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Review and Novel Framework with Hui–Walter Method and Bayesian Approach for Estimation of Uncertain Remaining Value in Refurbished Products

Ieva Dundulienė [1] and Robertas Alzbutas [1,2,*]

[1] Department of Applied Mathematics, Faculty of Mathematics and Natural Sciences, Kaunas University of Technology, Studentų Str. 50, LT-51368 Kaunas, Lithuania; ieva.dirdaite@ktu.edu

[2] Lithuanian Energy Institute, Breslaujos Str. 3, LT-44403 Kaunas, Lithuania

* Correspondence: robertas.alzbutas@ktu.lt

Abstract: Consumers' growing interest in sustainability and the consideration of purchasing second-hand products present conditions for developing and improving a new method for Remaining Value (RV) estimation. The remaining value refers to the value of an end-of-life product that has been inspected, repaired, if necessary, and prepared for resale. Through the literature review, the main blockers, trustworthiness, price, and quality, were identified as preventing consumers from purchasing used products. Trustworthiness could be ensured by evaluating used products in an automated and model-based manner. To enhance consumers' confidence, this study proposes a novel framework to assess the remaining value of non-new products by incorporating the diagnostic test results, even in the absence of a gold standard for model comparison and evaluation. This research expands the application of the Hui–Walter method beyond medical diagnostics by adapting it to sustainability-focused estimation. The proposed framework is designed to assist consumers in making data-informed purchase decisions and support retailers in assessing the market price while contributing to the environmental pillar of sustainability by reducing waste and resource consumption and extending the product lifetime. This work aligns with the United Nations Sustainable Development Goals 12 (Responsible Consumption and Production) and 13 (Climate Action) by providing quantifiable methods to extend the product lifecycle and minimize electronic waste. While this study focuses on developing the theoretical framework, future work will apply and validate this framework using empirical case studies and compare it with the remaining value estimation models.

Keywords: sustainability; refurbished products; remaining value; latent variables; Hui–Walter method; Bayesian approach; uncertainty estimation; machine learning; big data

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1. Introduction

Continuous interest in sustainability and consumers' positive attitude and willingness to purchase non-new electronic products have set requirements for model-informed solutions regarding remaining value estimation. Consumers express their interest in purchasing used products, followed by uncertainty related to the characteristics, trustworthiness, and risks of the used products. Advances in Artificial Intelligence have led to broader applications in different domains and disciplines.

Consumers often consider purchasing used or refurbished products. However, concerns regarding the quality of the product prevent consumers from making purchasing decisions. A primary factor contributing to this hesitation is a lack of awareness about the product's Remaining Useful Lifetime (RUL) and Remaining Value (RV). The Remaining Useful Lifetime (RUL) is defined as the duration that the product is expected to function as intended reliably. RUL estimation is usually applied in predictive maintenance or machinery management to estimate how long the product will function or predict the upcoming maintenance, deprecation process, and health index.

The Remaining Value (RV) extends the RUL concept by evaluating the economic value by estimating the product's monetary worth at a given point in time, reflecting both its physical condition and market factors.

Both metrics are inherently interconnected. RUL helps to evaluate the functional lifespan, while RV translates into an economical and measurable value that has an impact on consumer purchase decisions and sustainability outcomes.

Fundamentally, the evaluation of refurbished products is based on predictions under uncertainty. As a result, the models should be selected according to their ability to estimate uncertainty. For instance, Bayesian statistics or latent variables are selected to satisfy these requirements.

In addition, refurbished products align with the objectives of the United Nations Sustainable Development Goals (SDGs), particularly Responsible Consumption and Production (SDG12) and Climate Action (SDG13), by reducing the negative impact on the environment and minimizing the extraction of rare earth materials.

This research will review the primary motivators and detractors of buying used products. This is followed by a method review applied to the remaining value estimation and the Hui–Walter method adaptation for the remaining value estimation when the gold standard reference does not exist. Finally, the multistep framework is presented for the remaining value estimation. Ultimately, this research aims to fill the gap between consumer confidence and decision-making when purchasing used electronic products by providing a data-driven estimate.

2. Literature Review

The volume of electronic waste products is continuously increasing; in 2016, 53.6 million tons of e-waste was generated and only 17.4% of this was recycled; future predictions show that these numbers will continue to increase [1]. Consumers are aware of sustainability challenges and more frequently consider purchasing non-new products. There are multiple challenges related to perceived value, reliability, and uncertainty. Sustainability challenges are now being discussed and addressed; as a result, there will be many more applications in the future.

This section will review the definition of the different product processes used to extend the lifetime of products, the primary motivators and challenges raised by consumers when considering purchasing used products, the machine learning methods applied to solve remaining value and lifetime estimation problems, and the evolution of Hui–Walter method applications.

2.1. End-of-Life Product Refurbishment, Remanufacture, Reuse, and Recycling

Multiple definitions describe processes related to the usage of end-of-life products and their possible results: products can be refurbished, remanufactured, reused, and recycled. Each of these product states represents a different time and moment in the product life cycle and different problems related to the usage and value of the product.

Remanufacturing is defined as rebuilding a machine or subsystem by disassembling a product, replacing components, and reassembling, testing, and inspecting [2–4]. Reusing is the process of disassembling parts to reuse the parts of a product in new products. Recycling is the ultimate phase of the product when remanufacture and reuse are not applicable [3]. Refurbished products are inspected, repaired if necessary, and restored to working condition [5]. Refurbished products are usually not entirely disassembled, and some parts of the product might be replaced and tested. In the literature, refurbishment and remanufacturing are sometimes used interchangeably because the processes are similar. The difference between remanufacture and refurbishment is that the first should be delivered to the market and satisfy new product standards [6].

An increase in non-new product usage extends product lifecycles and significantly contributes to environmental improvement by requiring fewer resources to produce new products [2]. It is estimated that repairing and refurbishing products positively impacts Life Cycle Assessment (LCA) extension by extending the lifespan of devices. This extension helps to reduce global warming due to the increased acceptance of purchasing reconditioned products [7]. Furthermore, it was observed that refurbished products are more heterogeneous due to unknown usage patterns from previous users, but the goal after refurbishment is to meet defined standards.

Refurbishment significantly contributes to advancing all pillars of sustainability (environmental, economic, and social). Refurbished products help to reduce the waste and pollution associated with the manufacturing of new products, additionally lowering greenhouse gas emissions [8,9]. Furthermore, refurbishment supports circular economy models by extending the product lifecycle and efficient usage of resources, ensures that there are affordable products for society due to their lower price compared to new products, and raises awareness of sustainable consumption [10].

To summarize, refurbishment effectively prolongs a product's lifetime in a circular economy [7,11]. The refurbishment of smartphones is one way to extend smartphones' lifetime [12]. This is becoming a significant problem, as these items are frequently replaced, leading to a short product lifetime. This additionally brings environmental problems, as there is a demand to produce new products from raw materials. Moreover, the remaining value of refurbished products decreases with time; the pace of decrease depends on the quality and type of product [4].

2.2. Consumer Intention for Refurbished Electronics Products: A Comparative Analysis of Consumer Views and Concerns

Consumers' growing awareness of sustainability problems has prompted them to purchase used products. Consequently, researchers are analyzing the factors that best describe and shape consumer intentions when purchasing refurbished products, and are aiming to address their concerns and the risks associated with these products.

In a recent study, several factors that affect consumer decisions regarding the purchase of various refurbished electronics products were identified. The study concluded that purchase intentions are mainly influenced by the product's characteristics: category, perceived risk, and the knowledge that the product has been used before. As there is an increasing interest in the circular economy, more electronics will likely have to be refurbished due to consumer awareness [13].

A choice-based conjoint analysis revealed that consumers prioritize attributes such as product appearance, newness, as-new functionality, and replacement components that have direct personal use; this was instead of focusing on warranty or a reduced price in refurbished headphone cases [11].

A sentiment analysis of customers' reviews revealed different country-specific consumer perceptions of refurbished products [5]. Consumers share concerns regarding price, product quality, seller reliability, and warranty terms. Nevertheless, the importance of components varies by region, highlighting the importance of including the region in the price evaluation of the refurbished product price.

No single dominant feature consistently defines the value of refurbished products from a consumer perspective. One study demonstrated that customers' willingness to purchase refurbished products depends on the product category [14]. A systematic review concluded that price and quality perception are the most significant factors when considering purchasing refurbished products. Additionally, environmental awareness is becoming a more critical aspect as more value is added [15,16]. The statistical model indicated that the age of the product and the hardware conditions are essential factors for used laptops and mobile phones [17].

Despite an increase in the willingness of consumers to purchase used products due to financial and environmental benefits, there are still concerns related to unknown previous product contamination and usage patterns [11]. For example, when considering refurbished smartphones, consumers are more concerned about product-related improvements and battery life, as well as confidence that the product will continue to function after purchase [18]. Another study identified battery health, appearance, and functional product characteristics as the main concerns, while lower prices compared to brand-new items increased consumers' motivation to purchase used items [12]. Studies have indicated that remanufactured smartphones have higher price elasticity than new products while indicating that the volume of remanufactured products depends on the product's price [19]. Even though consumers advocate for repairability and memory size, the study showed that smartphone brands could significantly extend the economic lifespan of products, with some brands retaining value longer [20]. Research based on social media posts showed that the environmental aspect is slightly more important than the financial aspect. Additionally, research has highlighted the importance of characteristics such as the price, warranty, quality, and the seller's reputation [21].

Analyzing the sentiment of post-purchase reviews while applying natural language processing (NLP) models showed that consumers value the item's quality, which ideally should be evaluated not only by the user's sentiment but also by the product's actual performance [5,11].

Although most of the research focuses on the consumer side, a study highlighted the importance of the characteristics of the seller [22]. The analysis showed that the seller's reputation and the product's distribution simplify the consumer's decision to buy a used product. This allows customers to perceive the risk associated with the unknown product quality while considering the seller's credibility. The study highlighted that purchasing from online platforms often creates multilevel uncertainty and a lack of trust among consumers. This uncertainty is caused by the platform's trustworthiness and concerns related to the quality of the reconditioned products [23].

A case study based on the transactional data of refurbished smartphones from manufacturers identified that product age, condition, and thickness harm the remaining value of products. In contrast, software novelty, the screen size, and technical capabilities of the camera have the opposite effect [24]. Furthermore, manufacturer-refurbished smartphones were found to have a lower rate of value deprecation compared to third-party sellers. Smartphones generally have a short usage lifecycle, as newer models replace older products [12,24].

Price and warranty were identified as the most important motivators for purchasing refurbished smartphones [21]. Similarly, quality, cost savings, warranty, retailer reputation, and value-added services are key motivators for purchasing remanufactured bicycles [25]. Consumers also express their preferences for product evaluation from the perspective of sustainability while providing an eco-certificate [26].

The remaining useful lifetime and value estimation processes are thought to address several sustainability-related problems, such as lower resource consumption for product development. Due to reliable RUL or RUV estimation, consumers are more likely to feel confident about purchasing used products while minimizing the demand for new products and reducing the costs and resources required [22].

In summary, price was consistently identified as one of the top motivators for purchasing refurbished products. Additionally, what consumers are willing to pay correlated with the remaining value of the product, highlighting the factors of quality and reliability. Integrating multiple factors into the remaining value estimation helps consumers justify the price based on the quality and remaining functionality, and ensures trustworthiness.

2.3. Attempts to Assess Product History Traceability

Multiple researchers discussed the technology that best reflects consumers' need for complete information about the products used. In 2022, a digital lifecycle passport (DLP) concept was presented to overcome the problem of there being missing product lifecycle information [27]. DGP would reduce the environmental impact, provide a more accurate evaluation of the product's condition, value, and sustainability, and ensure transparency regarding the product's maintenance history, ownership, and repairs. It would also show the traceability of data, from the raw materials employed to the last use of the product. A recent study also covered the idea and requirements of a digital passport. The study showed that users would be interested in the usage and maintenance history of the product [28].

Transparency and trust are the main factors that prevent users from purchasing refurbished products. Blockchain-based product certification system technology was proposed to ensure refurbished products' trustworthiness and immutable product history [29]. The certification process eliminates the possibility of falsifying records and ensures transparent and traceable life cycle information for manufacturers and product owners. In general, certification might increase the willingness of the consumer to purchase refurbished products because of the trustworthiness certificate.

In 2023, a document presenting a scale for the grading of wireless products was published [30]. All the scales could be adjusted according to the business requirements. Additionally, the study suggested that the evaluation of refurbished electronics could benefit from a system similar to certified used car programs, which car manufacturers introduced in the previous century. The key difference between certified car programs and electronics is that vehicles are certified directly by the manufacturer. For reconditioned electronics, third-party testing companies typically handle the evaluation and certification process [23]. This underlines the importance of the standardized universal certification process and the need for transparency in the refurbished electronics process to build consumers' trust and reduce uncertainty. The grading and evaluation of the product by itself involves uncertainty due to software errors and product uniqueness [31].

In brief, consumers are more interested in purchasing used products, but risks related to the previous use and health of products are factors that prevent them from deciding to purchase used products. Certification allowing consumers to view the entire history of products would simplify the decision-making process.

2.4. Remaining Useful Lifetime and Value Estimation of Refurbished Products

Understanding the primary motivators for purchasing used products reveals how consumers make decisions. As previously discussed, the product's reliability, related to its remaining functionalities and quality, is one of the main reasons consumers fear purchasing used products. Estimating the RUL and RUV ensures that consumers can be confident of reliably using the product due to the RUL estimates. From the seller's side, the seller could provide a warranty because they are confident that the remaining lifetime is significantly longer than the warranty period.

In general, consumers lack confidence in used products, and the financial gain due to lower prices does not always outweigh the risks related to the unconfirmed quality of the product. As a result, estimating the remaining useful lifetime or value of a product in a timely manner would bring more confidence and fulfill consumers' expectations about the approved product quality.

The evaluation of refurbished products involves RUL and RUV estimation. RUL and RUV are usually analyzed in the literature as separate concepts by considering different product lifecycle stages. There is limited research on the combination of RUL and RUV estimations. Ideally, there would be a process that evaluates the remaining lifetime of the refurbished product and the highest remaining value at that time, which would ensure that products are sold when the RUL is sufficient and the RUV is not decreasing significantly. Furthermore, methods applied in the field of RUL and RUV determination by the domain and the required datasets will be presented with their limitations and potential improvements.

Multiple studies divide the RUL prediction methods into data-driven and model-driven approaches [32,33]. Traditionally, the model-driven strategy combines mathematical model definition, prior distribution assumptions, and data usage to obtain the posterior distribution and model prediction.

Generally, RUL estimation is focused on single-fault models, which could lead to lower accuracy compared to multi-fault models, which were recently proposed by improving the accuracy of the models [34].

2.5. The Role of Artificial Intelligence Methods for RUL and RUV Estimation

Artificial intelligence (AI) significantly contributes to the circular economy while solving real-world problems related to resource optimization and resource waste reduction. The circular economy is a sustainable alternative to the current linear economy and is widely discussed worldwide. A circular economy compensates for the drawbacks of the traditional linear economy by using resources multiple times. There is a lower amount of waste [13].

Deep learning (DL) methods are becoming more popular in RUL value prediction applications due to their ability to handle non-linear features and take into account uncertainty [35]. Uncertainty estimation is critical to evaluating the model's fit and helps prevent high costs due to the unpredicted failure of machinery processes.

Case study results have shown that Bayesian deep learning model applications are better than nonprobability component methods. The integration of uncertainty information improves model performance evaluation metrics [35]. Similarly, with epistemic and aleatoric uncertainty quantification and calibration in the RUL prediction process, these models provide a better estimate of product degradation and value over time. Additionally, it allows us to make data-informed decisions [36,37]. Furthermore, research has shown that integrating multisource data about expert knowledge, the product lifetime, and degradation helps improve the accuracy of RUL estimation. The application of the Bayes approach enabled the evaluation of the RUL and the estimation of model parameters, which could be updated in real time [38]. In multiple studies, the Wiener process was used to model the degradation process, and for the estimation of complex parameters, the Monte Carlo Markov Chain (MCMC) method was used [38,39].

Some studies have proposed creating online price recommendation systems for used cell phones. The primary goal of this platform is to collect various data and extract and prepare meaningful features from the pictures of the cell phone. This helps to identify the main features, buyers, and sellers, as well as the correct market price based on recent sales [40].

To mitigate risks and reduce financial loss, estimating the RUL with additional uncertainty estimation was suggested; this involved applying a bidirectional gated recurrent unit (BiGRU) network with the bootstrap method. The BiGRU method integrates forward and backward information flows, significantly improving model accuracy [41]. Furthermore, stochastic variables were more accurate in estimating the residual value of refurbished low-emission vehicles and outperformed other methods [42,43]. Remaining useful life (RUL) estimation is a significant problem that is analyzed for prognostic purposes to avoid unforeseen costs. Some authors proposed a multistep method that incorporates hybrid deep learning and a health indicator (HI) that incorporates equipment degradation to predict the RUL, taking into account the advantages of DL methods and a Convolutional Neural Network (CNN) to extract high-level spatial features and a Long Short-Term Memory (LSTM) neural network for handling long-term dependencies [32]. Another deep learning application proposed using Encoder Bi-directional Long Short-Team Memory Networks (Bi-LSTM) and Convolutional Neural Networks (CNNs) [44].

In addition, Bayesian Networks are widely used to predict RUL values. The Bayesian approach ensures that all prior information and data are integrated into the model. It is able to ingest additional values and parameters, and the model adapts to the new conditions [22]. This method continuously learns from newly ingested data, ensuring that the model performs well and adapts to market requirements.

A recent case study on the market price evaluation of remanufactured laptops applied a multistep additive model approach for price estimation. Multiple machine learning methods were applied to help set the price of used laptops. For the laptop value estimation, it was suggested that a brand-new product price be used as a proxy, and component evaluation was performed with the publicly available data for each new component [2].

In a different field, machine learning methods were applied to estimate the residual value of heavy construction equipment. Multiple ML models were compared, including Modified Decision Tree (MDT), LightGBM, and XGBoost. Here, the MDT stands for the modified decision tree structure, optimized for residual value prediction [45].

Other studies have assessed the application of the Gradient Boosted Trees method with the stacking ensemble method, which outperformed MLP, SVR, and CNN methods and significantly improved the accuracy of RUL prediction [46,47]. Another study showed that the Long Short-Term Memory (LSTM) neural network significantly improves RUL prediction compared to ML models, as LSTM can handle multiple sensor signals and remember long-term information periods [48]. A systematic literature review revealed that machine learning methods such as support vector machine (SVM), Random Forest (RF), Artificial neural networks (ANNs), deep learning, and k-means are often applied for predictive maintenance estimation processes [49].

There are special cases related to machinery that remain useful in estimating the useful lifetime of products and related challenges. For predictive maintenance, sensor data are usually analyzed; as a result, the time dependence component is of higher importance for estimation.

Moreover, the remaining value or lifetime of products is usually analyzed in parallel with the degradation process to compare values over time. For example, an integrated degradation modeling framework that considers model uncertainty and calibration and overcomes limitations related to data distribution and stochastic processes was proposed [50]. The health indicator is set to be an essential prerequisite for the evaluation of the degradation process. The unsupervised feature learning-based health indicator construction method is introduced to address multiple challenges, including manual feature selection, unlabeled data, and accurate RUL estimation. As a result, a multiple-step method is constructed; this first applies a multiscale convolutional autoencoder network that automatically extracts feature information from the data, then applies it to the data and evaluates the similarity to estimate health indicators [51].

In addition, computer vision could enrich the remaining value estimation process by incorporating information from images of products. The automatically detected product appearance evaluation results could be used as input for the other models while estimating the remaining value or life expectancy of the product. AI-enhanced systems significantly improve the efficiency of logistic operations, reduce error rates, and improve the speed and accuracy of product inspection [52].

Model evaluation and comparison are essential steps in the model implementation process. The literature review in Table 1 shows that root mean squared error (RMSE) and mean absolute error (MAE) metrics are commonly calculated to evaluate the prediction performance of RUL/RV models. As RUL and RUV estimation could be defined as a regression or time series problem, metrics are usually selected depending on this fact.

Table 1. Models' evaluation and comparison metrics.

Models' Evaluation Metric	Authors
MAE	[32,34–36,53]
RMSE	[32,34–36,41,44,47,54]
MAPE	[32,36,41]
Score	[36,44]

2.6. RUL and RUV Estimation with Unlabeled Data

The lack of sufficient and reliable manufacturing data has been identified as an essential challenge in RUL and RUV prediction [2]. As a result, there are additional requirements for RUL and RUV estimation models, as they need to overcome the lack of labeled data. Moreover, the quality and value of non-new products are unique, bringing additional inconvenience and uncertainty. Multiple possible scenarios were analyzed to address situations where data are missing or incomplete in the RUL and RUV estimation process.

To mitigate this problem, an Average Thresholded Confidence (ATC) method was proposed to minimize the impact of unlabeled data samples. A research gap was identified when machine learning methods failed on out-of-distribution (OOD) test data when there was a significant difference between the training and testing data, resulting in worse model performance. This ATC method allows one to evaluate the confidence of the model in model predictions by learning the confidence threshold from the model. Accuracy is estimated as the proportion of test examples for which the model confidence is higher than the threshold. The analysis showed that the ATC method is accurate with publicly available data sets and generated data samples [55].

Moreover, one possible solution to the problem of data not existing is the use of alternative data sources to evaluate the performance of ML models. Research has shown that the 3S testing framework (synthetic data for subgroup and shift testing) enriches the model evaluation process by ensuring that the model is evaluated in more scenarios and test cases [53].

Additionally, studies aiming to tackle the problems related to missing or scarce data have been conducted. One study presented an unsupervised method that constructs a health index that reflects the degradation process, allowing us to estimate the RUL and value accurately. Similarity-based models could be adjusted and applied to solve different problems, eliminating the labeled data requirements [56,57].

Another alternative is to divide the RUL estimation process into multiple stages to avoid human involvement in the process and use it as an unsupervised learning technique. This could be solved by constructing a health indicator (HI). Lastly, a convolutional network with a bidirectional long short-term memory (BiLSTM) network can be used to connect the HI with RUL values [54].

Transfer learning could be employed when the data in RUL predictions are incomplete. This is a common case in real practical scenarios. Transfer learning extracts data from the source dataset and transfers missing information to the incomplete target dataset. This enables one to mitigate the risk of there being missing data for RUL prediction. Some issues relate to the distribution difference between source and target data. As a result, adaptation techniques are used by applying the Doman Adversarial neural network (DANN) architecture with regularization [58].

Lastly, multiple semi-supervised learning methods and techniques can potentially solve missing data problems. One case study showed that no single method would best fulfill the requirements of the problem and fill in the missing values. There are multiple possible techniques depending on the situation. Appropriately, models should be selected individually based on the requirements and data characteristics [59].

Evaluation with no gold standard requires additional effort and brings some additional benefits [60]. Data labeling is a time-consuming and expensive process; as a result, there is an increasing need to overcome problems without investing in data labeling. Additionally, using data with approximate or imperfect gold standards might not ensure the reliability of the results.

The Hui–Walter method is a potential alternative for RUV estimation, allowing the handling of uncertainty estimation and latent variables when the direct evaluation of the product value is difficult. Initially, the Hui–Walter method was applied in medicine-related fields, but it could be expanded to informatics and the evaluation of sustainability problems.

Estimating the RUL and RV of refurbished products is a complex and multilevel process. Machine learning, deep learning, and Bayesian methods ensure that the models perform well and take uncertainty quantification into account. The potential application of the Hui–Walter method to evaluate used products is novel and necessary in the used products industry. The Hui–Walter method helps to estimate product uncertainty and values without having a gold standard.

2.7. Evolution and Application of the Hui Walter Method in the Absence of the Gold Standard

The Hui–Walter method was introduced to estimate the sensitivity and specificity of diagnostic tests without a gold standard to overcome the problem of the true disease status being unknown and there being no perfect reference test. In this section, the evolution of the Hui–Walter method will first be reviewed by highlighting its most critical challenges and differences compared to the classical Hui–Walter method and current applications of this method in different domains.

The Hui–Walter method was created to overcome situations with no gold standard. The gold standard concept is widely applied in the field of medicine. The tests should be compared with the known test value. In medicine, the gold standard is usually unavailable due to costs, the absence of technology, or ethical reasons related to patient health; as a result, latent class models are usually used to avoid and overcome all of the previously mentioned factors [61]. Virtual reference testing is a different way to evaluate the performance of the test when there is no perfect test [62]. The virtual reference is generated as a proxy for the gold standard and is derived by aggregating the performance of multiple tests. The gold standard in other subjects or domains refers to a high-quality labeled dataset used as a benchmark for evaluating other models.

To better understand the Hui–Walter method, it is essential to define the main components used to evaluate the tests: sensitivity, specificity, and prevalence. Prevalence is a value that ranges from zero to one and shows the percentage of the population with disease or infections. In the non-medical application of the Hui–Walter method, prevalence shows the percentage of products with a specific feature value [63]. Sensitivity refers to the test's ability to correctly identify disease in patients when they have it, while specificity evaluates whether non-disease cases are correctly identified.

Initially, the Hui–Walter method was introduced in 1980 to evaluate diagnostic tests without an actual value for comparison [64]. The key assumptions of the initial model are that the characteristics of two tests might be evaluated if multiple measures for the same individual have different prevalence rates and if the tests are conditionally independent [65]. Due to ethical reasons, the presence of disease cannot be validated either by intervention or using a test etalon if it does not exist or the actual disease status is unknown.

In the original paper, the Hui–Walter method was developed with multiple assumptions. Each individual undergoes multiple tests from different S populations, called Test 1 and Test 2. The characteristics of the tests are evaluated by applying the maximum likelihood method.

The Hui–Walter method is commonly applied, and the results are typically analyzed and presented in a binary table that summarizes the actual versus predicted test outcomes. The key elements of the Hui–Walter method include sensitivity (S_e), specificity (S_p), and prevalence π.

By following the Hui–Walter method, sensitivity and specificity metrics are the key indicators of the test's accuracy.

Sensitivity—the ability to correctly identify true positives, such as people with specific diseases.

$$S_e = \frac{TP}{TP + FN} \tag{1}$$

Specificity—the ability to correctly distinguish true negatives, such as people who do not have disease.

$$S_p = \frac{TN}{TN + FP} \tag{2}$$

In the case of the two tests in the T_1 and T_2 scenario, each test has its own sensitivity and specificity values: S_{e1}, S_{p1} for T_1 and S_{e2}, S_{p2} for T_2. For the test cases covered, Test T1 and T2 are positive:

$$P(T_1 = 1, T_2 = 1) = \pi Se_1 Se_2 + (1 - \pi)(1 - Sp_1)(1 - Sp_2) \tag{3}$$

Positive test T1 and negative T2:

$$P(T_1 = 1, T_2 = 0) = \pi Se_1 (1 - Se_2) + (1 - \pi)(1 - Sp_1) Sp_2 \tag{4}$$

Negative test T1 and positive T2:

$$P(T_1 = 0, T_2 = 1) = \pi(1 - Se_1) Se_2 + (1 - \pi) Sp_1 (1 - Sp_2) \tag{5}$$

Both tests are negative:

$$P(T_1 = 0, T_2 = 0) = \pi(1 - Se_1)(1 - Se_2) + (1 - \pi) Sp_1 Sp_2 \tag{6}$$

Likelihood function is constructed for the population:

$$L(\theta) = \prod_{i=1}^{n} P(T_{i1}, T_{2i} | \theta) \tag{7}$$

where n defines the number of individuals: T_{i1}, T_{2i}—tests results for individual I; θ—parameters S_{e1}, S_{p1}, S_{e2}, S_{p2}, π.

In the case of k tests, the probabilities are estimated as follows:

$$P(T_1, T_2, \ldots, T_k | D) = \prod_{i=1}^{k} P(T_j | D) \quad (8)$$

With an increasing number of tests, there are challenges related to identifiability. The likelihood function is constructed as follows:

$$L(\theta) = \prod_{i=1}^{n} P(T_{1i}, T_{2i,}, \ldots, T_{ki,}) \quad (9)$$

The initial method is based on latent class analysis (LCA), assuming that the disease's true state is an unknown latent variable. The method uses results from multiple tests to estimate the disease prevalence, test sensitivity, and specificity.

Researchers have attempted to expand the Hui–Walter method by introducing random effects into the latent class model to model the conditional dependence within tests. The incorporation of random effects into the latent class model takes into account the conditional dependence between tests. This ensures flexibility when the conditional independence assumption is violated. Conditional dependence might be identified due to the presence similarity among responses or multiple experts or tests relying on the subject-specific characteristics [66]. In the medical sphere, two tests might be dependent if the tests were based on the same biomarker. In refurbished product applications, tests could depend on test results based on the same factors.

The classical Hui–Walter method was extended by introducing the Bayesian estimation approach to evaluate test parameters' sensitivity, specificity, and prevalence. Bayesian methods allow one to incorporate prior knowledge and uncertainty into the model, which might be helpful in the absence of a gold standard. All posterior distributions are derived by applying the Gibbs sampler technique [67].

Later, in 2001, one study discovered that multiple populations, regardless of sample size, are insufficient to detect test metrics. Therefore, multiple populations with different prevalence levels should be selected [68]. Furthermore, Walter presented a Rule of Three concept that ensures model identifiability with latent class models and diagnostic tests. The main idea of this concept is that three observations or tests are required to ensure the identifiability of the method. Models are identifiable if all parameters can be estimated without considering additional assumptions.

The Hui–Walter paradigm was revisited in 2005, reconfirming the importance of the conditional independence assumption for estimating test statistics. The study showed that adding more tests or populations would not ensure conditional dependence. The experiments showed that minor differences between populations' prevalence are significant, as the lower the difference, the lower the precision of estimates. Lastly, Bayesian methods were recommended to address the poor precision, but prior distribution selection is essential to ensure unbiased results [65].

An extended version of the Hui–Walter method was applied by incorporating disease surveillance data into the model [69]. The results showed that the model's performance characteristics were improved by introducing surveillance data and allowing missing values. In the classical Hui–Walter method, missing values among diagnostic test values are not allowed.

In one of the latest applications of the Hui–Walter method, a comparison of the classical Bayesian Hui–Walter method and Bayesian logistic regression (LR) showed that the performance of the LR model is better when applied to conditionally independent tests and that the posterior estimates are more accurate [70]. Compared to the classical

HW model, the LR model includes multilevel data and individual values in the estimation process. The authors highlighted that the estimation of test characteristics is usually challenging to evaluate in real-world situations due to the unknown true prevalence of the disease. According to veterinary epidemiology processes, the mean of the posterior median and the 95% credibility interval have been calculated in practice.

There are multiple approaches to evaluating the results of tests solved with frequentist or Bayesian approaches. The main difference between these approaches is that the frequentist approach is easy to apply but usually lacks satisfying assumptions of normality or small data samples. The Bayesian approach enables interval prevalence calculations; expert or historical information is included in the estimation process [63].

The conditional independence assumption remains the central assumption that is violated in real-world problems, especially when there is a lack of a gold standard. Multiple authors admit that conditional independence assumptions remain problematic when applying methods to practical cases. Violating the conditional independence assumption leads to unreliable and biased sensitivity and specificity (parameters of uncertainty) estimates and wider confidence intervals.

The authors proposed a Bayesian approach to address conditional dependence issues when evaluating diagnostic test characteristics. The method assumes that the true disease status is a latent variable. The results and methodology depend on whether a priori information about the correlation between correlation tests is known beforehand. If a priori information about correlation is known with high precision, then the method adjusts to the more accurate estimate of the test prevalence and test characteristics. When the information about correlation is unknown, the method produces a wider uncertainty interval, reflecting the higher uncertainty of the test performance estimates [71].

When applying the Bayesian approach, setting the prior distribution based on historical data is important. The beta distribution is usually selected for the estimation of model parameters: prevalence, sensitivity, and sensitivity; this is due to the possible values ranging from 0 to 1 and the adaptability of the distribution to the actual values [72].

The simulations-based study demonstrates that integrating a conditional dependence assumption into Latent Class models is important for accurate sensitivity and specificity estimation. When conditional dependence exists and is assumed differently, then sensitivity and specificity estimates are biased [73]. Furthermore, the study highlighted that the violence of conditional independence will result in an identifiable solution. Therefore, the conditional independence assumption could not be violated [65].

The latest Hui–Walter method applications usually do not satisfy the conditional independence assumption. This issue is solved by including random effects or modeling the covariance between test results.

Although the Hui–Walter method was created to evaluate diagnostic test results in medicine, its application could be expanded to other disciplines and be unlimited. The ability to handle imperfect tests and estimate latent variables without a gold standard is valuable for industries like manufacturing. The latest applications of the Hui–Walter method include the Bayesian method, which enables the tackling of complex data and uncertainty estimation with confidence intervals.

Furthermore, a framework to evaluate non-new products' remaining value will be presented while applying concepts related to the Hui–Walter method.

3. A Framework for Estimating the Remaining Value of Refurbished Products by Applying the Hui–Walter Method

The ideas of the Hui–Walter method could be expanded and applied to other disciplines by employing the data collected from the diagnostic tests in the remaining value assessment process. There is a similarity between the application of the Hui–Walter method in medicine and refurbished products. Refurbished products, by definition, are unique, and remaining value estimation is a complex problem that cannot be unambiguously defined. Therefore, the Hui–Walter method could be expanded and adapted to the remaining value estimation of non-new products. The remaining value and quality of the refurbished product are evaluated by considering uncertainty. Uncertainty is present in both diagnostic tests and refurbished product evaluation due to the absence of a gold standard.

The Hui–Walter method is applied in medical diagnostics to assess the test performance when there is no gold standard, and it is crucial to evaluate the test's sensitivity, specificity, and prevalence. Similarly, there is no gold/reference standard in evaluating refurbished products, and the Hui–Walter method helps estimate the remaining value based on multiple data sources, such as battery diagnostic tests. As in medicine diagnostics, for remaining value estimation, at least two datasets of test or performance data should be collected. The application principles of the Hui–Walter method ensure more comprehensive refurbished product evaluation by integrating multiple data sources into the estimation process. As in diagnostic test settings for testing parameter estimation, the Bayesian approach is usually selected to handle complex posterior distributions. Bayesian methods could be applied to integrate prior usage knowledge into the value assessment for the remaining value estimation.

The test result values are the main difference between applying the Hui–Walter method in medical diagnostics and refurbished products. In the medical test setting, diagnostic tests usually have a binary value (positive/negative—there is or there is no disease), while in the evaluation of refurbished products, the results of the tests are continuous or ordinal (i.e., A, C). As a result, the remaining value estimation with the Hui–Walter method requires more complex calculations than the originally applied Hui–Walter method.

In a non-binary test setting, latent class variables represent predefined product conditions, such as the remaining value of a refurbished product. This latent class variable is used instead of the binary values originally used for the disease or no disease factor.

Let X denote the latent variable that represents the true remaining value of the product. $T_1, T_2, \ldots, T_i$ defines the test results T of the product *i*.

For each test T_k, where k = 1, …, i is expressed as a linear function of the latent remaining value and noise ε:

$$T_k = \lambda_k Z + \varepsilon_k \tag{10}$$

where λ_k is a loading factor that helps to combine test results k with the latent variable Z. $\varepsilon_k \sim N\left(0,\ \sigma_k^2\right)$ is a random error for the test k, where it is assumed that the error is distributed as a normal distribution with the zero mean and σ_k^2 variance.

Then, likelihood can be expressed by the following formula:

$$P(T_k|Z) = N\left(T_k\middle|\lambda_k Z,\ \sigma_k^2\right) \tag{11}$$

In the case of multiple tests, the joint likelihood is defined.

$$P(T_1,\ T_2,\ \ldots,\ T_i|Z) = \prod_{k=1}^{i} N\left(T_k\middle|\lambda_k Z,\ \sigma_k^2\right) \tag{12}$$

where $P(T_1,\ T_2,\ \ldots,\ T_i|Z)$ represents the joined probability, which is used to evaluate the test results given the true remaining value expressed as latent variable Z. The joined likelihood assumes that the conditional independence assumption of tests is not violated.

The posterior distribution of latent variable Z and parameters are calculated through the Bayes theorem.

$$P\left(Z\lambda_k,\ \sigma_k^2\middle|T_1,\ T_2,\ \ldots,\ T_i\right) \propto P(T_1,\ T_2,\ \ldots,\ T_i|Z)P(Z)\,P\left(\lambda_k,\ \sigma_k^2\right) \quad (13)$$

where $P\left(Z\lambda_k,\ \sigma_k^2\middle|T_1,\ T_2,\ \ldots,\ T_i\right)$ denotes the posterior distribution, $P(T_1,\ T_2,\ \ldots,\ T_i|Z)$ denotes the likelihood of estimating latent variable Z and parameters, P(Z) is the prior distribution of the latent variable Z, and $P\left(\lambda_k,\ \sigma_k^2\right)$ is the prior distribution of the parameters.

Estimating the posterior distribution, latent variable Z, and parameters used in the MCMC technique with Gibbs sampling allows the evaluation of posterior mean and credibility intervals, which defines the uncertainty of results.

$$\hat{RV} = E[Z|T_1,\ T_2,\ \ldots,\ T_i] \quad (14)$$

where $\hat{RV}$ is the estimated remaining values of the product, given the latent variables and test results.

The Hui–Walter problems formed are solved using either frequentist or Bayesian statistics. The application of MCMC enables uncertainty quantification because of the ensured posterior distribution of identified parameters through single-point estimates collected from likelihood functions. Additionally, the formulation of credibility intervals ensures that the decision-maker has the information to make decisions.

The application of the Bayesian approach enabled uncertainty estimation in the assessment of the remaining value of refurbished products. Multiple sources of uncertainty are related to reconditioned products, such as product uniqueness, passed usage history, and product characteristics. Diagnostic tests might have measurement errors. The Bayesian method helps to address uncertainties related to refurbished products. Firstly, the remaining product value is handled as a latent variable instead of the actual value. Uncertainty related to test results is handled as probabilistic, ensuring the integration of measurement errors into the RV estimation process. The Bayesian method integration enables the evaluation of remaining value and credibility intervals without a gold standard for refurbished products.

This paper presents an adapted and extended Hui–Walter method framework (Figure 1) that enables the estimation of the remaining useful lifetime for refurbished products. The framework consists of several key steps: data preprocessing and collection, the formation of latent variables, defined by Formula (10), the computation of the test's characteristics based on the adapted Hui–Walter method (11–13), and the evaluation (14) and comparison of results.

Firstly, for effective model development, at least two tests must be collected to represent the characteristics of the refurbished products. According to the original Hui–Walter method, these tests should ideally be conditionally independent to ensure that the evaluation of the tests remains unbiased. Unlike the original Hui–Walter method, the result of diagnostic tests of refurbished products has continuous values, such as battery health. As a result, the Hui–Walter method could not be directly applied to the collected data. Continuous test results are transformed into latent variables.

Constructing latent variables is one of the main requirements for model development. These variables enrich the model with product attributes like lifetime, battery, or product specification. Latent variables are evaluated through the Bayesian estimation by applying the Markov Chain Monte Carlo (MCMC) method.

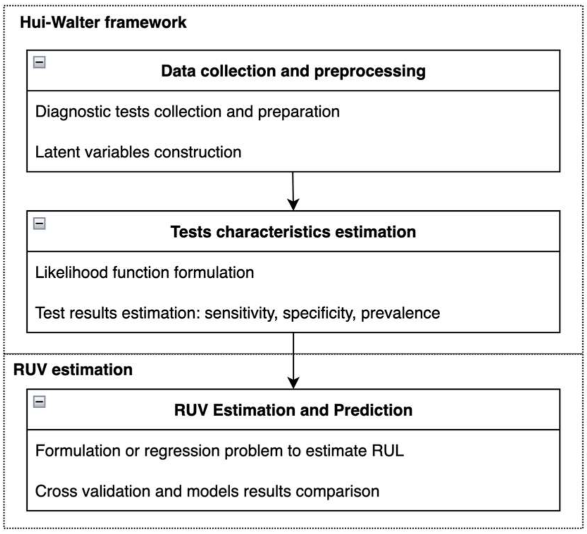

Figure 1. Refurbished products' remaining value estimation framework.

Further, for the evaluation of the results of the tests, a likelihood function is constructed to estimate the test parameters by applying Bayesian statistics. It also incorporates prior knowledge about the product history and adapts based on newly ingested data.

The final stage involves a regression task to evaluate the remaining product value by applying cross-validation to select hyperparameters and estimate model accuracy metrics.

To emphasize the methodological contribution of the proposed framework, we compare multiple methods that are commonly used for remaining value estimation under the uncertainty of refurbished products. Table 2 provides a structured comparison of several methods across multiple dimensions, including requirements for a gold standard or labeled data, the ability to handle and evaluate uncertainty, latent variables that uncover and integrate hidden information, adaptability, and alignment with sustainability objectives.

Table 2. Comparison of X and the proposed Hui–Walter and Bayesian approach for remaining value estimation under uncertainty.

Method	Gold Standard or Labeled Data	Uncertainty Handling	Latent Variables	Notes
Expert based	no	low	no	Subjective, lacks reproducibility
Rule-based	no	low	no	Lacks reproducibility, not adaptive
Machine Learning Regression	labeled data	medium	no	Requires labeled dataset
Bayesian Latent Class Model	no	high	yes	Generic, lacks the adaptability to sustainability and economic value
Original Hui–Walter Model	no	medium	yes	Designed for medical diagnostic tests
Proposed Hui–Walter Method Bayesian Approach Framework	no	high	yes	Adapted to sustainability and refurbished product evaluation

Expert-based or rule-based methods are simple but lack reproducibility and adaptability to changing conditions. Machine learning methods require labeled data, not covering uncertainty estimation. The Bayesian Latent Class Model deals with the lack of a gold standard and allows an evaluation of uncertainty and the limitations related to variable types. The original Hui–Walter method was not presented for non-medical applications and was designed for categorical values.

In contrast, the proposed Hui–Walter method using the Bayesian approach addresses these limitations by non-requiring data labeling and evaluates uncertainty and latent variables to reflect performance characteristics, formed for sustainability problem-solving. These features ensure that the proposed framework simplifies the remaining value estimation of refurbished products.

This framework's novelty lies in applying the Hui–Walter method in the non-medical field to remaining value estimation during second-hand product valuation. Additionally, it expands the application of the original Hui–Walter method by incorporating continuous diagnostic test data into the estimation process. The formation of latent class variables was selected to address the challenge of incorporating non-binary data into the framework. The inclusion of Bayesian methods and the MCMC technique enables remaining value estimation by additionally providing confidence intervals and enabling consumers to make more data-informed and interpretable decisions about remaining value trustworthiness.

4. Discussion

This research aimed to identify the primary barriers and concerns, as analyzed by multiple researchers, that prevent consumers from purchasing refurbished products. Additionally, it intended to fill the research gap by identifying a suitable method for remaining value estimation when a gold standard or reference does not exist for estimation. Based on these aims, the literature and integration of the Hui–Walter method into the remaining value estimation framework were analyzed.

The literature review revealed that machine learning and deep learning methods are usually applied to estimate the remaining value or lifetime of a product by incorporating historical data. However, this research uses diagnostic test results to present a novel remaining value estimation framework for refurbished products without a gold standard. Adapting the Hui–Walter method for remaining value estimation expands the applicability of this method and introduces a novel approach to the estimation process.

With the increasing integration of sensor data and the Internet of Things (IoT), it will be possible to make more accurate data-informed decisions in the remaining value estimation process because of the availability of data concerning device usage and maintenance history. However, it must ensure compliance with ethical standards, which ensure trustworthy and reliable solutions.

To ensure the adaptability of the framework, it is essential to develop a database system that integrates different data sources such as product specification, market demand, product description, and others into a single system. This would ensure that the Hui–Walter method operates with the most recent data sources.

Nonetheless, the proposed framework has multiple limitations. According to the assumptions of the Hui–Walter method, at least two diagnostic test results should be provided to evaluate the test characteristics and incorporate this information into the remaining value assessment process. The integration of diagnostic test results requires a different approach during the data preprocessing step while creating additional requirements for the datasets. On the other hand, integrating diagnostic test results allows for a more accurate evaluation of the remaining value and enables one to make more informed and data-based decisions. Nevertheless, combining Bayes and MCMC requires additional computational resources to ensure that RV estimates are closer to the true analytical solution.

The proposed remaining value estimation framework, which integrates the Hui–Walter method, is not limited to a single domain. Its application is widely adaptable, with the most prominent application fields being electronic appliances, such as smartphones, smartwatches, computers, and others.

Furthermore, the proposed framework presents an interdisciplinary collaboration between statisticians, circular economy practitioners, and consumers to include all aspects of the solution and challenges. The Hui–Walter method with the Bayesian approach empowers small enterprises to offer data-driven solutions while small enterprises are usually limited to labeled or unreliable data. As a result, the framework simplifies the RV estimation offering a practical and scalable solution with limited resources or expertise.

From a sustainability perspective, this framework enhances decision-making by enabling the quantification of uncertainty related to product value. The framework contributes to sustainable development and particularly aligns with Sustainable Development Goals SDG12 and SDG13 by promoting the responsible use of resources, increasing consumers' awareness and knowledge, and reducing our environmental impact.

Future research will focus on applying the Hui–Walter method for the estimation of multiple electronic products' remaining value and comparing the results with other RV methods. Additionally, this method could be adjusted and expanded by incorporating expert knowledge, allowing prior knowledge of the evaluation process.

5. Conclusions

In summary, increasing interest in the sustainability and reuse of electronic products has created opportunities to develop methods that simplify consumer decision-making in the second-hand market. Through a comprehensive review, we identified motivators and concerns related to the willingness of consumers to purchase used electronics products. A lack of trust and the price, reliability, and quality of products were the main factors that prevented consumers from making a decision. These concerns directly align with the sustainability challenges addressed by SDG 12 (Responsible Consumption and Production) and SDG 13 (Climate Action). Increasing consumer interest and the achievements of machine learning and statistical methods create opportunities for the development of simplified and data-driven approaches to evaluating the Remaining Useful Lifetime (RUL) and Remaining Value (RV) of refurbished products. The proposed framework extends the application of the Hui–Walter method, first used in medicine diagnostics, to sustainability and used electronics. This framework enables RV estimation, even without a gold standard or labeled data, enhances transparency, promotes more informed decisions, and helps to reduce waste. Since used products are unique due to unknown usage patterns, the long-term vision is to adapt the digitalized collected history of used electronics to simplify decision-making. The next step of this research is to apply this proposed framework to the selected data source and compare the results with the other methods used for remaining value estimation.

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References

1. Baldé, C.P.; Forti, V.; Gray, V.; Kuehr, R.; Stegmann, P. *The Global E-Waste Monitor–2017*; United Nations University: Shibuya, Tokyo; International Telecommunication Union: Genevan, Switzerland; International Solid Waste Association: Rotterdam, The Netherlands, 2017.
2. Turkolmez, G.B.; El Hathat, Z.; Subramanian, N.; Kuppusamy, S.; Sreedharan, V.R. Machine Learning Algorithms for Pricing End-of-Life Remanufactured Laptops. *Inf. Syst. Front.* **2024**, 1–19. [CrossRef]
3. Seaver, W.B. Design considerations for remanufacturability, recyclability and reusability of user interface modules. In Proceedings of the 1994 IEEE International Symposium on Electronics and the Environment, San Francisco, CA, USA, 2–4 May 1994; pp. 241–245. [CrossRef]
4. Schepler, X.; Absi, N.; Jeanjean, A. Refurbishment and Remanufacturing Planning Model for Pre-Owned Consumer Electronics. *Int. J. Prod. Res.* **2024**, *62*, 2499–2521. [CrossRef]
5. Seifian, A.; Shokouhyar, S.; Bahrami, M. Exploring Customers' Purchasing Behavior toward Refurbished Mobile Phones: A Cross-Cultural Opinion Mining of Amazon Reviews. *Environ. Dev. Sustain.* **2023**, *26*, 28131–28159. [CrossRef]
6. Chen, Y.; Chen, F. On the Competition between Two Modes of Product Recovery: Remanufacturing and Refurbishing. *Prod. Oper. Manag.* **2019**, *28*, 2983–3001. [CrossRef]
7. Pamminger, R.; Glaser, S.; Wimmer, W. Modelling of Different Circular End-of-Use Scenarios for Smartphones. *Int. J. Life Cycle Assess.* **2021**, *26*, 470–482. [CrossRef]
8. Bridgens, B.; Hobson, K.; Lilley, D.; Lee, J.; Scott, J.L.; Wilson, G.T. Closing the Loop on E-waste: A Multidisciplinary Perspective. *J. Ind. Ecol.* **2019**, *23*, 169–181. [CrossRef]
9. Zink, T.; Maker, F.; Geyer, R.; Amirtharajah, R.; Akella, V. Comparative Life Cycle Assessment of Smartphone Reuse: Repurposing vs. Refurbishment. *Int. J. Life Cycle Assess.* **2014**, *19*, 1099–1109. [CrossRef]
10. Kirchherr, J.; Reike, D.; Hekkert, M. Conceptualizing the Circular Economy: An Analysis of 114 Definitions. *Resour. Conserv. Recycl.* **2017**, *127*, 221–232. [CrossRef]
11. Wallner, T.S.; Magnier, L.; Mugge, R. Do Consumers Mind Contamination by Previous Users? A Choice-Based Conjoint Analysis to Explore Strategies That Improve Consumers' Choice for Refurbished Products. *Resour. Conserv. Recycl.* **2022**, *177*, 105998. [CrossRef]
12. Nasiri, M.S.; Shokouhyar, S. Actual Consumers' Response to Purchase Refurbished Smartphones: Exploring Perceived Value from Product Reviews in Online Retailing. *J. Retail. Consum. Serv.* **2021**, *62*, 102652. [CrossRef]
13. Wallner, T.S.; Haslbeck, J.M.B.; Magnier, L.; Mugge, R. A Network Analysis of Factors Influencing the Purchase Intentions for Refurbished Electronics. *Sustain. Prod. Consum.* **2024**, *46*, 617–628. [CrossRef]
14. Hamzaoui-Essoussi, L.; Linton, J.D. Offering Branded Remanufactured/Recycled Products: At What Price? *J. Remanufactur.* **2014**, *4*, 9. [CrossRef]
15. Singhal, D.; Jena, S.K.; Tripathy, S. Factors Influencing the Purchase Intention of Consumers towards Remanufactured Products: A Systematic Review and Meta-Analysis. *Int. J. Prod. Res.* **2019**, *57*, 7289–7299. [CrossRef]
16. Bigliardi, B.; Filippelli, S.; Quinto, I. Environmentally-Conscious Behaviours in the Circular Economy. An Analysis of Consumers' Green Purchase Intentions for Refurbished Smartphones. *J. Clean. Prod.* **2022**, *378*, 134379. [CrossRef]
17. Kwak, M.; Kim, H.; Thurston, D. Formulating Second-Hand Market Value as a Function of Product Specifications, Age, and Conditions. *J. Mech. Des.* **2012**, *134*, 032001. [CrossRef]
18. Mugge, R.; Jockin, B.; Bocken, N. How to Sell Refurbished Smartphones? An Investigation of Different Customer Groups and Appropriate Incentives. *J. Clean. Prod.* **2017**, *147*, 284–296. [CrossRef]
19. Phantratanamongkol, S.; Casalin, F.; Pang, G.; Sanderson, J. The Price-Volume Relationship for New and Remanufactured Smartphones. *Int. J. Prod. Econ.* **2018**, *199*, 78–94. [CrossRef]
20. Makov, T.; Fishman, T.; Chertow, M.R.; Blass, V. What Affects the Secondhand Value of Smartphones: Evidence from eBay. *J. Ind. Ecol.* **2019**, *23*, 549–559. [CrossRef]
21. Sharifi, Z.; Shokouhyar, S. Promoting Consumer's Attitude toward Refurbished Mobile Phones: A Social Media Analytics Approach. *Resour. Conserv. Recycl.* **2021**, *167*, 105398. [CrossRef]
22. Hostens, E.; Eryilmaz, K.; Vangilbergen, M.; Ooijevaar, T. Bayesian Networks for Remaining Useful Life Prediction. *PHM Soc. Eur. Conf.* **2024**, *8*, 11. [CrossRef]
23. Lee, S.M.; Lee, S.J. Consumers' Initial Trust toward Second-Hand Products in the Electronic Market. *J. Comput. Inf. Syst.* **2005**, *46*, 85–98. [CrossRef]
24. Zhou, L.; Gupta, S.M. Value Depreciation Factors for New and Remanufactured High-Technology Products: A Case Study on iPhones and iPads. *Int. J. Prod. Res.* **2020**, *58*, 7218–7249. [CrossRef]
25. Vafadarnikjoo, A.; Mishra, N.; Govindan, K.; Chalvatzis, K. Assessment of Consumers' Motivations to Purchase a Remanufactured Product by Applying Fuzzy Delphi Method and Single Valued Neutrosophic Sets. *J. Clean. Prod.* **2018**, *196*, 230–244. [CrossRef]

26. Harms, R.; Linton, J.D. Willingness to Pay for Eco-Certified Refurbished Products: The Effects of Environmental Attitudes and Knowledge. *J. Ind. Ecol.* **2016**, *20*, 893–904. [CrossRef]
27. Plociennik, C.; Pourjafarian, M.; Nazeri, A.; Windholz, W.; Knetsch, S.; Rickert, J.; Ciroth, A.; Precci Lopes, A.D.C.; Hagedorn, T.; Vogelgesang, M.; et al. Towards a Digital Lifecycle Passport for the Circular Economy. *Procedia CIRP* **2022**, *105*, 122–127. [CrossRef]
28. Jensen, S.F.; Kristensen, J.H.; Adamsen, S.; Christensen, A.; Waehrens, B.V. Digital Product Passports for a Circular Economy: Data Needs for Product Life Cycle Decision-Making. *Sustain. Prod. Consum.* **2023**, *37*, 242–255. [CrossRef]
29. Regueiro, C.; Gómez-Goiri, A.; Pedrosa, N.; Semertzidis, C.; Iturbe, E.; Mansell, J. Blockchain-Based Refurbishment Certification System for Enhancing the Circular Economy. *Blockchain Res. Appl.* **2024**, *5*, 100172. [CrossRef]
30. CTIA. *Wireless Device Grading Scales Criteria and Definitions*; CTIA Certification: Washington, DC, USA, 2023.
31. Yanıkoğlu, İ.; Denizel, M. The Value of Quality Grading in Remanufacturing under Quality Level Uncertainty. *Int. J. Prod. Res.* **2021**, *59*, 839–859. [CrossRef]
32. Kong, Z.; Cui, Y.; Xia, Z.; Lv, H. Convolution and Long Short-Term Memory Hybrid Deep Neural Networks for Remaining Useful Life Prognostics. *Appl. Sci.* **2019**, *9*, 4156. [CrossRef]
33. Wang, T.; Mrad, N. Probabilistic Analysis for Remaining Useful Life Prediction and Reliability Assessment. *IEEE Trans. Reliab.* **2022**, *71*, 1207–1218. [CrossRef]
34. Lin, J.; Lin, Z.; Liao, G.; Yin, H. A Novel Product Remaining Useful Life Prediction Approach Considering Fault Effects. *IEEE/CAA J. Autom. Sin.* **2021**, *8*, 1762–1773. [CrossRef]
35. Benker, M.; Furtner, L.; Semm, T.; Zaeh, M.F. Utilizing Uncertainty Information in Remaining Useful Life Estimation via Bayesian Neural Networks and Hamiltonian Monte Carlo. *J. Manuf. Syst.* **2021**, *61*, 799–807. [CrossRef]
36. Lin, Y.-H.; Li, G.-H. A Bayesian Deep Learning Framework for RUL Prediction Incorporating Uncertainty Quantification and Calibration. *IEEE Trans. Ind. Inf.* **2022**, *18*, 7274–7284. [CrossRef]
37. Xuan, Q.L.; Adhisantoso, Y.G.; Munderloh, M.; Ostermann, J. Uncertainty-Aware Remaining Useful Life Prediction for Predictive Maintenance Using Deep Learning. *Procedia CIRP* **2023**, *118*, 116–121. [CrossRef]
38. Zhao, Q.; Jia, X.; Guo, B.; Cheng, Z. Real-Time Bayes Estimation of Residual Life Based on Multisource Information Fusion. In Proceedings of the 2018 Prognostics and System Health Management Conference (PHM-Chongqing), Chongqing, China, 26–28 October 2018; pp. 215–222.
39. Wang, X.; Balakrishnan, N.; Guo, B. Residual Life Estimation Based on a Generalized Wiener Degradation Process. *Reliab. Eng. Syst. Saf.* **2014**, *124*, 13–23. [CrossRef]
40. Idrees, A.M.; Taie, S. Online Price Recommendation System for Shopping Used Cell Phones. *Res. J. Appl. Sci. Eng. Technol.* **2016**, *13*, 15–23. [CrossRef]
41. She, D.; Jia, M. A BiGRU Method for Remaining Useful Life Prediction of Machinery. *Measurement* **2021**, *167*, 108277. [CrossRef]
42. Niu, H.; Zeng, J.; Shi, H.; Zhang, X.; Liang, J. Degradation Modeling and Remaining Useful Life Prediction for a Multi-Component System with Stochastic Dependence. *Comput. Ind. Eng.* **2023**, *175*, 108889. [CrossRef]
43. Wróblewski, P.; Lewicki, W. A Method of Analyzing the Residual Values of Low-Emission Vehicles Based on a Selected Expert Method Taking into Account Stochastic Operational Parameters. *Energies* **2021**, *14*, 6859. [CrossRef]
44. Liu, H.; Liu, Z.; Jia, W.; Lin, X. A Novel Deep Learning-Based Encoder-Decoder Model for Remaining Useful Life Prediction. In Proceedings of the 2019 International Joint Conference on Neural Networks (IJCNN), Budapest, Hungary, 14–19 July 2019; pp. 1–8.
45. Shehadeh, A.; Alshboul, O.; Al Mamlook, R.E.; Hamedat, O. Machine Learning Models for Predicting the Residual Value of Heavy Construction Equipment: An Evaluation of Modified Decision Tree, LightGBM, and XGBoost Regression. *Autom. Constr.* **2021**, *129*, 103827. [CrossRef]
46. Zhevnenko, D.; Kazantsev, M.; Makarov, I. Interaction Models for Remaining Useful Lifetime Estimation. *J. Ind. Inf. Integr.* **2023**, *33*, 100444. [CrossRef]
47. Singh, S.K.; Kumar, S.; Dwivedi, J.P. A Novel Soft Computing Method for Engine RUL Prediction. *Multimed. Tools Appl.* **2019**, *78*, 4065–4087. [CrossRef]
48. Wu, J.; Hu, K.; Cheng, Y.; Zhu, H.; Shao, X.; Wang, Y. Data-Driven Remaining Useful Life Prediction via Multiple Sensor Signals and Deep Long Short-Term Memory Neural Network. *ISA Trans.* **2020**, *97*, 241–250. [CrossRef]
49. Carvalho, T.P.; Soares, F.A.A.M.N.; Vita, R.; Francisco, R.D.P.; Basto, J.P.; Alcalá, S.G.S. A Systematic Literature Review of Machine Learning Methods Applied to Predictive Maintenance. *Comput. Ind. Eng.* **2019**, *137*, 106024. [CrossRef]
50. Lin, Y.-H.; Ding, Z.-Q. An Integrated Degradation Modeling Framework Considering Model Uncertainty and Calibration. *Mech. Syst. Signal Process.* **2022**, *166*, 108389. [CrossRef]
51. Guo, L.; Yu, Y.; Duan, A.; Gao, H.; Zhang, J. An Unsupervised Feature Learning Based Health Indicator Construction Method for Performance Assessment of Machines. *Mech. Syst. Signal Process.* **2022**, *167*, 108573. [CrossRef]

52. Schlüter, M.; Lickert, H.; Schweitzer, K.; Bilge, P.; Briese, C.; Dietrich, F.; Krüger, J. AI-Enhanced Identification, Inspection and Sorting for Reverse Logistics in Remanufacturing. *Procedia CIRP* **2021**, *98*, 300–305. [CrossRef]
53. van Breugel, B.; Imrie, F.; Seedat, N. Can You Rely on Your Model Evaluation? Improving Model Evaluation with Synthetic Test Data. *Adv. Neural Inf. Process. Syst.* **2023**, *36*, 1889–1904.
54. Xu, Z.; Bashir, M.; Liu, Q.; Miao, Z.; Wang, X.; Wang, J.; Ekere, N. A Novel Health Indicator for Intelligent Prediction of Rolling Bearing Remaining Useful Life Based on Unsupervised Learning Model. *Comput. Ind. Eng.* **2023**, *176*, 108999. [CrossRef]
55. Garg, S.; Balakrishnan, S.; Lipton, Z.C. Leveraging unlabeled data to predict out-of-distribution performance. *arXiv* **2022**, arXiv:2201.04234.
56. Schwartz, S.; Montero Jiménez, J.J.; Vingerhoeds, R.; Salaün, M. An Unsupervised Approach for Health Index Building and for Similarity-Based Remaining Useful Life Estimation. *Comput. Ind.* **2022**, *141*, 103716. [CrossRef]
57. Xue, B.; Xu, H.; Huang, X.; Zhu, K.; Xu, Z.; Pei, H. Similarity-Based Prediction Method for Machinery Remaining Useful Life: A Review. *Int. J. Adv. Manuf. Technol.* **2022**, *121*, 1501–1531. [CrossRef]
58. Siahpour, S.; Li, X.; Lee, J. A Novel Transfer Learning Approach in Remaining Useful Life Prediction for Incomplete Dataset. *IEEE Trans. Instrum. Meas.* **2022**, *71*, 1–11. [CrossRef]
59. Chawla, N.V.; Karakoulas, G. Learning From Labeled And Unlabeled Data: An Empirical Study Across Techniques And Domains. *J. Artif. Intell. Res.* **2005**, *23*, 331–366. [CrossRef]
60. Liu, Z.; Li, Z.; Mhlanga, J.C.; Siegel, B.A.; Jha, A.K. No-Gold-Standard Evaluation of Quantitative Imaging Methods in the Presence of Correlated Noise. *Proc. SPIE-Int. Soc. Opt. Eng.* **2022**, *12035*, 120350M. [PubMed]
61. Walter, S.D. Estimation of Diagnostic Test Accuracy: A "Rule of Three" for Data with Repeated Observations but without a Gold Standard. *Stat. Med.* **2021**, *40*, 4815–4829. [CrossRef] [PubMed]
62. Hahn, A.; Schwarz, N.G.; Frickmann, H. Comparison of Screening Tests without a Gold Standard—A Pragmatic Approach with Virtual Reference Testing. *Acta Trop.* **2019**, *199*, 105118. [CrossRef]
63. Messam, L.L.M.; Branscum, A.J.; Collins, M.T.; Gardner, I.A. Frequentist and Bayesian Approaches to Prevalence Estimation Using Examples from Johne's Disease. *Anim. Health Res. Rev.* **2008**, *9*, 1–23. [CrossRef]
64. Hui, S.L.; Walter, S.D. Estimating the Error Rates of Diagnostic Tests. *Biometrics* **1980**, *36*, 167. [CrossRef]
65. Toft, N.; Jørgensen, E.; Højsgaard, S. Diagnosing Diagnostic Tests: Evaluating the Assumptions Underlying the Estimation of Sensitivity and Specificity in the Absence of a Gold Standard. *Prev. Vet. Med.* **2005**, *68*, 19–33. [CrossRef]
66. Qu, Y.; Tan, M.; Kutner, M.H. Random Effects Models in Latent Class Analysis for Evaluating Accuracy of Diagnostic Tests. *Biometrics* **1996**, *52*, 797–810. [CrossRef] [PubMed]
67. Joseph, L.; Gyorkos, T.W.; Coupal, L. Bayesian Estimation of Disease Prevalence and the Parameters of Diagnostic Tests in the Absence of a Gold Standard. *Am. J. Epidemiol.* **1995**, *141*, 263–272. [CrossRef] [PubMed]
68. Johnson, W.O. Screening without a "Gold Standard": The Hui-Walter Paradigm Revisited. *Am. J. Epidemiol.* **2001**, *153*, 921–924. [CrossRef] [PubMed]
69. Bermingham, M.L.; Handel, I.G.; Glass, E.J.; Woolliams, J.A.; Bronsvoort, B.M.D.C.; McBride, S.H.; Skuce, R.A.; Allen, A.R.; McDowell, S.W.J.; Bishop, S.C. Hui and Walter's Latent-Class Model Extended to Estimate Diagnostic Test Properties from Surveillance Data: A Latent Model for Latent Data. *Sci. Rep.* **2015**, *5*, 11861. [CrossRef]
70. Ni, H.; Koop, G.; Klugkist, I.; Nielen, M. Evaluation of Bayesian Hui-Walter and Logistic Regression Latent Class Models to Estimate Diagnostic Test Characteristics with Simulated Data. *Prev. Vet. Med.* **2023**, *217*, 105972. [CrossRef]
71. Dendukuri, N.; Joseph, L. Bayesian Approaches to Modeling the Conditional Dependence Between Multiple Diagnostic Tests. *Biometrics* **2001**, *57*, 158–167. [CrossRef]
72. Sun, A.; Zhou, X. Estimation of Diagnostic Test Accuracy Without Gold Standards. *Stat. Med.* **2025**, *44*, e10315. [CrossRef]
73. Keddie, S.H.; Baerenbold, O.; Keogh, R.H.; Bradley, J. Estimating Sensitivity and Specificity of Diagnostic Tests Using Latent Class Models That Account for Conditional Dependence between Tests: A Simulation Study. *BMC Med. Res. Methodol.* **2023**, *23*, 58. [CrossRef]