrikas, esantis tarp kondensatoriaus plokščių, šiek tiek padidina lauką kiaurymėje, o didėjant ERS

dielektrinei konstantai laukas kiaurymėje mažėja. Pateikti lauko pasiskirstymo įvairiuose kiaurymės lygiuose grafikai. Gautos analizinės išraiškos, leidžiančios apskaičiuoti elektrinio lauko stiprio modulį ir kryptį bet kuriame kiaurymės taške, jei ERS santykinė dielektrinė pastovioji yra tarp 1 ir 7. Ištirtas laukas kondensatoriuje, sudarytame iš kelių identiškų sluoksnių, kurių kiekviename kuriamas priešingos krypties laukas. Nustatyta, kad laukas bet kuriame sluoksnyje yra mažesnis negu analogiškame vieno sluoksnio kondensatoriuje. Lauką viename daugiasluoksnio kondensatoriaus sluoksnyje galima padidinti lyginant ir su viensluoksniu kondensatoriumi, jeigu įtampa veikia tik viename iš vidinių sluoksnių. Il. 8, bibl. 1 (lietuvių kalba, santraukos lietuvių, anglų ir rusų k.)

R. Bansevicius, J.A. Virbalis. Investigation of the Electric Field of the Plane Capacitor Round Hole // Electronics and Electrical Engineering. - Kaunas: Technologija, 2004. - No.5(54). - P.9-14.

The round hole of the plain capacitor, filled by electrorheological fluid ERF can be applied for construction of the valves controlling by electric field. It is important to know distribution of electric field in the hole, when the difference between the plates of capacitor is created. The electric field of hole was investigated using finite element method. The two- and three-dimensional models were used. The modelling results confirm analytical expressions obtained previously for calculation of electric field strength on the axis of hole and in any point of hole. The capacitive circuit was presented for evaluation of influence of permittivities of dielectric between the plates and ERF to the strength of electric field in the hole. The dielectric material between the plates of capacitor fractionally amplifies the field of hole. But the field of hole decreases with increment of ERF permittivity. The dependences of electric field strength on the distance at axis in several layers of hole are presented. The approximated expressions are obtained for calculation of value and direction of electric field strength in any point of hole, if relative permittivity of ERF is in interval 1-7. The field of hole of multilayer capacitor with opposite fields was investigated. The field in any layer of such capacitor is lesser than in analogical monolayer capacitor. The field in one layer of multilayer capacitor can be amplified in comparison with monolayer capacitor, if the difference of potentials is connected to alone layer only. (Ill. 8, bibl. 1; in Lithuanian; summaries in Lithuanian, English, Russian).

Р. Бансявичюс, Ю.А. Вирбалис. Исследование электрического поля в круглом отверстии плоского конденсатора // Электроника и электротехника. - Каунас: Технология, 2004. - №.5(54). - С.9-14.

Отверстие, в плоском конденсаторе, заполненное электрореологической жидкостью ЭРЖ может быть применено для конструирования клапанов, управляемых электрическим полем. Поэтому актуально знать, как в таком отверстии распределена напряженность электрического поля после подключения напряжения к конденсатору. Это поле было исследовано методом конечных элементов. Применялась как двухмерная, так и трехмерная модель. Результаты моделирования подтвердили правильность раннее полученных выражений для расчета поля, как на оси, так и в любой точке отверстия воздушного конденсатора. Предложена схема емкостной цепи, позволяющая оценить влияние диэлектрических проницаемостей диэлектрика, заполняющего конденсатор и ЭРЖ на электрическое поле в отверстии. Установлено, что диэлектрик между платами конденсатора несколько увеличивает поле в отверстии, а увеличение диэлектрической проницаемости ЭРЖ уменьшает поле в отверстии. Получены аналитические выражения, позволяющие рассчитать величину и направление электрического поля в любой точке поля, если относительная диэлектрическая проницаемость ЭРЖ находится в пределах 1-7. Исследовано поле в конденсаторе образованном из нескольких идентичных слоев, в каждом из которых создается поле противоположного направления. Установлено, что поле в каждом из слоев меньше, чем в аналогичном однослойном конденсаторе. Поле в одном слое многослойного конденсатора можно увеличить по сравнению с однослойным конденсатором, если напряжение подключить только к одному из средних слоев. Ил. 8, библ. 1 (на литовском языке; рефераты на литовском, английском и русском яз.).