PROCEEDINGS RESEARCH CENTER OF THE PUBLIC INSTITUTION VIBROMECHANIKA

Journal of Vibroengineering

2000 No4 (5)

ISSN 1392-8716

Contents

Aims and Scope

Original papers dealing with modern developments in the areas of theory and applications of mechanical, mechatronic and biomechatronic system are welcomed. The following subjects are indicated as principal topics:

vibroacoustic and wave processes
analysis and synthesis of nonlinear vibration systems
generation of vibrations and waves
vibrostabilization and control of motions
transformation of motion by vibrations and waves
technology of vibrations and waves
vibration measurement, vibro-identification, vibro-diagnostics and monitoring

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J. Vibroengineering is supported by

Academy of Sciences of Lithuania
Vilnius Gediminas Technical University
Kaunas University of Technology
Kaunas Magnus Vitautas University
Vilnius University
Vilnius Pedagogical University
Lithuanian University of Agriculture

ALL PABLISHED MATERIALS ARE REVIEWERED

The Unbalanced Rotor With the Dynamically Self Regulating System

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Abstract: The system analysed consists from the rotor which may rotate on the case suspensed by elastic elements to the immovable basis. The case of the passive vibroexciter is attached by a rotational link to the rotor. Approximately by the analytical method the steady state regimes are determined. For the regimes which it is easier to implement the conditions of existence and stability are determined.

Keywords: an unbalanced rotor, a passive vibroexciter, the conditions of existence and stability.

NOTATION

A and B	the amplitude of harmonic vibrations of the case of the system according to the axis OX and OY respectively
C_x, C_y, C_z, H_x, H_y, H_z,	the coefficients of stiffness and viscous friction of elastic dissipative elements
I	the moment of inertia about the relative axis of rotation of the member
m	the mass
p	the eigenfrequency
r	the disbalance
x,y	the coordinates of the case of the system
z	the deviation of the mass of the passive exciter from the relative axis of rotation
z_s	the statical position of this mass
α	the angle of rotation of the case of the passive exciter wit respect to the axis OX
ε	a dummy small parameter at the end of calculations assumed equal to one
φ	the angle of rotation of the rotor
ω	the frequency of excitation of the case of the system
Ω	the frequency of the case of the passive exciter
$-$	the sign of averaging with respect to time, for example $\overline{\alpha}$
$\cdot = \frac{d}{dt}$	
$/ = \frac{d}{d\alpha}$	

1 INTRODUCTION

The unbalanced system with passive vibroexciter the case of which is attached to the rotor by a rotational link is analysed. Various steady state regimes may exist in this system. This problem has something in common with the dynamics of similar systems investigated earlier [1–3].But in the systems of analysed type some easily implementable specific regimes may exist.

The purpose of this work is to determine the specific qualities of the system in case of simple synchronous regimes, to determine the conditions of their existence and stability.

2 INVESTIGATION

The system analysed consists of the basis 1, the unbalanced rotor 2, the case 3 of the self regulating system with the mass 4. The case 1 is attached to the immovable basis by the elastic members with the coefficients of the stiffness C_x and C_y. The mass 4 is attached to the case 3 by an elastic member with the coefficient of stiffness C_z.

The coordinates of the corresponding points are denoted by

$$O_1(x, y), A(x_A, y_A) \text{ and } B(x_B, y_B)$$

First of all the expressions of the kinetic and potential energies and of the dissipative function of the system are obtained

$$T = 0{,}5\{m(\dot{x}^2 + \dot{y}^2) + I_1\dot{\varphi}^2 + I_2\dot{\alpha}^2 + 2(m_A + m_B)r \cdot \dot{\varphi}$$

$$\cdot(-\dot{x}\sin\varphi + \dot{y}\cos\varphi) + m_B\{\dot{z}^2 + z^2\dot{\alpha}^2 + 2\dot{x} \cdot$$

$$(\dot{z}\cos\alpha - z\dot{\alpha}\sin\alpha) + 2\dot{y}(\dot{z}\sin\alpha + z\dot{\alpha}\cos\alpha) +$$

The Journal was received on 1 August 2000 and was accepted for publication on 30 August 2000.

$$+2r\dot{\varphi}[\dot{z}\sin(\alpha-\varphi)+z\dot{\alpha}\cos(\alpha-\varphi)]\} \quad (1)$$

$$\Pi = 0{,}5\left[c_x x^2 + c_y y^2 + c_z(z - z_s)\right] \quad (2)$$

$$D = 0{,}5\left(H_x \dot{x}^2 + H_y \dot{y}^2 + H_z \dot{z}^2\right) \quad (3)$$

where

$$m = m_0 + m_A + m_B$$

$$I_1 = I_0 + (m_A + m_B)r^2$$

m_0 – the mass of the member 1, m_A – the mass of the member 2 and the reduced mass of the member 3, m_B – the mass of the member 4 concentrated at point B, I_0 – the moment of inertia of the member 2 about the point O_1,

$$r = O_1A, \quad z = AB.$$

C_x, C_y, C_z, H_x, H_y, H_z – the corresponding coefficients of stiffness and viscous friction, z_s – the possition of statical equilibrium.

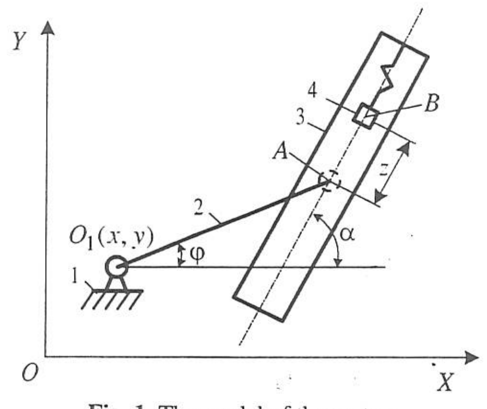

Fig. 1. The model of the system

On the basis of (1) – (3) the differential equations of motion are obtained

$$m\ddot{x} + H_x\dot{x} + c_x x = X$$

$$m\ddot{y} + H_x\dot{y} + c_y y = Y$$

$$\ddot{z} + H_z / m_B + \left(p^2 - \dot{\alpha}^2\right)z = Z$$

$$I_2\ddot{\alpha} + \Lambda = 0$$

$$I_1\ddot{\varphi} + \Phi = 0 \quad (4)$$

where

$$X = (m_A + m_B)r\left(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi\right) -$$

$$-m_B\left[\left(\ddot{z} - z\dot{\alpha}^2\right)\cos\alpha - \left(z\ddot{\alpha} + 2\dot{z}\dot{\alpha}\right)\sin\alpha\right]$$

$$Y = -(m_A + m_B)r\left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right) -$$

$$-m_B\left[\left(\ddot{z} - z\dot{\alpha}^2\right)\sin\alpha + \left(z\ddot{\alpha} + 2\dot{z}\dot{\alpha}\right)\cos\alpha\right]$$

$$Z = -\ddot{x}\cos\alpha - \ddot{y}\sin\alpha - r\left[\ddot{\varphi}\sin(\alpha - \varphi) - \dot{\varphi}^2\cos(\alpha - \varphi)\right]$$

$$\Lambda = m_B z\{z\ddot{\alpha} + 2\dot{z}\dot{\alpha} + r\left[\ddot{\varphi}\cos(\alpha - \varphi) + \dot{\varphi}^2\sin(\alpha - \varphi)\right] -$$

$$-\ddot{x}\sin\alpha + \ddot{y}\cos\alpha\}$$

$$\Phi = (m_A + m_B)r(-\ddot{x}\sin\varphi + \ddot{y}\cos\varphi) +$$

$$+m_B\left[\left(\ddot{z} - z\dot{\alpha}^2\right)\sin(\alpha - \varphi) + \left(z\ddot{\alpha} + 2\dot{z}\dot{\alpha}\right)\cos(\alpha - \varphi)\right] \quad (5)$$

Further the steady state regimes are analysed in the case when

$$x = A\cos\omega t, \quad y = B\sin\omega t \quad (6)$$

$$\alpha = \Omega t + \overline{\alpha} \quad (7)$$

by taking into account that α and φ perform small oscillation about their stationary positions, that is

$$\Lambda = \varepsilon\Lambda$$

$$\Phi = \varepsilon\Phi \quad (8)$$

where ε – a dummy small parameter at the end of calculations assumed equal to one. Then the steady state regimes are sought with the help of the power series with respect to ε, namely:

$$\overline{\alpha} = \overline{\alpha}_0 + \varepsilon\overline{\alpha}_1 + \varepsilon^2\overline{\alpha}_2 \ldots \quad (9)$$

and in this way similarly, where α_1, α_2, ... are periodic function of time t.

Case 1.

$\varphi = \omega t$

In this case by taking into account (4–9) the following differential equation is obtained for z_0

$$\ddot{z}_0 + \left(p^2 - \Omega^2\right)z_0 = p^2 z_s - \ddot{x}\cos\alpha - \ddot{y}\sin\alpha +$$

$$+r\omega^2\cos[(\Omega - \omega)t + \overline{\alpha}] \quad (10)$$

From it follows

$$z_0 = \frac{p^2}{p^2-\Omega^2} z_s + \frac{0{,}5\omega^2(A+B)+r\omega^2}{p^2-\omega^2-(\Omega-\omega)^2}\cos\left[(\Omega-\omega)t+\overline{\alpha}\right]+$$
$$+\frac{0{,}5\omega^2(A-B)}{p^2-\omega^2-(\Omega+\omega)^2}\cos\left[(\Omega+\omega)t+\overline{\alpha}\right] \quad (11)$$

and

$$\Lambda_0 = \Lambda\Big|_{\substack{\varphi=\omega t\\ \alpha=\Omega t+\overline{\alpha}}} = \Lambda_0\left[(\Omega-\omega)t+\overline{\alpha}, (\Omega+\omega)t+\overline{\alpha},\right.$$
$$\left.2(\Omega-\omega)t+2\overline{\alpha},\ 2(\Omega+\omega)t+\overline{\alpha},\ 2\Omega t+2\overline{\alpha},\ 2\omega t,\ 2\omega t+\overline{\alpha}\right] \quad (12)$$

is a periodic function of the indicated arguments.

By limiting the analysis with the quasilinear case it is obtained that the average value of $\overline{\Lambda}_0$ is equal to zero only at

$$\Omega = 0, \Omega = \omega, \Omega = -\omega \quad (13)$$

Case 1.1.

$$\Omega = 0 \quad (14)$$

From (12) it is obtained

$$\overline{\Lambda}_0 = \overline{\Lambda}\Big|_{\substack{\varphi=\omega t\\ \alpha=\Omega t+\overline{\alpha}}} =$$
$$= m_B \frac{0{,}125\omega^4(2r+A+B)(A-B)}{p^2-2\omega^2}\sin 2\overline{\alpha} \quad (15)$$

The stationary values are at $\overline{\Lambda}_0 = 0$, that is

$$\overline{\alpha}_1 = n\pi$$

$$\overline{\alpha}_2 = (n+0{,}5)\pi \quad (16)$$

The conditions of stability are

$$G\cos 2\overline{\alpha} > 0 \quad (17)$$

where

$$G = \frac{0{,}5\omega^4(2r+A+B)(A-B)}{p^2-2\omega^2} \quad (18)$$

In the case when

$$G > 0 \quad (19)$$

$\overline{\alpha}_1$ are stable regimes and $\overline{\alpha}_2$ are unstabile ones. When $G<0$ the stable and unstable regimes interchange, that is $\overline{\alpha}_1$ are unstable $\overline{\alpha}_2$ and are stable.

Case 1.2

$$\Omega = \omega \quad (20)$$

From (12) it is obtained

$$\overline{\Lambda}_0 = m_B \frac{0{,}5\omega^2}{p^2-\omega^2}(2r+A+B)\left[p^2 z_s \sin\overline{\alpha} + \right.$$
$$\left. + 0{,}25\omega^2(2r+A+B)\sin 2\overline{\alpha}\right] = 0 \quad (21)$$

and the condition of stability is

$$m_B \frac{0{,}5\omega^2}{p^2-\omega^2}(2r+A+B)\left[p^2 z_s \cos\overline{\alpha} + \right.$$
$$\left. + 0{,}5\omega^2(2r+A+B)\cos 2\overline{\alpha}\right] > 0 \quad (22)$$

and the condition of stability with respect to z is

$$p^2 - \omega^2 > 0 \quad (23)$$

From the latter relationships it followes that the rotational mations may exist also when

$$z_s = r = 0$$

or one of them is equal to zero.

When $z_s = 0$ in the steady state regime in the interval $\overline{\alpha} \in (0, 2\pi)$ there are four stationary values of $\overline{\alpha}$.

$$\overline{\alpha}_1 = n\pi$$

$$\overline{\alpha}_2 = (n+0{,}5)\pi \quad (24)$$

When $z_s \neq 0$ in separate cases in the interval $\overline{\alpha} \in (0, 2\pi)$ there may be only two stationary values of $\overline{\alpha}$.

Case 1.3.

$$\Omega = -\omega \quad (25)$$

From (12) it is obtained

$$\overline{\Lambda}_0 = m_B \frac{0{,}5\omega^2}{p^2-\omega^2}(A-B)\left[p^2 z_s \sin\overline{\alpha} + \right.$$
$$\left. + 0{,}25\omega^2(A-B)\sin 2\overline{\alpha}\right] = 0 \quad (26)$$

In this case the equation is analogous to the case 1.2, but insread of the member $2r + A + B$ there is a member $A–B$.

Case 2.

$$\varphi = 0 \quad (27)$$

In this case by taking into account (4-9) the following differential equation is obtained for z_0

$$\ddot{z}_0 + \left(p^2 - \Omega^2\right) z_0 = p^2 z_s - \ddot{x}\cos\left(\Omega t + \overline{\alpha}\right) - \ddot{y}\sin\left(\Omega t + \overline{\alpha}\right)$$

From it follows

$$z = \frac{p^2}{p^2 - \Omega^2} z_s + \frac{0{,}5\omega^2 (A+B)}{p^2 - \Omega^2 - (\omega - \Omega)^2} \cos\left[(\Omega - \omega)t + \overline{\alpha}\right] +$$

$$+ \frac{0{,}5\omega^2 (A-B)}{p^2 - \Omega^2 - (\omega + \Omega)^2} \cos\left[(\Omega + \omega)t + \overline{\alpha}\right] \qquad (28)$$

Case 2.1.

$$\Omega = 0 \qquad (29)$$

From (28) it is obtained

$$\overline{\Lambda}_0 = m_B \frac{0{,}125\omega^4 \left(A^2 - B^2\right)}{p^2 - \omega^2} \sin 2\overline{\alpha} = 0 \qquad (30)$$

The conditions of stability are

$$m_B \frac{0{,}5\omega^2 \left(A^2 - B^2\right)}{p^2 - \omega^2} \cos 2\overline{\alpha} > 0$$

$$p^2 - \omega^2 > 0 \qquad (31)$$

The stationary values of $\overline{\alpha}$ are

$$\overline{\alpha}_1 = n\pi$$

$$\overline{\alpha}_2 = (n + 0{,}5)\pi .$$

When $A^2 > B^2$

$\overline{\alpha}_1$ are stable regimes

$\overline{\alpha}_2$ are unstable ones.

When $A^2 < B^2$ the $\overline{\alpha}_1$ and $\overline{\alpha}_2$ according to the stability interchange, that is $\overline{\alpha}_1$ are unstable $\overline{\alpha}_2$.

Case 2.2.

$$\Omega = \omega$$

Here

$$\overline{\Lambda}_0 = m_B \frac{0{,}25\omega^2 (A+B)}{p^2 - \omega^2} \left[2p^2 z_s \sin\overline{\alpha} + \right.$$

$$\left. + 0{,}5\omega^2 (A+B) \sin 2\overline{\alpha}\right] = 0$$

and

$$\frac{\partial\overline{\Lambda}_0}{\partial\overline{\alpha}} = m_B \frac{0{,}25\omega^2 (A+B)}{p^2 - \omega^2} \left[2p^2 z_s \cos\overline{\alpha} + \right.$$

$$\left. + \omega^2 (A+B) \cos 2\overline{\alpha}\right] \qquad (32)$$

Case 2.3.

$$\Omega = -\omega$$

Here

$$\overline{\Lambda}_0 = m_B \frac{0{,}25\omega^2 (A-B)}{p^2 - \omega^2} \left[2p^2 z_s \sin\overline{\alpha} + \right.$$

$$\left. + 0{,}5\omega^2 (A-B) \sin 2\overline{\alpha}\right] = 0$$

and

$$\frac{\partial\overline{\Lambda}_0}{\partial\overline{\alpha}} = m_B \frac{0{,}25\omega^2 (A-B)}{p^2 - \omega^2} \left[2p^2 z_s \cos\overline{\alpha} + \right.$$

$$\left. + \omega^2 (A-B) \cos 2\overline{\alpha}\right] \qquad (33)$$

Case 3.

$$\dot{x} = \dot{y} = 0,\ \dot{\varphi} = \omega,\ \alpha = \omega t + \overline{\alpha} \qquad (34)$$

According to (4-9)

$$\ddot{z}_0 + \left(p^2 - \omega^2\right) z_0 = p^2 z_s + r\omega^2 \cos\overline{\alpha}_0 .$$

$$I_2 \ddot{\overline{\alpha}} + m_B z \left(2\dot{z}_0 \omega + r\omega^2 \sin\overline{\alpha}_0\right) = 0 \qquad (35)$$

From it follows

$$\overline{\Lambda}_0 = m_B r z \omega^2 \sin\overline{\alpha} = 0$$

$$\frac{\partial\overline{\Lambda}_0}{\partial\overline{\alpha}} = m_B r z \omega^2 \cos\overline{\alpha} > 0$$

The regimes

$$\alpha_1 = 2n\pi$$

are stable at

$$p^2 - \omega^2 > 0$$

and in this case

$$\alpha_2 = (2n + 1)\pi$$

are unstable ones.

3 CONCLUSIONS

It is determined that the unbalanced rotor witch the passive vibroexciter on an elastic foundation in case of the steady state regimes has specific qualities.

The conditions of existence and stability of the stationary motions of the case of the passive vibroexciter about the positions of equal librinum and of the synchronous rotational motions are determined. The obtained analytical inequalities reveal the specific qualities of the system.

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