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Contents

The Conductive Fluid, Confined in the Infinite Rigid Cylinder, Surface Vibration Under the of Variable Electrical Field

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Abstuct: The electrohydrodynamic conductive fluid surface vibration confined in an rigid cylinder and having a circular interface has been investigated . It is found that there exits a critical electric field beyond which the intensive vibration of surface is exits. When the surface tension factor prevails in the system the resonanse curve is directed at the smaller side of velocity of electrical field and amplitude of oscilations decrease while increasing the velocity of electrical field. When the viscous factor prevails in the investigated system, the rezonance curve is symetrical with respect to maximum amplitude.

Keywords: electrohydrodynamic, surface vibration, rezonance.

The investigated model is showh in picture 1. In a circural hermetic cylinder there is conductive liquid, which takes up volume $0 \le r \le r_0$; $0 \le v \le 2\pi$; $-h_0 \le z \le 0$ .In distance h_1 from the surface of liquid the metalic electrode is placed. The space between the electrodes is filled with gass and supplied with alternating voltage.

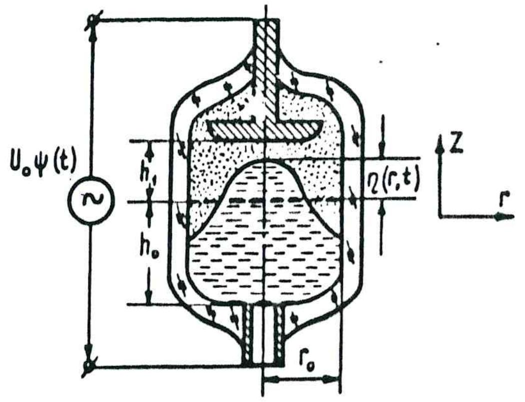

Fig. 1. The investigated model

In equilibrian stage the meniscus of conductive liquid has hemisphere form, and the lines of eletrical forces are perpendicular to both surfaces. The motion of liquid surfaces harmonized in all volume and the potentional of electric field is harmonized in between electrodical space. Taking into consideration all the mentioned presumtions the vibrations of the surface of conductive liquid we can describe in the following equation system[1]:

$$\Delta\varphi = 0, \quad \Delta U = 0 \, , \tag{1}$$

$$F(r,z,t) = \rho\frac{\partial\varphi}{\partial t} - \rho q\eta +$$

$$2\mu\frac{\partial^2\varphi}{\partial t^2} + \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\left[1+\left(\frac{\partial\eta}{\partial r}\right)^2\right]^{-\frac{1}{2}} . \tag{2}$$

$$\frac{\partial\varphi}{\partial r} = 0, \; when \; r = r_0, \quad \frac{\partial\varphi}{\partial z} = 0, \; when \; z = -h_o \, , \tag{3}$$

$$\frac{\partial\varphi}{\partial z} = -\frac{\partial\eta}{\partial t}, \; U = 0, \; when \; z = \eta \, , \tag{4}$$

$$U = U_0\psi(t), \; when \; z = h_1 \tag{5}$$

where Δ - Laplacian operator in cylindrical co-ordinate system, φ, η -the potencial of liquid velocity and the amplitude of surface vibration, U - the potential of electrical field ρ ,μ, q, α -correspondingly the density of liquid, dynamic viscosity, normal acceleration and surface tension coefficient.

We'll investigate the symmetrical surface vibrations acordingly to z axis. With respect to boundary conditions (3) φ and η we search for solutions in certain type[2]:

$$\varphi = \sum_{n=1}^{\infty} C_0 \frac{\partial\beta_n(t)}{\partial t}\frac{ch k_n(z+h_0)}{k_n sh k_n h_0} I_0(k_n r) .$$

$$\eta = \sum_{n=1}^{\infty} C_0 \beta_n(t) I_0(k_n r),$$

where $k_n = \frac{\xi_n}{r}$, ξ_n -n-order Bessel function $I_0(k_n r)=0$ root.

In order to find unknown coefficients $\beta_n(t)$, φ and η expressions we put into kinematic equation (2). After simple matematical operations and using equations (4) and (5) we get:

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$$\frac{\partial^2\beta_n(t)}{\partial t^2}+2\nu k_n^{\ 2}\frac{\partial\beta(t)}{\partial t}+\left(qk_n+\frac{\alpha}{\rho}k_n^{\ 3}-\frac{\varepsilon U_0^2 k_n^{\ 2}\psi^2(t)}{4\pi\rho h_1^2 thk_n h_1}\right)\beta_n(t)thk_n h_0-\gamma_1\beta_n^{\ 3}(t)=0, \qquad (6)$$

$$\text{where } \gamma_1=\frac{3C_0\alpha k_n^4 thk_n h_0}{\rho r_0^2 I_0^2(k_n r)}\int_0^{r_0}\left[k_n r I_0^2(k_n r)I_1^2(k_n r)+\frac{2}{3}I_0(k_n r)I_1^3(k_n r)\right]dr, \quad \nu=\frac{\mu}{\rho}.$$

We shall investigate the case, when the electrical field between the electrodes changes accordingly cosinus law, that is $\psi(t)=\cos\Omega t$. In this case the equation (6) can be written:

$$\frac{\partial^2\beta_n(t)}{\partial t^2}+\left[\delta_0+\delta_1\beta_n^2(t)\right]\frac{\partial\beta_n(t)}{\partial t}+\left(\omega_n^2+g_n^2\cos 2\Omega t\right)\beta_n(t)-\gamma_1\beta_n^3(t)=0\ , \qquad (7)$$

$$\text{where } g_n^2=\frac{\varepsilon U_0^2 k_n^2 thk_n h_0}{8\pi\rho h_1^2 thk_n h_1},\quad \omega_n^2=\left(\frac{\rho q+\alpha k_n^2}{\rho}-\frac{\varepsilon k_n U_0^2}{8\pi\rho h_1^2}cthk_n h_1\right)k_n thk_n h_0.$$

$\delta_0=2\nu k_n^2, \delta_1$ – constant, the value of which is fixed in the way of experiments.

When $q_n\cos 2\Omega t=0$ we get equation of surface free oscilations, when the uniform electrical field is supplied between electrodes:

$$\varpi_n=\left(\omega_n^2-\frac{3}{4}\gamma\beta_0^2\right)^{\frac{1}{2}}.$$

We notice from this equation that indirect factors and constant component of electrical field decrease the velocity of free oscilations. If we want to find $\beta_n(t)$ when $\Omega\approx\omega_n$ the solution of equation (7) search in the following form:

$\beta_n(t)=a\sin\Omega t-b\cos\Omega t=\beta_0\sin(\Omega t-\varphi)$,

where $\beta_0=\left(a^2+b^2\right)^{\frac{1}{2}},\quad \varphi=arctg\frac{b}{a}$.

Using this expression in equation (7) after simp operations, we get the following equation of constrain oscilations amplitude when $\Omega\approx\omega_n$:

$$\beta_0=\left\{\frac{3}{2}\gamma_1\left(\frac{\omega_n^2}{\Omega^2}-1\right)-\frac{1}{2}\delta_0\delta_1\pm\sqrt{\frac{3}{2}\gamma_1\delta_1\delta_0\left(1-\frac{\omega_n^2}{\Omega^2}\right)+\frac{1}{4}\delta_1\Omega^2\left[\frac{g_n^4}{4\Omega^4}-\left(1-\frac{\omega_n^2}{\Omega^2}\right)^2\right]+\frac{9}{16}\left[\frac{g_n^4}{\Omega^4}-4\frac{\delta_0^2}{\Omega^2}\right]}\right\}$$
$$\left(\frac{9}{8}\frac{\gamma_1^2}{8\Omega^2}+\frac{1}{8}\delta_1^2\right)^{-\frac{1}{2}}.$$

In cases when viscousity factor or nonlinear surface tension factor is small the equation (8) can be written:

$$\beta_0=\left\{\pm\frac{2}{\delta_1\Omega}\left[g_n^4-4\left(\Omega^2-\omega_n^2\right)^2\right]^{\frac{1}{2}}-4\frac{\delta_0}{\delta_1}\right\}^{\frac{1}{2}},$$

when $\gamma_1=0$ (9)

$$\beta_0 = \frac{2}{\sqrt{3\gamma_1}}\left[\left(\omega_n^2 - \Omega^2\right) \pm \left(\frac{g_n^2}{4} - \Omega^2\delta_0^2\right)^{\frac{1}{2}}\right]^{\frac{1}{2}},$$

when $\delta_1=0$, (10)

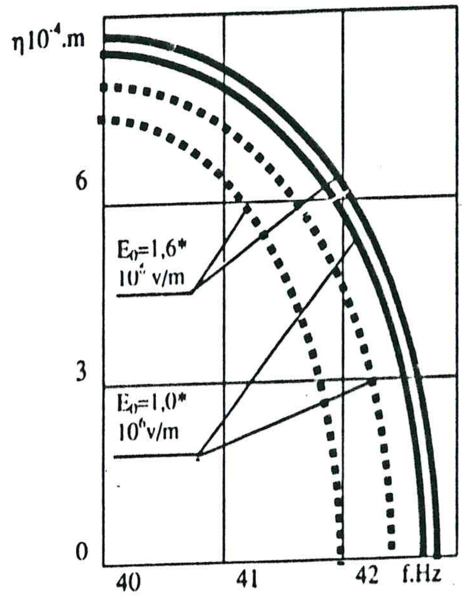

Fig. 2. Resonanse liquid surface vibration amplitude curve, when $\delta_1 = 0$, $r_0=3{,}1.10^{-3}$m, $h_0=10^{-2}$m, $h_1=3{,}8.10^{-4}$m.

While the surface tension factor prevails in the system the resonanse curve (fig.2)is directed at the smaller side of Ω meanings and the amplitude of oscilations decreases while increasing the velocity of electric field. In this case to excite the surface vibrations we can only when β_0^2 is a whole quantity that is:

$$E_0 \triangleright \left(\frac{16\pi\rho\omega_n\delta_0}{\varepsilon k_n^2}\frac{thk_n h_1}{thk_n h_0}\right)^{\frac{1}{4}}.$$

$$\omega_n^2 - \frac{\delta_0^2}{2} - \left(\frac{g_n^4}{4} - \delta_0\omega_n^2 + \frac{\delta_0}{4}\right)^{\frac{1}{2}} < \Omega^2 < \omega_n^2 -$$

$$-\frac{\delta_0^2}{2} + \left(\frac{g_n^4}{4} - \delta_0^2\omega_n^2 + \frac{\delta_0^4}{4}\right)^{\frac{1}{2}}.$$

When viscouc factor prevails in the system (fig.3) the rezonance curve is symmetrical with respect to maximum amplitude.

Conclutions

Using received expressions we can calculate dynamical characteristics of commutators with liquid metalic contacts depending on their geometrical parameters and supplied characteristics of electric field.

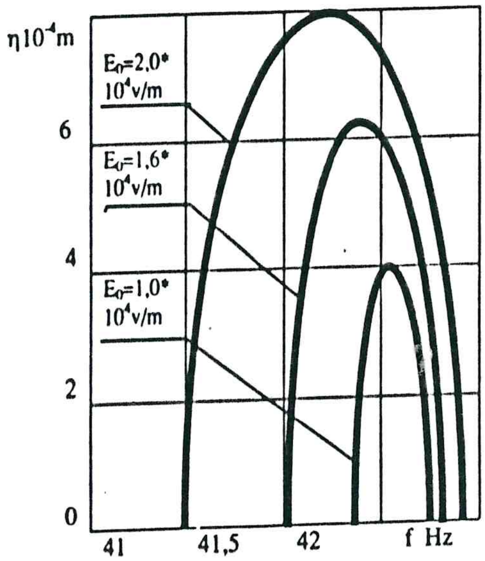

Fig. 3. Resonanse liquid surface vibration amplitude curve, when $\gamma_1 = 0$, $r_0=3{,}1.10^{-3}$m, $h_0=10^{-2}$m, $h_1=3{,}8.10^{-4}$m.

If we compleate this condition, then area of described Ω^2 meanings exists, in which we can excite intensive oscilations of surface. The boundaries of this area we find from condition $\beta_0^2=0$. Then follows:

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