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Contents

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Multiple synchronization and unperiodical motion in a system with unbalanced rotors

E. Astrauskiene[1], K. Ragulskis[1], I. Tiknevichiene[1], L. Ragulskis[2]
[1]Kaunas University of Technology
[2]Universitas Vytauti Magni

Introduction

The simple synchronisation of mechanical systems is deeply investigated in the research works of I.Blechman [1]. The multiple synchronisation of some systems was investigated by K.Ragulskis [2 - 4] and his coworkers.

The dynamical system is analysed as an essentially nonlinear one in which the multiple synchronisation may exist. For the investigation of the latter as an essentially nonlinear system two small parameters are used with the help of which the conditions of existence and stability are determined. More complicated processes and regimes of chaotic type are analysed by numerical methods.

The analysis is performed for the case of a plane system. This system consists of a case which in the plane may perform two orthogonal longitudinal motions. The case is excited by two unbalanced rotors, which rotate in the opposite directions.

Investigation

The kinetic and potential energies of the analysed system are described by the equations

$$T = 0{,}5\left(m_x\dot{x}^2 + m_y\dot{y}^2 + I_1\dot{\varphi}_1^2 + I_2\dot{\varphi}_2^2\right) + $$
$$+ m_1 r_1\dot{\varphi}_1\left(-\dot{x}\sin\varphi_1 + \dot{y}\cos\varphi_1\right) + $$
$$+ m_2 r_2\dot{\varphi}_2\left(\dot{x}\sin\varphi_2 + \dot{y}\cos\varphi_2\right), \quad (1)$$

$$\Pi = 0{,}5\left(c_x x^2 + c_y y^2\right) + m_1 g\left(y + r_1\sin\varphi_1\right) + $$
$$+ m_2 g\left(y + r_2\sin\varphi_2\right), \quad (2)$$

and the dissipative function is of the form

$$D = 0{,}5\left(H_x\dot{x}^2 + H_y\dot{y}^2\right). \quad (3)$$

here

$m_x = m_1 + m_2$; $m_y = m_0 + m_1 + m_2$; $I_1 = I_{10} + m_1 r_1^2$, $I_2 = I_{20} + m_2 r_2^2$, $\dot{} = \frac{d}{dt}$, m_0 – the mass of the carrying body, m_i – the mass of the i-th disbalance (i = 1,2), r_i – the radius-vector of the i-th disbalance, I_i – the reduced moment of inertia of the i-th member, x, y – the orthogonal displacements of the carrying body, φ_i – the angles of rotation of unbalanced rotors.

The mathematical model of the system with introduced two small parameters μ and ε which take into account the nonlinearities of the system is the following system of essentially nonlinear differential equations

$$\begin{cases} m_x\ddot{x} + H_x\dot{x} + c_x x = m_1 r_1\left(\ddot{\varphi}_1\sin\varphi_1 + \dot{\varphi}_1^2\cos\varphi_1\right) - m_2 r_2\left(\ddot{\varphi}_2\sin\varphi_2 + \dot{\varphi}_2^2\cos\varphi_2\right); \\ m_y\ddot{y} + H_y\dot{y} + c_y y = m_1 r_1\left(\ddot{\varphi}_1\cos\varphi_1 - \dot{\varphi}_1^2\sin\varphi_1\right) - m_2 r_2\left(\ddot{\varphi}_2\cos\varphi_2 - \dot{\varphi}_2^2\sin\varphi_2\right); \\ I_1\ddot{\varphi}_1 + \varepsilon m_1 r_1\left(-\ddot{x}\sin\varphi_1 + \ddot{y}\cos\varphi_1\right) + \varepsilon m_1 r_1 g\cos\varphi_1 = \varepsilon M_1\left(\dot{\varphi}_1\right); \\ I_2\ddot{\varphi}_2 + \varepsilon m_2 r_2\left(\ddot{x}\sin\varphi_2 + \ddot{y}\cos\varphi_2\right) + \varepsilon m_2 r_2 g\cos\varphi_2 = \varepsilon M_2\left(\dot{\varphi}_2\right), \end{cases} \quad (4)$$

where the parameters μ and ε at the end of calculations are made equal to one.

The local steady state regime is analysed

$$\overline{\dot{\varphi}_1} = \omega, \quad \overline{\dot{\varphi}_2} = \frac{1}{2}\omega,$$

where the dashes over the letters mean averaging with respect to time.

The periodic solutions of the system (4) are sought in the form of the following series:

$$x = x_0 + \varepsilon x_1 + \ldots, \quad y = y_0 + \varepsilon y_1 + \ldots,$$
$$\varphi_1 = \varphi_{10} + \varepsilon\varphi_{11} + \ldots, \quad \varphi_2 = \varphi_{20}(\mu) + \varepsilon\varphi_{21}(\mu) + \ldots, \quad (5)$$

the last of which corresponds to the regime of multiple synchronisation.

By taking into account only the linear part with respect to μ for the case of the conservative system and equating the coefficients at equal powers of ε when $H_x = H_y = 0$ the differential equations are obtained, from which it is determined

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$$x_0 = \frac{\omega^2 r_1 \mu_{x_1}}{p_x^2 - \omega^2}\cos\omega t + \frac{0{,}25\omega^2 r_2 \mu_{x_2}}{p_x^2 - 0{,}25\omega^2}\cos(0{,}5\omega t + \alpha); \qquad (6)$$

$$y_0 = \frac{\omega^2 r_1 \mu_{y_1}}{p_y^2 - \omega^2}\sin\omega t + \frac{0{,}25\omega^2 r_2 \mu_{y_2}}{p_y^2 - 0{,}25\omega^2}\sin(0{,}5\omega t + \alpha); \qquad (7)$$

$$\dot{\varphi}_{10} = \omega t; \qquad (8)$$

$$\varphi_{20} = 0{,}5\omega t + \alpha + \mu\left[D\cos(0{,}5\omega t + \alpha) - \frac{32 M_2 \varepsilon}{D I_2 \omega^2 (8 + D^2)}\cdot\sin(0{,}5\omega t + \alpha) + \frac{16\varepsilon M_2\left[(2 + D^2)\mu_{x_1}(p_y^2 - \omega^2) + 2\mu_{y_1}(p_x^2 - \omega^2)\right]}{I_2\omega^2(8 + D^2)\left[8\mu_{x_1}(p_y^2 - \omega^2) - (8 + D^2)2\mu_{y_1}(p_x^2 - \omega^2)\right]}\cdot\cos 2(0{,}5\omega t + \alpha)\right]. \qquad (9)$$

From the conditions of periodicity of φ_{11} and it is obtained

$$\cos 2\alpha = \frac{2g(p_x^2 - \omega^2)(p_y^2 - \omega^2)\left[8M_2 + D I_2 \omega^2 (L + 0{,}5KD)\right]}{I_2 r_1 \omega^6 D^2\left[\mu_{y_1}(p_x^2 - \omega^2) - \mu_{y_1}(p_y^2 - \omega^2)\right]}; \qquad (10)$$

$$M_1 < \frac{m_1 r_1 r_2 \omega^4}{16}\left|\frac{\mu_{y_2}(p_x^2 - 0{,}25\omega^2) - \mu_{x_2}(p_y^2 - 0{,}25\omega^2)}{(p_x^2 - 0{,}25\omega^2)(p_y^2 - 0{,}25\omega^2)}\right|. \qquad (11)$$

In the equations (6 – 11) it is denoted

$$p_x = \sqrt{\frac{c_x}{m_x}},\quad \mu_{x_1} = \frac{m_1}{m_x},\quad \mu_{x_2} = \frac{m_2}{m_x},\quad p_y = \sqrt{\frac{c_y}{m_y}},\quad \mu_{y_1} = \frac{m_1}{m_y},\quad \mu_{y_2} = \frac{m_2}{m_y},\quad D = \frac{4 m_2 r_2 g}{I_2 \omega^2},\quad L = -\frac{32 M_2}{D I_2 \omega^2 (8 + D^2)},$$

$$K = \frac{16 M_2}{I_2 \omega^2 (8 + D^2)}\cdot\frac{(2 + D^2)\mu_{x_1}(p_y^2 - \omega^2) + 2\mu_{y_1}(p_x^2 - \omega^2)}{8\mu_{x_1}(p_y^2 - \omega^2) - (8 + D^2)\mu_{y_1}(p_x^2 - \omega^2)}.$$

According to (4 – 11) the condition of stability of steady state multiple synchronisation is obtained

$$\frac{\mu_{x_2}(p_y^2 - 0{,}25\omega^2) - \mu_{y_2}(p_x^2 - 0{,}25\omega^2)}{(p_x^2 - 0{,}25\omega^2)(p_y^2 - 0{,}25\omega^2)}\sin\alpha > 0. \qquad (12)$$

The conditions of existence and stability (11) and (12) define the limits of variation of the parameters of the system with which it is possible to implement the regime of multiple synchronisation.

For numerical investigation the analysed system is reordered into the following one

$$\begin{cases} \ddot{x} + h_x\dot{x} + p_x^2 x = X; \\ \ddot{y} + h_y\dot{y} + p_y^2 y = Y; \\ J_1\ddot{\varphi}_1 + M_{v_1} + m_1 g r_1\cos\varphi_1 = M_1(\dot{\varphi}_1); \\ J_2\ddot{\varphi}_2 + M_{v_2} + m_2 g r_2\cos\varphi_2 = M_2(\dot{\varphi}_2); \end{cases} \qquad (13)$$

where

$$X = \mu_{x_1} r_1\left(\ddot{\varphi}_1\sin\varphi_1 + \ddot{\varphi}_1^2\cos\varphi_1\right) - \mu_{x_2} r_2\left(\ddot{\varphi}_2\sin\varphi_2 + \ddot{\varphi}_2^2\cos\varphi_2\right),$$

$$Y = -\mu_{y_1} r_1\left(\ddot{\varphi}_1\cos\varphi_1 - \ddot{\varphi}_1^2\sin\varphi_1\right) - \mu_{y_2} r_2\left(\ddot{\varphi}_2\cos\varphi_2 - \ddot{\varphi}_2^2\sin\varphi_2\right),$$

$$M_{v_1} = m_1 r_1 \left(-\ddot{x} \sin \varphi_1 + \ddot{y} \cos \varphi_1 \right), \qquad M_{v_2} = m_2 r_2 \left(\ddot{x} \sin \varphi_2 + \ddot{y} \cos \varphi_2 \right), \tag{14}$$

$$M_i = A_i - B \dot{\varphi}_i . \tag{15}$$

The system is analysed when

$$A_1 = \text{const}; \; A_2 = \text{var}.$$

The performed calculations in the sufficiently wide for practice interval of change of parameters confirm the coincidence according to the zones of existence and stability and the characteristics of steady state synchronous regimes. But other types of regimes may be analysed only numerically. Here some of their graphical realtionships are presented (fig. 1 – 4) for the following parameters

$$h_x = h_y = 0{,}2; \; p_x = \sqrt{0{,}5}; \; p_y = \sqrt{0{,}8};$$
$$\mu_1 = \mu_2 = 0{,}5; \; r_1 = r_2 = 0{,}5; \; I_1 = I_2 = 1{,}0; \; g = 0;$$
$$B_1 = B_2 = 0{,}6 .$$

From the presented graphical realtionships it follows that non steady state regimes play an important role in the dynamics of such systems.

Conclusions

1. The dynamical qualities for the case of a complicated mechanical system the case of which may move in a plane according to two orthogonal coordinates and which is excited by

2. two unbalanced rotors rotating in the opposite directions are revealed. It is shown that in the system complicated steady state regimes may exist, such as chaos, combined synchronisation.

3. The zones of existence of regimes of separate types are shown by analytical realtionships and graphically.

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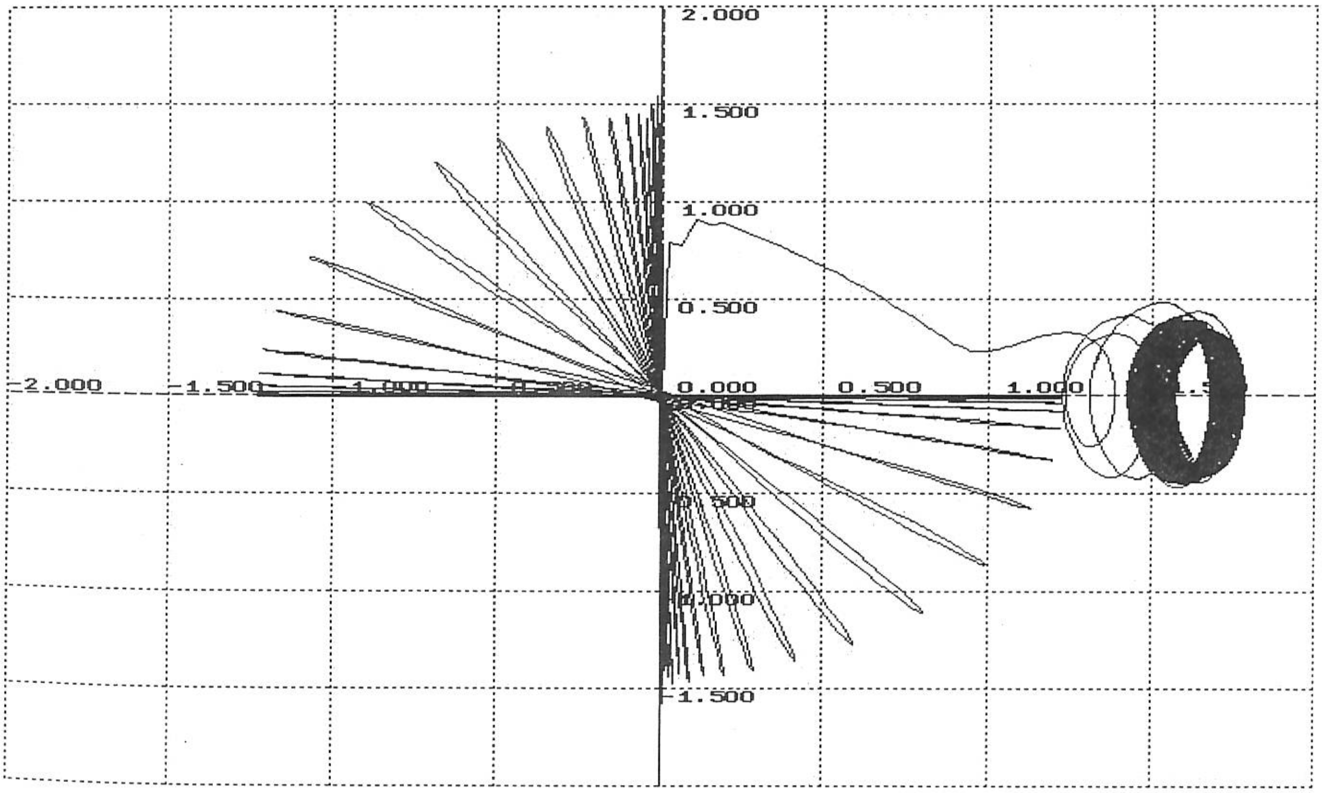

Fig. 1 The transient to steady state process of the synchronous regime at $\omega > p_y$; $A_1 = A_2 = 1{,}0$

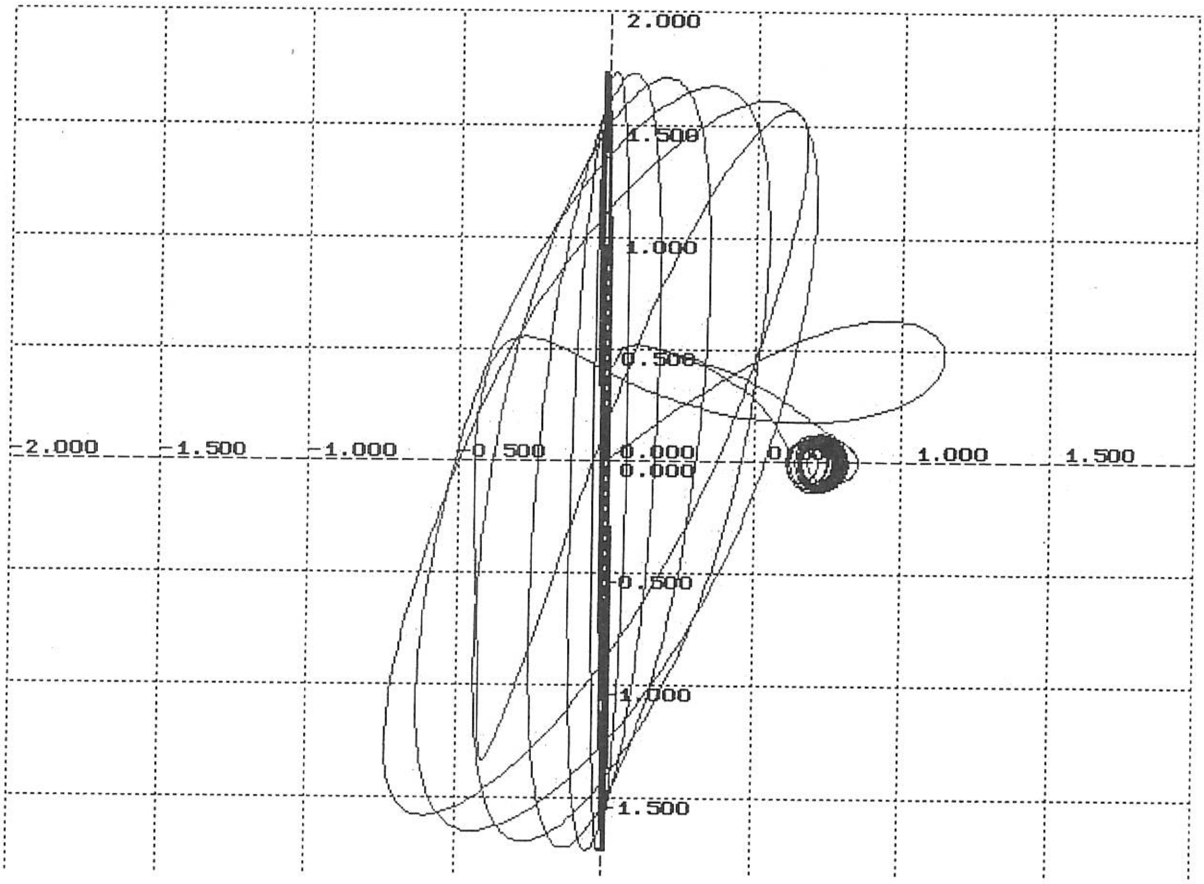

Fig 2 The steady nonperiodic state regime at $\omega > p_y$; $A_1 = 1{,}1$; $A_2 = 1{,}0$

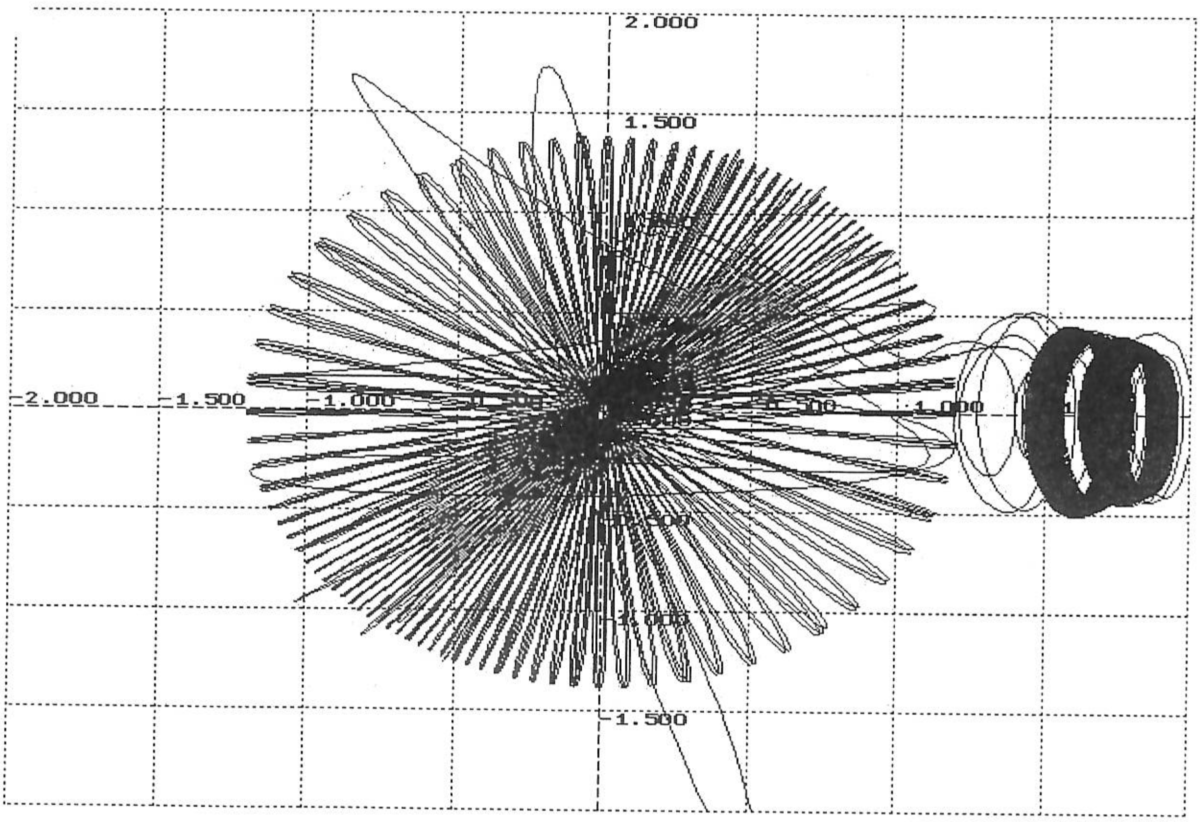

Fig. 3 The transient to steady state process of the synchronous regime at $p_x < \omega < p_y$; $A_1 = 0{,}501$; $A_2 = 0{,}5$

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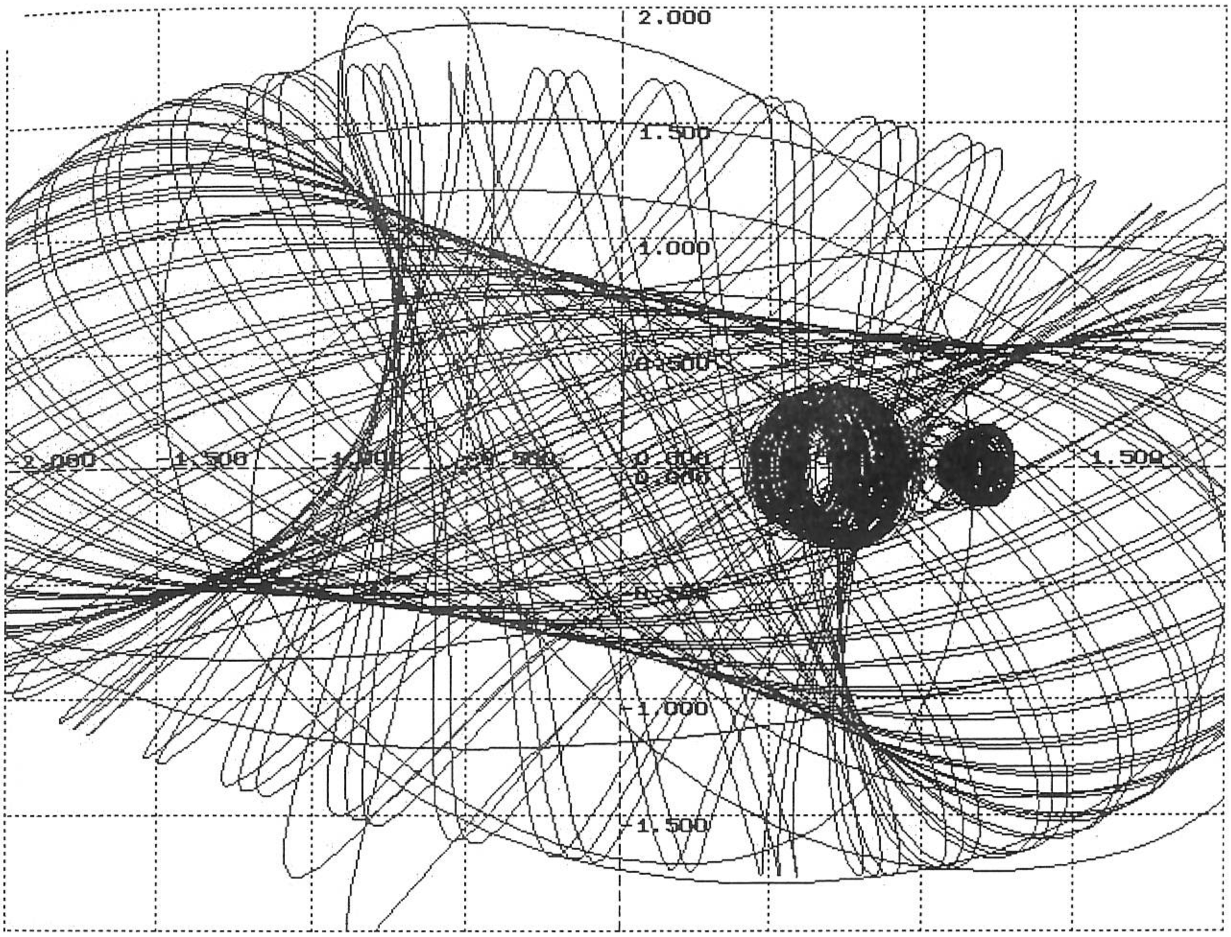

Fig. 4 The steady state nonperiodic process at $p_x < \omega < p_y$; $A_1 = 0{,}8$; $A_2 = 0{,}5$