EXCITATION ZONES OPTIMIZATION FOR PIEZOELECTRIC STRUCTURES

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This paper presents a study of optimization of the electrodes dislocation of piezoelectric actuators. The following conditions of optimization problem are considered: to unify excitation voltage forms, to achieve reverse motion and maximum coefficient of efficiency. Realisation of this requirement means that generator has to supply voltage of constant frequency, amplitude and phase, reverse motion must be achieve only through changing the polarity of the electric flux and the surface of the piezodrive is fully covered with electrodes. When all aforementioned conditions are achieved, any mode shape of piezoelectric actuator could be obtained by changing polarity of the voltage, supplied to the particular electrode. In order to find the sign of the polarity, corresponding to the certain modal shape, the comparison must be done between directions of the vector of amplitude of the equivalent mechanical force and particular eigenvector of the actuator. If the direction of displacement eigenvector of the piezodrive and the vector of amplitudes of the equivalent mechanical force is the same or similar i.e. the angle is within the limits $[-\pi/2, \pi/2]$, then the polarity of the voltage, supplied to particular electrode have initial sign. In other case the sign of polarity must be changed. FEM modelling is performed in calculating process. The results of calculations of excitation zones dislocation of the piezoelectric actuators are shown under two optional conditions of fixing.

1. Introduction

Recent advances in development, theory and applications of new smart materials, structures and devices, including materials with extremely high piezoelectric properties extended the area of the researches of new systems with high levels of integration and multifunctionality. Particular interest is concentrated in development piezoelectric drives as high precision and sensitiveness system, that enables the creation of a time constant positioning drives, micro manipulators, micro pumps, transducers for materials physical properties measurement, transmission of motion into vacuum chambers without power losses, converters, vibration concentrators, scanners, etc.

The performance of such devices strongly depends on the features of actuator, the main part of the piezoelectric system. Many different constructions of the actuators (such as beam, plate, cylinder, disc and etc.) are used in order to achieve particular characteristics of movement of the actuator and final link of kinematics pair. The shape and optimal location of electrodes on the surface of actuator have the great importance to vibration mode of the actuator. Using different configurations of electrodes, vibration of main and higher resonance modes of piezoelectric actuator could be achieved. In case of optimal electrodes configuration, needless harmonics of actuator could be eliminated and also the concentration of mechanical stresses could be reduced. These facts are very important in case of multicomponent oscillations of actuator, that is analysed in this paper. Using finite element method (FEM) calculations are carried out. FEM allows achieving the precision of calculations of the optimal excitations zones locations within the limits of the area of finite element.

2. Finite element model of piezoelectric actuator

In most cases piezoelectric actuators are resonance system, that operate in first or higher resonance frequency. Synthesis of needful field of the oscillations must be obtained by using the particular shapes, geometrical parameters, boundary conditions and also topology of excitation zones locations of the actuator. Different beam, plate, shell shaped piezoelectric actuators are widely used in small power ultrasonic motors. Various kinds of resonance oscillations of actuator: longitudinal, flexural, rotational, shear and so on, could be obtained using different geometry of electrodes [4]. In order to achieve needful resonance oscillations of actuator, particular electrodes must be excited. So question is how to obtain optimal electrodes configuration, corresponding to the certain resonance oscillations. This problem is especially important, in case of multicomponent oscillation of the actuator. In order to derive optimal law of coupled oscillations, not only electrodes configuration must be find but also resonance frequency and phase must be derived. Excitation zones configuration in case of multicomponent movement also must guarantee independent excitation of separate component oscillation. Excitation zones configuration problem of the plate shaped actuator was analysed in [4, 6], but usually configuration of the electrodes is obtained in empirical way. So there is no common analytical method for this problem solving especially in case of multicomponent oscillation.

Analysis of the piezoelectric actuator must be carried out appreciating the electric occurrence in the system. Based on FEM, every node of the element has one additional DOF used for electric potentials in modelling process. Equation (2.1) fully define piezoeffect:

$$\begin{aligned} \{\sigma\} &= \left[c^E\right]\{\varepsilon\} - [e]^T\{E\}, \\ \{D\} &= [e]\{\varepsilon\} - [\ni^s]\{E\}, \end{aligned} \tag{2.1}$$

where $[c^E]$, $[e]$, $[\ni^s]$ — accordingly matrix of stiffness for a constant electric field; matrix of piezoelectric constants; matrix of dielectric constants, evaluated at the constant strains; $\{\sigma\}$, $\{\varepsilon\}$, $\{D\}$, $\{E\}$ — accordingly vectors of stresses, strains, electric induction and electric field. Here:

$$[e] = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \tag{2.2}$$

$$[\text{э}^s] = \begin{bmatrix} \text{э}_{11} & 0 & 0 \\ 0 & \text{э}_{11} & 0 \\ 0 & 0 & \text{э}_{33} \end{bmatrix}. \tag{2.3}$$

The solution applied for the equations of motion, suitable for the piezoelectric actuator, can be derived from the principle of minimum potential energy by means of variational functionals [3]. The basic dynamic FEM equation of motion for ultrasonic transducer that is fully covered with electrodes can be expressed as:

$$\begin{aligned} [M]\left\{\ddot{\delta}\right\} + [C]\left\{\dot{\delta}\right\} + [K]\{\delta\} - [T]\{\varphi\} &= \{R\}, \\ [T]^T\{\delta\} + [S]\{\varphi\} &= \{Q\}, \end{aligned} \tag{2.4}$$

where $[M]$, $[K]$, $[T]$, $[S]$, $[C]$ — accordingly matrices of mass, stiffness, electroelasticity, capacity, damping; $\{\delta\}$, $\{\varphi\}$, $\{R\}$ — accordingly vectors of nodes displacements, potentials, external mechanical forces.

Here

$$[K] = \int_V [B]^T \left[c^E\right] [B] \, dV, \tag{2.5}$$

$$[T] = \int_V [B]^T [e] [B_E] \, dV, \tag{2.6}$$

$$[S] = \int_V [B_E]^T [\varepsilon^s] [B_E] \, dV, \tag{2.7}$$

$$[M] = \varrho \int_V [N]^T [N] \, dV, \tag{2.8}$$

$$[C] = \alpha[M] + \beta[K], \tag{2.9}$$

where $[N]$ – matrix of shape function used for evaluation of displacements; $[B]$, $[B_E]$ — matrices of shape functions derivatives, applied for evaluation of displacements and potential accordingly. Damping matrix $[C]$ is derived using mass and stiffness matrices by assigning constants α and β. In general case matrices

$[B]$ and $[B_E]$ - can be expressed as follow:

$$(2.10) \qquad [B] = [a][N] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} [N],$$

$$(2.11) \qquad [B_E] = \nabla[N_E] = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} [N_E],$$

where $[N_E]$ — matrix of shape function used for evaluation of potentials.

Generally mechanical and electrical boundary conditions are determined in analysis of the piezoelectric body. Mechanical displacement and stresses of the surface, electric charge and potential can be defined as boundary conditions in certain surface area. We are interested in solution applied for the piezoelectric actuator when vector of external mechanical forces is equal to zero e.g. $\{R\} = 0$. This particular condition is related with the case of indirect piezoelectric effect analysis of the actuator.

3. Optimization analysis

In order to obtain optimal electrodes configuration of the actuator, the following conditions to the piezoelectric system were accepted: to unify excitation voltage forms, to achieve reverse motion and maximum coefficient of efficiency. Realisation of the first requirement means that current source has to generate voltage of stable frequency, amplitude and phase, in order to simplify construction of the machine, second — that reverse motion must be achieve by changing the polarity of the electric flux and third — that the actuator must be fully covered with the electrodes. When all aforementioned conditions are achieved, any mode shape of piezoelectric actuator could be obtained by changing polarity of the voltage, supplied to the particular electrode. In order to find the sign of the polarity, corresponding to the certain modal shape, the comparison between directions of the vector of amplitude of the equivalent mechanical force and particular eigenvector of the actuator must be done. If the direction of displacement eigenvector of the piezoelectric actuator and the vector of amplitudes of the equivalent mechanical force is the similar i.e. the angle is within the limits $[-\pi/2, \pi/2]$, then the polarity of the voltage, supplied to particular electrode have initial sign. In other case the sign of polarity must be changed. Depends on the sign of $\cos\gamma_i^e$ the polarity of voltage is defined, where γ_i^e — the angle between the vector of amplitudes of the equivalent mechanical forces and eigenvector of the actuator.

Due to the first condition of optimization to unify excitation voltage, the potential of electrodes of all piezoelements must be equal as shown in Eqs. (3.1):

$$\begin{aligned} \{\varphi\}^e &= \{U\}^e \,\mathrm{sg}^e \sin\omega_k t, \\ \mathrm{sg}^e &= \begin{cases} 1, \\ -1, \end{cases} \end{aligned} \tag{3.1}$$

where $\{U\}^e$, ω_k — accordingly vector of element excitation voltage amplitude and k resonance frequency. Active frequency of the piezoelectric actuator is closed to the resonance, so the electric flux must have the same excitation voltage frequency. Modal shapes and natural frequencies are obtained by reducing equations (2.4), into standard eigenvalue form:

$$\begin{aligned} [M]\left\{\ddot{\delta}\right\} + [K]\{\delta\} - [T]\{\varphi\} &= \{0\}, \\ [T]^T\{\delta\} + [S]\{\varphi\} &= \{0\}. \end{aligned} \tag{3.2}$$

The eigenvalue (natural frequencies) and normalised displacement eigenvectors are derived from the modal solution of the piezoelectric system [3] and used in further optimization analysis process.

Let's return to Eqs. (2.4). When the vector of the external mechanical forces $\{R\}$ is setting to zero, we obtain the following expression:

$$[M]\left\{\ddot{\delta}\right\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{-F\}. \tag{3.3}$$

Here

$$\{F\} = \sum_e [L]^e[T]^e\{U\}^e \mathrm{sg}^e \sin\omega_k t = \sum_e \{F\}^e \sin\omega_k t, \tag{3.4}$$

where $\{F\}$ — vector external equivalent mechanical forces; $\{F\}^e$ — vector of amplitude of external equivalent mechanical force at the nodes of finite element in global coordinate system; $\{T\}^e$ — matrix of electroelasticity of finite element; $\{L\}^e$ — matrix of transformation between local and global element coordinates.

Here

$$\{F\}^e = [L]^e[T]^e\{U\}^e \mathrm{sg}^e. \tag{3.5}$$

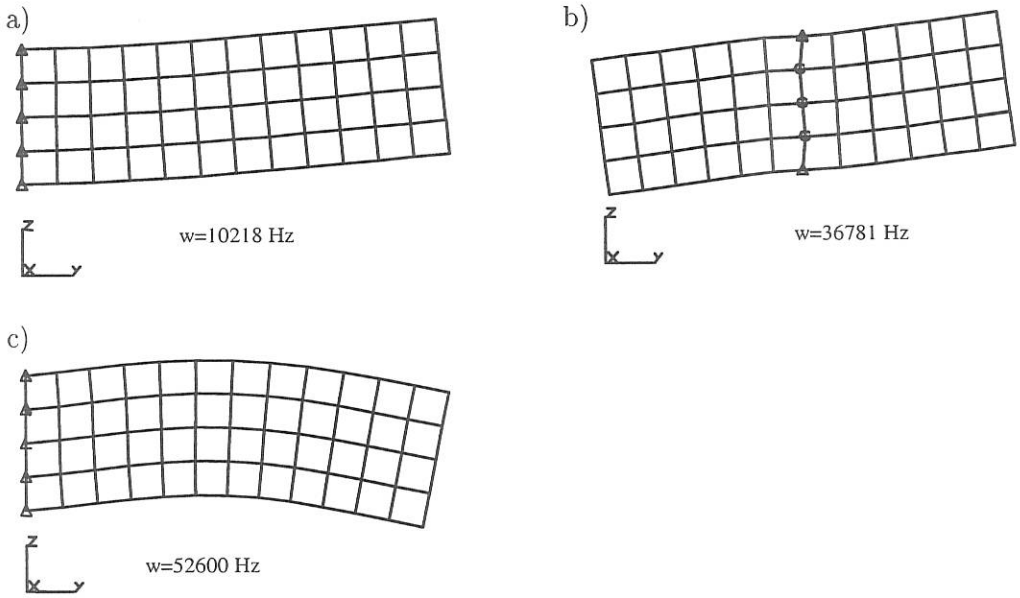


Fig. 1. The modal shapes of the plate shaped piezoelectric actuator: a) first modal shape, when constrained nodes of the left side; b) first modal shape, when constrained the centre nodes; c) second modal shape, when constrained nodes of the left side. Vector of polarization is perpendicular to the plane of paper.

Rys. 1. Kształty modalne piezoelektrycznego aktuatora w formie płytki: a) pierwszy kształt modalny, gdy zamocowane węzły znajdują się z lewej strony; b) pierwszy kształt modalny, gdy zamocowane węzły znajdują się pośrodku; c) drugi kształt modalny, gdy zamocowane węzły znajdują się z lewej strony. Wektor polaryzacji jest prostopadły do płaszczyzny rysunku

The solution of the basic dynamic FEM equation of motion for piezoelectric actuator could be written in following form:

$$\{\delta\} = [\Delta_0]\{z(t)\}, \tag{3.6}$$

where $[\Delta_0]$ — normalised eigenvectors; $z(t)$ — coefficient of proportional.

Here

$$[\Delta_0] = [\{\delta_0\}_1, \{\delta_0\}_2, ..., \{\delta_0\}_n]. \tag{3.7}$$

Normalised eigenvectors could be obtained from Eqs. (3.2):

$$\{\delta_0\}_i^T[M]\{\delta_0\}_j = \begin{cases} 0, & i \neq j, \\ 1, & i = j. \end{cases} \tag{3.8}$$

In the similar way as in (3.8), we obtain following equations:

$$\begin{aligned} [K]\{\delta_0\}_i &= \omega_i^2[M]\{\delta_0\}_i, \\ \{\delta_0\}_i^T[K]\{\delta_0\}_j &= \begin{cases} 0, & i \neq j, \\ \omega_i^2, & i = j. \end{cases} \end{aligned} \tag{3.9}$$

Matrix of damping is proportional to the mass matrix, so following equation is obtained:

$$\{\delta_0\}_i^T[C]\{\delta_0\}_j = \begin{cases} 0, & i \neq j, \\ 2\omega_i h_i, & i = j. \end{cases} \tag{3.10}$$

Having Eqs. (3.8), (3.9), (3.10), we can obtain coefficient of proportional:

$$\ddot{z}_i + 2\omega_i h_i\, \dot{z}_i + \omega_i^2 z_i = -\{\delta_0\}_i^T\{F\}, \qquad i = 1, 2, ..., n. \tag{3.11}$$

Solving Eq. (3.11) we could calculate coefficient $z_k(t)$, that correspond to k natural frequency of actuator.

Now lets make the analysis of effective job of the external forces when electrodes of the actuator are excited. Referring to the third condition of optimization problem to obtain the maximum of coefficient of efficiency, the effective job of the actuator must be maximised $A_k^{\mathrm{ef}} \Rightarrow \max\limits_{e}$. The average of the effective job, corresponding to the k natural frequency could be obtained as follow:

$$A_k^{\mathrm{ef}} = \frac{\omega_k}{2\pi} \int\limits_0^{2\pi/\omega_k} \{\delta_0\}_k^T\{F\} z_k(t)\, dt. \tag{3.12}$$

If we put Eqs. (3.1) and (3.4) into (3.12), the following expression of the effective job could be obtained:

$$A_k^{\mathrm{ef}} = \frac{\omega_k}{2\pi} \int\limits_0^{2\pi/\omega_k} \sum_e \{\delta_0\}_k^T\{F\}^e \sin\omega_k t\, z_k(t)\, dt. \tag{3.13}$$

Equation (3.13) could be rewritten in the form:

$$A_k^{\mathrm{ef}} = \sum_e \{\delta_0\}_k^T\{F\}^e H_k. \tag{3.14}$$

Here

$$H_k = \frac{\omega_k}{2\pi} \int\limits_0^{2\pi/\omega_k} \sin\omega_k t\, z_k(t)\, dt. \tag{3.15}$$

Variable H_k from Eq. (3.15) is a time independent and referring to the particular conditions, $\{\delta_0\}_k$ is constant value, so only the multiplication of the vector of the amplitudes of the equivalent mechanical forces and eigenvector of piezoelectric actuator must be maximised. In order to achieve maximum value, $\cos\gamma_k^e$ — the cosine of the angle of aforementioned vectors must be calculated and maximised:

$$\cos\gamma_k^e = \frac{\{\delta_0\}_k^T\{F\}^e}{|\delta_0|_k|F^e|}, \tag{3.16}$$

where $\cos\gamma_k^e$ — the cosine of the angle between the vector of the amplitudes of the equivalent mechanical forces and eigenvector of piezoelectric actuator.

Based on Eq. (3.16), the $\cos\gamma_k^e$ depends only on the direction of the vector of the equivalent mechanical forces in finite element. Referring to Eq. (3.5), the direction of the vector could be changed by changing the polarity of the voltage. The maximum of the oscillations amplitudes, accordingly to the certain modal shapes of the piezoelectric actuator, can be achieved when polarity of the excitation electrical signal is identical to the sign of $\cos\gamma_k^e$.

4. Processing and results

Calculations, based on Eq. (3.16), were carried out with plate shaped actuators, that are widely used in rotational and linear motion type ultrasonic motors. Two different conditions of fixing of the actuator were selected: the centre nodes were constrained in the first case and the nodes of the left side were constrained in

a)

1.2	3.7	7.2	0.5	-6.8	-4.4	-1.7
0.3	1.5	2.6	0.9	-2.0	-2.3	-1.4
1.5	2.3	2.0	-0.9	-2.6	-1.5	-0.3
1.7	4.4	6.8	-0.5	-7.2	-3.7	-1.1

b)

-3.2	-2.7	-2.5	-2.3	-1.8	-1.1	-0.4
-2.5	-2.4	-2.2	-1.9	-1.4	-0.9	-0.3
-3.4	-2.6	-2.2	-1.9	-1.4	-0.8	-0.2
-3.0	-2.2	-1.8	-1.3	-1.0	-0.6	-0.1

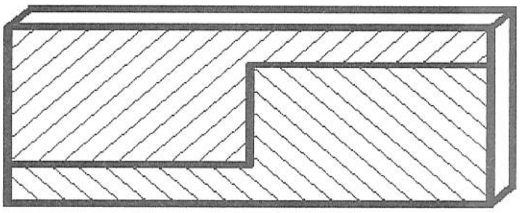
c)

1.2	3.7	0.7	0.5	0.3	0.14	0.03
-0.1	-0.1	-0.1	-0.0	-0.0	-0.0	-0.0
0.03	0.2	0.2	0.14	0.1	0.1	0.03
-1.3	-1.0	-0.7	-0.5	-0.2	-0.1	-0.0

Fig. 2. Values ($*10^{-2}$) of $\cos\gamma_k^e$ and configuration of electrodes of the piezoelectric actuators under two options of fixing conditions: a) constrained the centre nodes, first modal shape; b) constrained nodes of the left side, first modal shape; c) constrained nodes of the left side, second modal shape. Vector of polarization is perpendicular to the plane of paper.

Rys. 2. Wartości ($*10^{-2}$) $\cos\gamma_k^e$ i konfiguracja elektrod aktuatorów piezoelektrycznych dla dwóch warunków zamocowania: a) zamocowane węzły środkowe, pierwszy kształt modalny; b) zamocowane węzły po lewej stronie, pierwszy kształt modalny; c) zamocowane węzły po lewej stronie, drugi kształt modalny. Wektor polaryzacji jest prostopadły do płaszczyzny rysunku the second case. The results of calculations of $\cos\gamma_k^e$ for all cases are given in the centres of gravity of the element (see Fig. 2). Accordingly to the signs of $\cos\gamma_k^e$, electrodes topologies are determined. Zones of the electrodes, with different polarities of the excited voltage, are separated with bold line. As we mentioned in the previous section of this paper, the excitation voltage polarity, supplied to the particular zones, must be identical to the sign of $\cos\gamma_k^e$. Calculated topologies of electrodes have strictly determined zones and are similar to the electrodes configuration, created by the intuition of engineer (see Fig. 3) and practically used in the real actuators. Configuration of electrodes strongly depends on constraining conditions of the actuator.

a)



b)

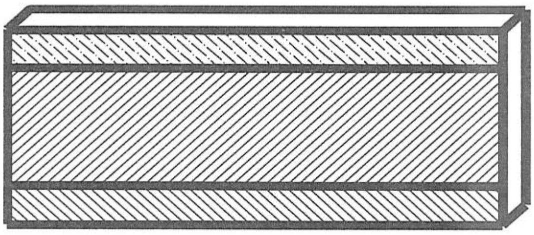


Fig. 3. Configuration of electrodes of the piezoelectric actuator created according intuition of engineer for the same type of the actuators as shown: a) in Fig. 2 case (a); b) in Fig. 2 case (c). Vector of polarization is perpendicular to paper plane.

Rys. 3. Konfigurecja elektrod aktuatora piezoelektrycznego utworzona zgodnie z intuicją projektanta dla tego samego typu aktuatorów, jaki pokazano: a) na Rys. 2, przypadek (a); b) na Rys. 2, przypadek (c). Wektor polaryzacji jest prostopadły do płaszczyzny rysunku

5. Conclusion

There are not common methods, developed for the actuators and other ultrasonic system design. The calculation algorithm of optimal electrodes configuration of the piezoelectric actuator provided in this paper is especially important in case of the multicomponent oscillation of the structures. This method is based on FEM and allows achieving precision of calculations according to the limits of the area of finite element. Using this optimization algorithm, the problem of durability and reliability of the actuators could be solved.

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OPTYMALIZACJA STREF WZBUDZENIA W STRUKTURACH PIEZOELEKTRYCZNYCH

Streszczenie

Praca przedstawia optymalizację rozmieszczenia elektrod w aktuatorach piezoelektrycznych. Rozważa się następujące warunki zagadnienia optymalizacyjnego: ujednolicenie kształtu napięcia wzbudzjącego, uzyskanie ruchu wstecznego i maksymalnego współczynnika sprawności. Spełnienie tych wymagań oznacza, że generator musi dostarczać napięcia o stałej częstotliwości, amplitudzie i fazie; ruch wsteczny musi być uzyskany wyłącznie przez zmianę biegunowości przepływu, a powierzchnia napędu piezoelektrycznego musi być całkowicie pokryta elektrodami. Gdy wszystkie wymienione wyżej warunki są spełnione, można uzyskać dowolny kształt działania aktuatora przez zmianę biegunowości napięcia dostarczanego do danej elektrody. Aby znaleźć znak biegunowości odpowiadającej danemu kształtowi modalnemu, należy dokonać porównania między kierunkami wektora amplitudy odpowiedniej siły mechanicznej i szczególnego wektora własnego aktuatora. Jeśli kierunek wektora własnego przemieszczeń napędu piezoelektrycznego i wektora amplitud równoważnej siły mechanicznej jest taki sam lub podobny, tzn. kąt mieści się w granicach $[-\pi/2, \pi/2]$, wtedy biegunowość napięcia dostarczanego do danej elektrody ma znak początkowy. W przeciwnym przypadku znak biegunowości musi zostać zmieniony. Modelowanie MES dokonuje się w trakcie procesu obliczeniowego. Wyniki obliczeń dotyczące rozmieszczenia stref wzbudzenia aktuatorów piezoelektrycznych są pokazane przy dwóch przykładowo założonych warunkach zamocowania.

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