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Contents

Aims and Scope

Original papers dealing with modern developments in the areas of theory and applications of mechanical, mechatronic and biomechatronic system are welcomed. The following subjects are indicated as principal topics:

vibroacoustic and wave processes
analysis and synthesis of nonlinear vibration systems
generation of vibrations and waves
vibrostabilization and control of motions
transformation of motion by vibrations and waves
technology of vibrations and waves
vibration measurement, vibro-identification, vibro-diagnostics and monitoring

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ALL PABLISHED MATERIALS ARE REVIEWERED

Research on Flexible Technological Rotor Dynamics Applying Various Dynamic Models

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Abstract

The rotor analyzed in this report possesses 12 disks of different masses and rotates on hydrodynamic sliding bearings. Each of 12 disks has its own unbalance, which due to various reasons may alter during the exploitation. Unbalance elimination of each separate disk is a difficult technical task. Therefore the rotor specifications give two balancing planes with their allowed unbalances. Rotor operating speed is between the first and the second critical speed. Rotor dynamics has been investigated by present – day modeling and experimental testing methods taking into consideration the well-known theoretical assumptions [1,2,3].

Some results from work "Reduction of turbocompresors vibroaktivity" sponsored by Lithuanian state Science and Studies Fund in 1995 – 2000 have been used in this paper.

Keywords: flexible rotor, vibrations

1 Estimation of vibrations damping

Damping of rotor vibrations, using method finite elements (FEM), has been estimated in two ways:

- by describing vibrations damping with coefficients characterizing internal and external friction dependent on the medium and material characteristics;
- by describing vibrations damping with a proportional damping matrix [2].

Damping of vibrations described by applying coefficients characterizing internal and external friction. When analyzing vibrations damping in a flexible rotor both damping due to internal friction and damping due to external friction can be highlighted. The effect of external friction forces (the medium effect) has been estimated by determining damping coefficient.

$$P_{išt}(u) = c_{išt}\dot{u}\,, \qquad (1)$$

here: $P_{išt}(u)$ – external resisting force, $\dot{u}$ – speed of vibrations, $c_{išt}$ – damping coefficient.

Vibrations damping due to internal friction in a rotor has been evaluated by introducing an additional damping coefficient which varies in accordance whit vibrations amplitudes. When calculating we assumed that damping of vibrations due to internal friction is not strong and does not depend on vibrations frequency. In this way, a mean damping coefficient specifying energy amount scattered because of hysteresis occurrences in the rotor material can be calculated on the basis of following expression:

$$c_{vdt,vid} = \frac{\delta}{\pi}\sqrt{mk}\,, \qquad (2)$$

here: δ – logarithmic decrement of vibrations, m – mass of rotor, k – stiffness of rotor.

Since a logarithmic decrement of rotor depends on physical properties and internal stresses of rotor material i.e. on vibrations amplitude it can be expressed as follows:

$$\delta = \frac{2^{n+1}\upsilon(n-1)A^{n-1}}{n(n+1)}, \qquad (3)$$

here: n, υ – coefficients characterizing the material hysteresis loop and dependent on material physical properties only, A – vibrations amplitude value. Coefficients are determined according to expressions:

$$n = 1 + \frac{\ln\dfrac{\delta_1}{\delta_2}}{\ln\dfrac{\sigma_{01}}{\sigma_{02}}}; \qquad (4)$$

$$\upsilon = \frac{\delta_1(n+1)nE^{n-1}}{2^{n+1}(n-1)\sigma_{01}^{n-1}}; \qquad (5)$$

here: δ_1, δ_2 – strongest material deformation of two adjacent fluctuations of a specimen under free vibrations; σ_{01}, σ_{02} – strongest material stresses of the above mentioned adjacent specimen fluctuations; E – modulus of material elasticity. Coefficient values of structural steel are n=2,279; υ=43,2.

The expressions of damping coefficients having been written in a matrix form and having been added, damping matrix is obtained:

$$\mathbf{C} = \frac{2^{n+1}\upsilon(n-1)A_0^{n-1}}{\pi n(n+1)}\mathbf{\Lambda} + \mathbf{C}_{isr}\,, \qquad (6)$$

here: $\mathbf{\Lambda}$ – matrix characterizing the properties of rotor material. Its elements are calculated in accordance with expression $\sqrt{m_{ij}\cdot k_{ij}}$, here $m_{ij}>0$, $k_{ij}>0$ (i, j=1, 2, 3,.., n). The symbols of matrix $\mathbf{\Lambda}$ elements are the same as those of stiffness matrix elements. $\mathbf{C}_{isr}$ – matrix of damping due to external friction coefficients.

It is evident that damping depends on the vibrations amplitude raised to a certain power. It is of great importance in a case of flexible rotors whose vibrations amplitudes of various elements may differ significantly.

Damping of vibrations described by applying a proportional damping matrix. In this case damping matrix proportional to masses and stiffness matrices is used [2].

$$\mathbf{C} = \alpha_1\mathbf{M} + \alpha_2\mathbf{K}\,, \qquad (7)$$

here α_1, α_2 – coefficients of proportionality.

The properties of viscous friction are defined by a dynamic-response factor characterizing dispersion of vibration energy in the rotor material of sliding bearings.

$$Q = \frac{\omega_r}{\Delta\omega}, \qquad (8)$$

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here: $\Delta\omega = (\omega_r - \omega_1) + (\omega_r - \omega_2)$, ω_r – resonance frequency, ω_1 – before resonance frequency under which vibrations amplitude of a tested object accounts for 0,71 of the resonance amplitude value, ω_2 – after resonance frequency under which vibrations amplitude of a tested object accounts for 0,71 of the response amplitude value.

In this case damping matrix expression will be:

$$\mathbf{C} = \frac{\omega_r}{Q}\mathbf{M} \cdot \qquad (9)$$

When damping is determined in this way it is assumed to be linear and does not depend on either vibrations amplitude or frequency.

2 Dynamic model

Equation of rotor vibrations is obtained by using finite elements model. By applying the first version of damping description a dynamic equation of a tested rotor is written in the following way [4]:

$$(\mathbf{M}+\mathbf{M}')\ddot{\mathbf{U}}+\omega\mathbf{G}\dot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}+\mathbf{P}_K+\mathbf{P}_C, \qquad (10)$$

by applying the second damping description version a slightly changed rotor dynamics equation is used:

$$(\mathbf{M}+\mathbf{M}')\ddot{\mathbf{U}}+\omega\mathbf{G}\dot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}, \qquad (11)$$

here **M** – matrix of rotor masses, **M'** – matrix of masses characterizing the revolutions of rotor cross-sections around their axes, **G** – gyroscopic matrix, **C** – damping matrix, **K** – stiffness matrix, **U** – matrix of rotor elements displacements, **F** – matrix of acting forces on a rotor, $\mathbf{P}_K$, $\mathbf{P}_C$ – matrix estimating hydrodynamic forces [4], ω – angular frequency or rotor revolution.

Matrix elements of acting forces on the rotor are calculated by flowing expression:

$$F_i = (D_i\omega^2 + m_i\frac{\omega^2}{\omega_{krit}^2 - \omega^2}e_i)\sin\frac{\pi x_i}{l_r}\sin\omega t, \qquad (12)$$

here D_i – unbalance of an element, m_i – mass of an element, e_i – eccentricity of an element, x_i – distance from the point O to the investigated element, l_r – length of a rotor, ω_{krit} – first circular critical speed of a rotor.

3. Determination of rotor free vibrations frequencies

When the left-hand side of the equation set describing rotor vibrations is put equal to zero, natural oscillation frequencies of rotor elements can be determined.

$$(\mathbf{M}+\mathbf{M}')\ddot{\mathbf{U}}+\omega\mathbf{G}\dot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=0. \qquad (13)$$

Natural frequencies of that system are calculated by setting up following coefficients matrix **H** [3]:

$$\mathbf{H} = \begin{bmatrix} \mathbf{Q} & \mathbf{E} \\ -(\mathbf{M}+\mathbf{M}')^{-1}(\omega\mathbf{G}+\mathbf{C}) & (\mathbf{M}+\mathbf{M}')^{-1}\mathbf{K} \end{bmatrix}, \qquad (14)$$

here: **Q** – zero matrix, **E** – unit matrix.

Natural frequencies of the system are determined by solving problem of proper values:

$$\det(\mathbf{H} - \lambda\mathbf{E}) = 0 \,. \qquad (15)$$

The proper values problem having been solved, λ is matrix-column of complex proper values is obtained whose complex part is frequencies of the set natural vibrations.

When calculating, it is assumed that there rotor is rotating at 180 rad/s frequency. Frequencies of rotor natural vibrations are given in Table 1.

Table 1.

Natural vibrations frequencies of rotor applying FEM dynamic model

No	Rotor natural frequencies, rad/s	No	Rotor natural frequencies, rad/s	No	Rotor natural frequencies, rad/s
1	54,3	27	962,7	53	1991,3
2	54,4	28	962,7	54	1991,3
3	121,6	29	1110,9	55	2050,4
4	121,8	30	1110,9	56	2050,4
5	186,2	31	1187,0	57	2194,7
6	186,2	32	1187,0	58	2194,7
7	195,7	33	1309,2	59	2616,2
8	195,7	34	1309,2	60	2616,2
9	533,0	35	1336,3	61	2822,7
10	533,0	36	1336,3	62	2822,7
11	596,6	37	1538,2	63	3218,1
12	596,6	38	1538,2	64	3218,1
13	641,4	39	1540,5	65	3395,8
14	641,6	40	1540,5	66	3395,8
15	705,3	41	1596,5	67	4282,3
16	705,3	42	1596,5	68	4282,3
17	825,1	43	1689,6	69	5784,4
18	825,1	44	1689,6	70	5784,8
19	854,7	45	1728,1	71	6210,2
20	854,7	46	1728,1	72	6210,2
21	914,2	47	1735,3	73	7581,4
22	914,2	48	1735,3	74	7581,4
23	936,8	49	1737,6	75	9400,4
24	936,8	50	1737,6	76	9400,4
25	954,1	51	1774,4		
26	954,1	52	1774,4		

In order to estimate thoroughly dynamic behavior of a tested rotor its free vibrations frequencies have been calculated:

a) a rotor as a rigid, elastically suspended body (Table 2). In this case natural frequencies of rotor vibrations I supports in X, Y directions and revolutions around the supports are calculated according to following expressions:

$$p_{x,y} = \sqrt{\frac{2k_{x,y}}{m}}, \qquad (16)$$

$$p_{x\varphi,y\varphi} = \sqrt{\frac{k_{x\varphi,y\varphi}}{mr_{x,y}^2}}, \qquad (17)$$

here: k – stiffness of a rotor support in a corresponding directions, m – mass, r – moment of inertia of a rotor in a corresponding direction.

Table 2.

Natural vibrations frequencies of a rotor as a rigid body

Rotor natural frequencies, rad/s			
p_x	p_y	$p_{x\varphi}$	$p_{y\varphi}$
55,53	56,68	122,69	123,07

b) a rotor as flexible body. In this case natural frequencies (rotor critical speed) can be calculated by applying a classical dynamic model. In this case natural vibrations frequencies of rotor transverse vibrations in X and Y directions and natural vibrations frequencies of rotor revolutions around the corresponding planes are calculated on the basis following expressions [3]:

$$p_{x,y}^{\ *} = n^2\pi^2\sqrt{\frac{EJ_{x,y}}{l^3 m}}, \tag{18}$$

$$p_{x\varphi,y\varphi}^{\ *} = \frac{n\pi}{r^*}\sqrt{\frac{GW}{lm}}, \tag{19}$$

here: n = 1, 2, 3...; l – length of rotor; E – modulus of material elasticity; $J_{x,y}$ – modulus of inertia of rotor cross-section with respect to a corresponding axis; m – mass of rotor; G – modulus of shearing; W – modulus of resistance of rotor cross-section; r^* – radius of rotor cross-section inertia.

Table 3 gives the results of calculation of natural vibrations frequencies by applying a classical dynamic model. For comparison, the values of rotor first critical speed obtained when measuring rotor vibrations during the shutdown of a turbine and the data from the turbine specifications are presented.

Table 3.

Natural frequencies of transverse vibrations of a rotor as a flexible body

	p_x	p_y	$p_{x\varphi}$	$p_{y\varphi}$	Determined experimentally	Given in specifications
Firsts	192,4	192,4	3107	3107	186 - 189	188,5
Second	769,6	769,6	6214	6214	-	-

4. Comparison of experimental and theoretical data obtained by modeling steam turbine vibrations

For experimental investigation steam turbine K-15-41-1 of turbo compressor K-1290-121-1 operating in ammonia production shop in "Achema" company has been chosen. The machine vibrations have been measured following the requirements of ISO 10816 standard in rotor supports of turbo compressor and steam turbine in axial and radial directions on ten measurement points i.e. in all bearings of the equipment. Measurement was carried out by vibration control device "Vibrospect FFT (Pruftechnik AG, Germany) which is in "Achema" diagnostic laboratory. It has software for rotor balancing and vibrations diagnostics [5].

Figs 1 and 2 present amplitude-frequency characteristics of vibrations obtained from the first steam turbine support:

- Fig. 1 – by applying a dynamic model set up by means of FEM when damping by the first described method and the results obtained by experimentally measuring support vibrations during shutdown of a steam turbine;
- Fig. 2 – by applying s dynamic model set up by means of FEM when damping by second described method.

In order to compare amplitude-frequency characteristics, the characteristic segments of rotor support vibrations are given separately (Fig. 3). They are obtained in the frequency range, which approximately corresponds to the turbine operation speeds i.e. from, 300 to 400 rad/s. This Figure also presents the amplitude-frequency characteristic of rotor vibrations obtained by applying a classical dynamic model [5].

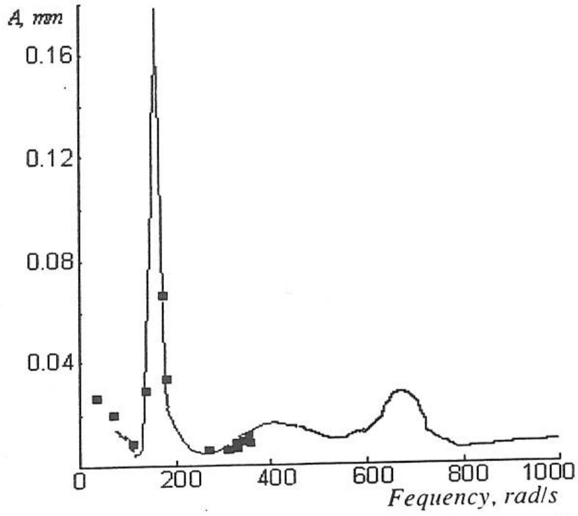

Fig. 1. Amplitude-frequency characteristic of rotor first support vibrations obtained by applying a dynamic model set up by means of FEM, damping being estimated by the first method. The dots indicate vibration amplitudes obtained by measuring vibrations experimentally during shutdown of a tested machine.

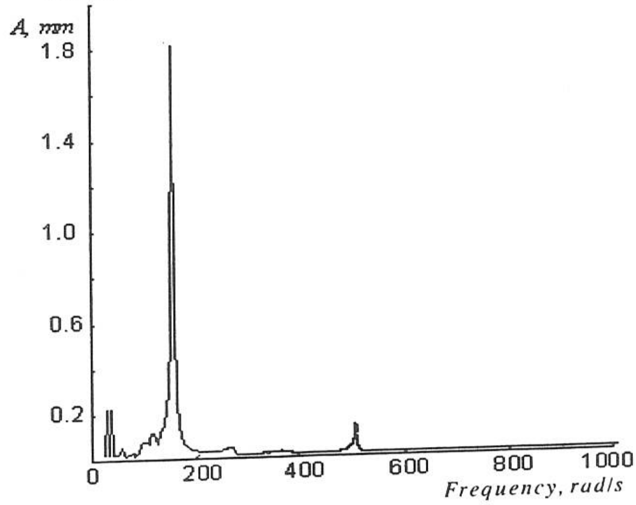

Fig. 2. Amplitude-frequency characteristic of rotor first support vibrations obtained by applying a dynamic model set up by means of FEM having estimated damping by the second method.

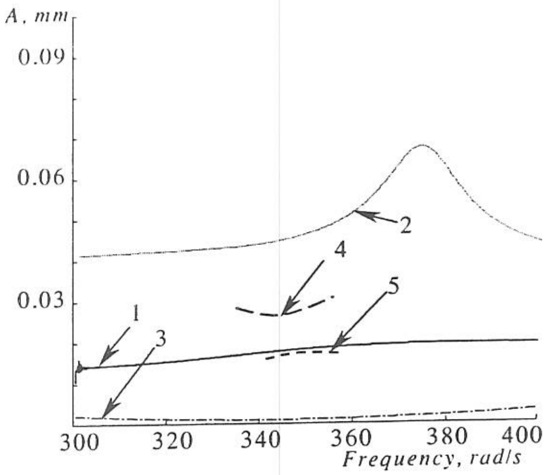

Fig. 3. Segment of an amplitude-frequency characteristic of steam turbine rotor support vibrations in the zone of rotor operation frequencies: 1 – dynamic model set up by means of FEM when damping is estimated by the first method; 2 – dynamic model set up by means of FEM with damping being estimated by the second method; 3 – classical dynamic model; 4 – experimentally obtained curve when measuring vibrations at 1H point before balancing the rotor; 5 – experimentally obtained curve when measuring vibrations at 1H point after balancing the rotor.

5. Conclusions

1 The data from Table 1 and 3 indicate that the tested rotor after having reached the operating speed of 300 – 400 rad/s is flexible as it works between the first and second critical speeds.

2 After the analysis of the data (Figs 1 and 2) it is evident that the values of resonance vibrations amplitudes when applying FEM model with a proportional damping matrix and with damping calculated with respect to internal and external friction differ 10 times. Under resonance regimes the characteristic is significantly steeper. When analyzing the curves given in Fig. 2 it is evident that the vibration level rise near the double critical speed (about 375 rad/s), due to the sliding bearing effect [4], shows up considerably stronger with the application of a dynamic model having a proportional damping matrix than when applying a model with damping calculated on the basis of internal and external friction.

3 Then comparing characteristics obtained by modeling with those obtained experimentally (Fig.3) it is evident that by taking a dynamic model with a proportional damping matrix the vibration amplitudes in the operating frequencies zone (Curve 2) are obtained 3 times greater than those really measured at the allowed rotor unbalance (Curve 5). Whereas, when applying a dynamic matrix with damping being calculated on the basis of internal and external friction, the amplitudes obtained by modeling (Curve 1) and those really measured of rotor supports are of the same order. Applying a classical dynamic model the obtained amplitude-frequency characteristics of rotor vibrations (Curve 3) significantly differ from the experimentally obtained values, in this way, when applying this dynamic model the value of the rotor second critical speed cannot even approximately be determined.

4 Analyzing the vibration amplitude-frequency characteristics in the wide range of frequencies and comparing them with the experimentally obtained data, it is evident that the real steam turbine vibrations are best represented by a dynamic model set up by means of FEM and whose damping is estimated by the first method.

5 Referring to the investigation results a dynamic model set up by means of FEM and estimating internal and external friction acting on the steam turbine rotor is recommended for the analysis of flexible rotor dynamics; for the analysis of rigid and quasi-rigid rotors dynamics a dynamic model set up by means of FEM with a proportional damping matrix can be used.

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