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DOVILĖ KARALIENĖ

ANALYSIS OF ALGEBRAIC ESTIMATES OF ELECTROCARDIOGRAPHIC AND ULTRASONIC SIGNALS

Summary of Doctoral Dissertation
Technological Sciences, Informatics Engineering (07T)

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DOVILĖ KARALIENĖ

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1. GENERAL CHARACTERISTICS OF THE RESEARCH

The introduction outlines the relevance of the topic and describes its aim, objectives and tasks. The introduction also briefly presents the main results of the thesis, their practical significance and scientific novelty.

1.1. Relevance of the Topic

Computing machinery and technology development provides an opportunity to accumulate increasing amounts of information and to perform more complex and informative signal analysis. However, with the increasing amount of information and knowledge, various diagnostic apparatus supply, with the development of multi-modal signals and data storage, it is becoming more and more difficult to carry out adequate data analysis and generalization of the accrued volume of information. Problems occur with new signals and the information stored inside; these problems are particularly acute in the areas where signals, such as biomedical, seismic, ultrasonic and other sources, are extremely complicated.

Electrocardiographic (ECG) and ultrasonic signals are analyzed in the thesis. ECG signals are physiological system (human heart) generated signals. The main feature of physiological systems is their complexity 'hidden' in biomedical signals. Processing these signals from complex system positions opens up opportunities to understand their generating system components and dynamic interfaces. This discovery prompted the complex system theory applications from the molecular to the organism level. These signals are called *complex* signals.

There are not enough conventional methods for heart-generated non-linear and non-stationary signal analysis. For the purpose of ECG signal analysis, nonlinear dynamic methods are applied based on deterministic chaos and complex systems theories. These theories can greatly extend and enhance the signal analysis capabilities by quality and quantity; moreover, the knowledge of dynamics that affects the complex signal is crucial to the perception of the process as well as the selection of the appropriate research method. This area of research has been recently developed in many fields of science.

Other signals analyzed in the thesis are ultrasonic signals. These signals are used in the medical industry, when examining human internal organs as well as diagnosing diseases, in the non-destructive material control, when examining various material properties, rough surfaces, pipe, train, rails, nuclear reactor internal surfaces and many other areas.

A wide range of spectral and time domain analysis methods were developed for ultrasonic signal analysis. The methods are selected depending on the aim of the analysis, their generating process characteristics and signal properties. The ultrasonic signal identification, i.e. the methods of mathematical modeling, is examined in the dissertation. Signal identification allows filtering the signals, determining parameters such as the duration of the signal, the power, energy and other signal characteristics. Some of the most popular methods found in scholarly literature for signal identification are Fourier and Prony's methods. Prony's method describes signals by a fixed number of exponential function linear derivatives with constant coefficients. Unlike Fourier, Prony's method allows to identify non-periodical fading finite length signals without losing information about the phase. This is a particularly important feature when the method is used in order to determine the signal delay time. However, a major drawback of these methods is the unknown number of model components (a model order) which identify the signal; moreover, it is limited by the exponential function coefficients of stability, which, in some cases, results in increasing the signal approximation error. Nevertheless, Prony's method is applicable in many fields, such as biomedicine, non-destructive material control, genetics, financial, etc. fields. Not surprisingly, a wide variety of modifications of Prony's method are developed and applied for the above outlined areas of analysis.

1.2. The Aim and Objectives of the Dissertation Research

The object of the research is the identification algorithms for signals approximated by linear recurring sequence.

The aim of the research is to modify Prony's method for the identification of signals that can be approximated by the sum of exponential functions with polynomial coefficients by using the optimal number of model components based on approximation errors and convergence speed.

In order to achieve this aim, the following tasks are to be solved in the dissertation:

1. To review signal identification methods which will be used for a comparison with the constructed method.
2. To modify Prony's identification method for the signals that can be approximated by linear derivatives of exponential functions with polynomial coefficients.
3. By using a modified Prony's method, to create such an algebraic signal interpolation algorithm, that the signal would be optimally identified by the linear derivative of an exponential function.
4. To create prototypes of program systems with the following possibilities:
   - to identify electrocardiographic signal parameters of certain fragments and to perform complexity analysis of the identified parameters (algebraic estimates);
   - to identify the ultrasonic signal and its starting point and to perform analysis of the identified signal parameters (algebraic estimates).

1.3. Methods, Data and Software of the Research

1. Algebraic analysis theory and its application to create signal identification techniques are used in the research;
2. Models and identification methods approximating a signal by the sum of exponential functions are used for the created signal identification algorithm study and comparison
3. *Matlab* v. R2009a software package is applied for calculations and the realization of developed algorithms and prototypes.
4. Electrocardiographic and ultrasound signal data is used for experiments of the developed prototypes of signal identification program systems.

1.4. Scientific Novelty and Practical Importance of the Research

1. An extended (modified) Prony's method identifies signals by an exponential function with polynomial coefficients of linear formations. Thus, the method allows, in certain cases, the signals to be identified more precisely than by using other Prony's methods describing signals by the exponential function with constant coefficients models. This allows performing more precise analysis in many application areas of Prony-type methods.
2. The created algebraic signal interpolation algorithm determines the optimal number of components of the exponential function that describe the signal. As a result, the drawback of the unknown number of model components when applying other types of Prony's methods is eliminated.
3. The created algebraic interpolation algorithm can be applied not only in order to interpolate the unknown signal values, but also to extrapolate them. This feature can be applied to time series forecasting tasks.
4. New methods are created for the identification and analysis of electrocardiographic and ultrasonic signals.

1.5. Statements Presented for the Defense

1. A new Prony signal (time series) identification model was proposed which ensures approximation of the analyzed signals (time series) by the sum of exponential functions with polynomial coefficients;
2. An extended Prony-type interpolation algorithm was suggested which ensures optimal parameter identification of the signal (time series) which can be described by linear recurrent sequences;
3. Prototypes of program systems were proposed which ensure analysis of electrocardiographic and ultrasound signal algebraic estimates.

1.6. Approbation of Dissertation Results

The topic of the thesis is covered in 7 scientific publications, including 3 publications in the main list of the Institute for Scientific Information (ISI) publications with a citation index, 1 publication in nationally (in Lithuania) recognized periodical publication, and 3 publications in international conference proceedings.

The results were presented and discussed at 4 scientific conferences.

1.7. Scope and Structure of the Thesis

The dissertation consists of the introduction, 5 main chapters, conclusions, literature sources, a list of publications and annexes. The dissertation contains 138 pages, 66 pictures, 13 tables, and 117 sources of cited references.

Chapter 1 consists of a brief discussion of the signal properties, a review of signal identification methods suggested by other authors, the theoretical background of the linear recurrent sequence and the concept of the minimal order of a sequence. Also, this part introduces the analyzed electrocardiographic and ultrasonic signal characteristics, application areas, problems occurring in the course of analysis and the research methods applied as their solution at the end of this chapter.

Chapter 2 provides the theoretical foundations of the concept of the minimal order of a sequence and its application for the fragment of a sequence identification algorithm. This chapter, presents constructed algorithms and their application for the sequence (consisting of several different linear recurrent sequences) fragmentation.

Chapter 3 introduces the theory of linear recurrent function and its application. This part presents the new extended Prony's interpolation method and its comparison with other well-known interpolation methods.

Chapter 4 is devoted to the prototype of the program system for electrocardiographic signal complexity analysis. This part presents a new ECG complexity analysis method based on the extended Prony's interpolation algorithm. At the end of the chapter, the practical usage of the prototype for veloergometry test data is shown.

Chapter 5 introduces application of an extended Prony's model and an interpolation method for the ultrasonic signal identification. A prototype of the program system for signal identification and analysis is also presented. At the end of the chapter, the designed prototype applications are provided for experimental ultrasonic signal data.

2. SIGNAL IDENTIFICATION TECHNIQUE AND METHODS

In this dissertation, the discrete-time real valued (complex valued) signal and its identification methods are investigated. The discrete-time signal is a sequence or a series of signal values defined in discrete points of time t. The distance in time between each point of time is the time-step which can be denoted $\Delta t,\ t_j = j\Delta t,\ j = 0,1,\ldots,N-1$ (where N is the number of available observations). The time series can be written in the following way:

$$y_0 = f(t_0), y_1 = f(t_1), \ldots, y_j = f(t_j),\ j = 0,1,\ldots,N-1 .$$

In order to make the notation simple, we can write the signal $f(t)$.

In this work, we shall exclusively deal with deterministic signals as their physical description is known completely, either in a mathematical form or in a graphical form (Lyons, 2011).

2.1. Signal Identification Technique

Currently, identification is an integral part of the modern industrial control and automation schemes. The task of identification, however, appears in almost every walk of life. Identification is the task of using input-output data when seeking to build an empirical model; it is a mathematical abstraction of the process shown in Figure 2.1. The main object of interest is the deterministic (input-output) component of the model because it captures the dynamics of the physical process.

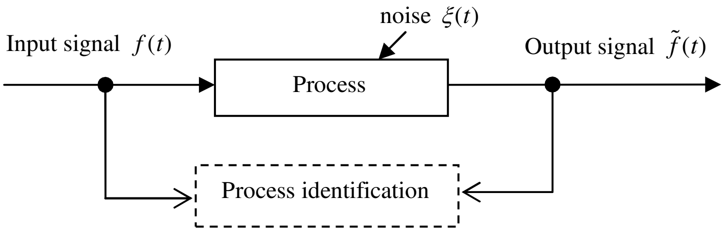

Figure 2.1. Process identification by using the 'black-box' model

Most modern processes of interest are complex to the extent that precludes a fundamental approach. The natural recourse has been towards data-driven approaches since they assume minimal prior knowledge and largely depend on input-output observations for developing empirical models. In this dissertation, we are constructing new mathematical methods for the development of these 'black-box' models . It should be noted that the 'black-box' modeling approach, the input-output relationship is estimated from experimental data only.

For many physical and mathematical applications, under the assumption that a given signal is the superposition of many individual components, it is often desirable to decompose this signal into individual components (James Hu *et al.*, 2013):

$$f(t) = \sum_{k=1}^{m} f_k(t), \quad t = 0,1,....,N-1; \tag{2.1}$$

where f_k – k-th signal component, $m \in N$ – a number of components.

In scholarly literature, this signal identification technique is also named *signal decomposition* (James Hu *et al.*, 2013; Chacko and Ari, 2012; Boßmann *et al.*, 2012) and the number of components (m) is named the model order (Yin, Zhu and Ding, 2011).

The general model expression of identifying output signal (2.1) (Potts and Tasche, 2010) is as follows:

$$\tilde{f}(t) = \sum_{k=1}^{m} f_k(t) + \xi(t), \quad t = 0,1,....,N-1; \tag{2.2}$$

where $\xi(t)$ is the Gaussian (white) noise term.

One of the tasks of the dissertation is to obtain a signal model as close as possible to $f(t)$ with minimizing the effect of $\xi(t)$ and to find the optimal value of m.

2.2. Signal Identification Methods

The most widely used signal identification techniques are Fourier-based (Shou-peng and Pei-wen, 2006) methods which decompose a signal into the sum of many, possibly infinite, simple sine and cosine functions.

Fourier expansions

$$F_F(t) = \sum_{k=-\infty}^{\infty} \mu_k \exp\left(i\frac{2\pi k}{T} t \right);$$

are successfully exploited for the approximation of function $f(t)$ in a variety of theoretical and practical applications. Fourier techniques involve the assumption that a signal is either infinite in duration or repetitive within some fundamental period over all the time.

However, a recorded signal is always finite in duration, most likely aperiodic, and even damped. In contrast to Fourier, Prony's method computes an approximation to function $f(t)$ by using only a finite number of damped complex exponentials (Osborne and Smyth, 1995, Ravanbod, Karimi and Amindavar, 2013):

$$F_P(t) = \sum_{k=1}^{m} \mu_k \exp(\lambda_k t);$$

where $m \in N$ and $\mu_k, \lambda_k \in C$. Prony-type methods are successfully exploited for effective and accurate approximation of different functions and signals (Martin, Miller and Pearce, 1989; Fuite, 2007; Giesbrecht and Labahn, 2009; Steedlt, 1992) (even though the determination of m remains problematic in general).

Other popular signal decomposition techniques include signal deconvolution (Boßmann *et al.*, 2012), wavelet (Boßmann *et al.*, 2012), cosine transform (Li, Wang, Huang and Lu, 2010), empirical mode decomposition (Janušauskas, Jurkonis, Lukoševicius, Kurapkienė and Paunksnis, 2005), etc.

As mentioned previously, the goal of this dissertation is to modify Prony's method for the identification of signals. Accordingly, in this research, we use the extended Prony-type scheme in order to interpolate a function on an equispaced grid. The extended Prony‘s scheme comprises complex exponentials and polynomials:

$$F_{PP}(t) = \sum_{k=1}^{m} Q_k(t)\exp(\lambda_k t);$$

where $m \in N$; $\lambda_k \in C$ and $Q_k(t)$ are polynomials in t with complex coefficients and non-negative integer powers of t.

In this dissertation, we present a new Prony-type signal identification method which identifies signals by using an extended Prony’s model. The proposed method is based on the concept of the minimal order of the linear recurring sequence and linear recurring functions.

3. LINEAR RECURRING SEQUENCES AND THEIR APPLICATIONS

3.1. The Concept of the Minimal Order of Linear Recurring Sequence

Let us consider the following sequence:

$$y_0, y_1, y_2, ... := (y_j; j \in Z_0);$$

where elements y_j can be real or complex numbers. Then, a sequence of Hankel matrices reads:

$$H_n := (y_{i+j-2})_{1\le i,j\le n} = \begin{bmatrix} y_0 & y_1 & \cdots & y_{n-1} \\ y_1 & y_2 & \cdots & y_n \\ & & \cdots & \\ y_{n-1} & y_n & \cdots & y_{2n-2} \end{bmatrix}; n = 1,2,....$$

The Hankel transform (the sequence of determinants of Hankel matrices) $(d_n; n \in N)$ reads:

$$d_n := \det H_n.$$

Definition 3.1. The minimal order of the recurring sequence $(y_j; j \in Z_0)$ is $m \in Z_0$; $m < +\infty$

$$rank(y_j; j \in Z_0) = m$$

if the sequence of determinants of Hankel matrices has the following structure:

$$(d_1, d_2, ..., d_m, 0, 0, ...); \qquad (3.1)$$

where $d_m \ne 0$ and $d_{m+1} = d_{m+2} = ... = 0$ (Kurakin *et al.*, 1995; Kurakin, 2001).

For example, let $y_j := j^2$, $j \in Z_0$. Then, $rank(j^2; j \in Z_0) = 3$ because the sequence of determinants of Hankel matrices reads $(0,-1,-8,0,0,\ldots)$.

Let $rank\left(y_j; j \in Z_0\right) = m$. Then the characteristic polynomial for the sequence $(y_j; j \in Z_0)$ is defined as (Kurakin *et al.*; 1995, Kurakin, 2001):

$$\hat{d}_m := \det \hat{H}_m := \begin{vmatrix} y_0 & y_1 & \cdots & y_m \\ y_1 & y_2 & \cdots & y_{m+1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{m-1} & y_m & \cdots & y_{2m-1} \\ 1 & \rho & \cdots & \rho^m \end{vmatrix} = 0. \tag{3.2}$$

The expansion of the determinant in Equation (3.2) yields an m-th order algebraic equation for the determination of roots of the characteristic polynomial:

$$A_m\rho^m + A_{m-1}\rho^{m-1} + \ldots + A_1\rho + A_0 = 0; \tag{3.3}$$

where $A_m \neq 0$ because $d_m \neq 0$.

Theorem 3.1. Let the minimal order of sequence $\left(y_j; j \in Z_0\right)$ be m and the multiplicity indexes of roots $\rho_1, \rho_2, \ldots, \rho_l$ of the characteristic polynomial (Equation 3.3) be $m_1, m_2, \ldots, m_l$ accordingly; $\sum_{r=1}^{l} m_r = m$. Then the following equality holds true (Kurakin *et al.*; 1995, Kurakin, 2001):

$$y_j = \sum_{r=1}^{l}\sum_{k=0}^{m_r-1} \mu_{rk} \binom{j}{k} \rho_r^{j-k}; \tag{3.4}$$

where $\mu_{rk}, \rho_r \in C$; $\mu_{rm_r-1} \neq 0$ (Navickas and Bikulčienė, 2006).

We must note that $\mu_{rk}\binom{j}{k}\rho_r^{j-k} = 0$ if $\binom{j}{k} = 0$ which is true when $0 \leq j < k$. Moreover, $0^0 = 1$; $0^1 = 0^2 = \ldots = 0$.

The opposite statement holds as well. If Equation (3.4) holds true, then

$$rank(y_j; j \in Z_0) = m_1 + m_2 + \ldots + m_l.$$

Rigorous proof of this theorem is given in Navickas and Bikulčienė (2006).

In general, in Expression (3.4), one of the roots can be equal to 0, for example, with index m_0.

Theorem 3.2. Let the minimal order of the sequence $\left(y_j; j \in Z_0\right)$ be m, the index of root $\rho_0 = 0$ is m_0 and multiplicity indexes of roots $\rho_1, \rho_2, \ldots, \rho_l$ of the characteristic polynomial (Eq. 3.3) are $m_1, m_2, \ldots, m_l$ accordingly; $\sum_{r=0}^{l} m_r = m$. Then, the following equality holds true:

$$y_j = \sum_{k=0}^{m_0-1} \mu_{0k}\binom{j}{k} 0^{j-k} + \sum_{r=1}^{l}\sum_{k=0}^{m_r-1} \mu_{rk}\binom{j}{k}\rho_r^{j-k}; \tag{3.5}$$

where $\mu_{0m_0-1}, \ \mu_{rm_r-1} \neq 0, \ r = 1, \ldots, l.$

Definition 3.2. Sequence $(y_j; j \in Z_0)$ is a *linear recurring sequence* (LRS) if elements of the sequence can be expressed in the form of Equation (3.4) or Equation (3.5).

Corollary 3.1. Equation (3.4) can be rewritten in the following form:

$$\sum_{r=1}^{l}\sum_{k=0}^{m_r-1}\mu_{rk}\binom{j}{k}\rho_r^{j-k} = \sum_{r=1}^{l}\left(\sum_{k=0}^{m_r-1}\hat{\mu}_{rk}\, j^k\right)\rho_r^j; \tag{3.6}$$

where $\rho_1, \rho_2, ..., \rho_l \neq 0$ and $\mu_{rm_r-1} \neq 0$.

Remark 3.1. We should note that coefficients μ_{rk} are determined in order to fit the initial conditions of the recurrence (roots $\rho_1, \rho_2, ..., \rho_l$ are defined by Equation (3.2)):

$$\sum_{r=1}^{l}\sum_{k=0}^{m_r-1}\binom{j}{k}\rho_r^{j-k}\mu_{rk} = y_j;\ j = 0,1,...,m-1 \tag{3.7}$$

or

$$\sum_{k=0}^{m_0-1}\mu_{0k}\binom{j}{k}0^{j-k} + \sum_{r=1}^{l}\sum_{k=0}^{m_r-1}\mu_{rk}\binom{j}{k}\rho_r^{j-k} = y_j;\ j = 0,1,...,m-1. \tag{3.8}$$

This system of linear equations has a unique solution (Navickas and Bikulčienė, 2006).

Remark 3.2. Let $rank(y_j; j \in Z_0) = m$ and the first $2m$ elements of that series be known. Then, it is possible to use Equation (3.3), Equation (3.7) (or Equation (3.8)) and Equation (3.5) (or Equation (3.6)) to calculate all the elements of that sequence.

Corollary 3.2. In the application of Theorem 3.1 or Theorem 3.2, it is important to distinguish the following:

(a) $\omega_j^{(0)} = \sum_{k\,:\,|\rho_k| = 1}\mu_k\rho_k^j,\ j \in Z_0$ – stationary component;

(b) $\omega_j^{(1)} = \sum_{k\,:\,|\rho_k| > 1}\mu_k\rho_k^j,\ j \in Z_0$ – stimulant component;

(c) $\omega_j^{(-1)} = \sum_{k\,:\,|\rho_k| < 1}\mu_k\rho_k^j,\ j \in Z_0$ – inhibitory component.

of time series which consists of the complexity of the content of time series:

$$y_j = \omega_j^{(-1)} + \omega_j^{(0)} + \omega_j^{(1)}$$

It must be also stated that roots $\rho_1, \rho_2, \rho_3, ..., \rho_l$ of the characteristic equation (Equation (3.3)) can be placed on the unit circle (Figure 3.1).

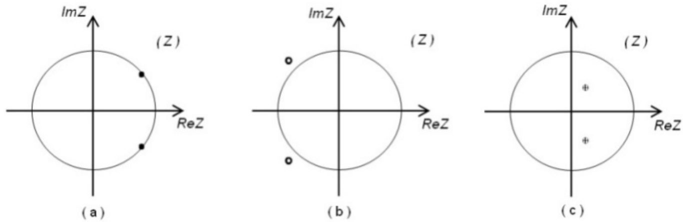

Figure 3.1. (a) – stationary – on the unit circle $|z| = 1$; (b) – stimulant – outside of the unit circle $|z| = 1$; (c) – inhibitory roots are inside the unit circle $|z| = 1$

Remark 3.3. The determination of the minimal order of sequence $\{y_j\}$ by using Definition 3.1 has a cost of

$$\sum_{j=1}^{m} O(j^3) = O(m^4)$$

flops. Construction of characteristic polynomial Equation (3.3) needs $O(m^4)$ flops, and $O(m^2)$ flops are required to find the roots of this polynomial. A system of linear equations Equation (3.6) or Equation (3.7) can be solved by using the Gaussian elimination method; the process has a cost of $2m^3/3$ flops.

3.2. LRS Theory Application for the Fragment of a Sequence

In practical applications, for example, in signal processing, finite sets of sequences are usually used. In this dissertation, we present LRS theory applications for the fragment of a sequence.

Let us consider a continuous sequence $Y := (y_0, y_1, y_2, ..., y_{L-1}) = (y_j; j := \overline{0, L-1})$, where elements can be real or complex numbers $y_j \in C$, $j \in \overline{0, L-1}$, L – a number of elements of the analyzed fragment (the length of the fragment).

Definition 3.2. A fragment of sequence Y is a finite set of elements written in the following order:

$$S(Y, a, b) = (y_a, y_{a+1}, y_{a+2}, ..., y_{a+L-1});$$

where $L = b - a + 1$, $L \in N$, $a \le b$, $a, b \in Z_0$, a is the start, b is the end position of the fragment.

On the basis of the concept of the minimal order of LRS, an algebraic fragment identification method was developed which can be applied for the construction of the LRS expression of time series. The developed fragment identification algorithm is shown in Figure 3.2.

The proposed algebraic fragment identification algorithm was used for the construction of other identification methods and algorithms:

- Fragment identification algorithm when the LRS expression of the fragment part is known. This property could be applied for solving interpolation and extrapolation (forecasting) problems;
- Minimal order of LRS identification method. This method could be applied for finding the optimal number of a signal model (Equation 2.2) components;
- Fragmentation algorithm for a sequence consisting of several different LRS.

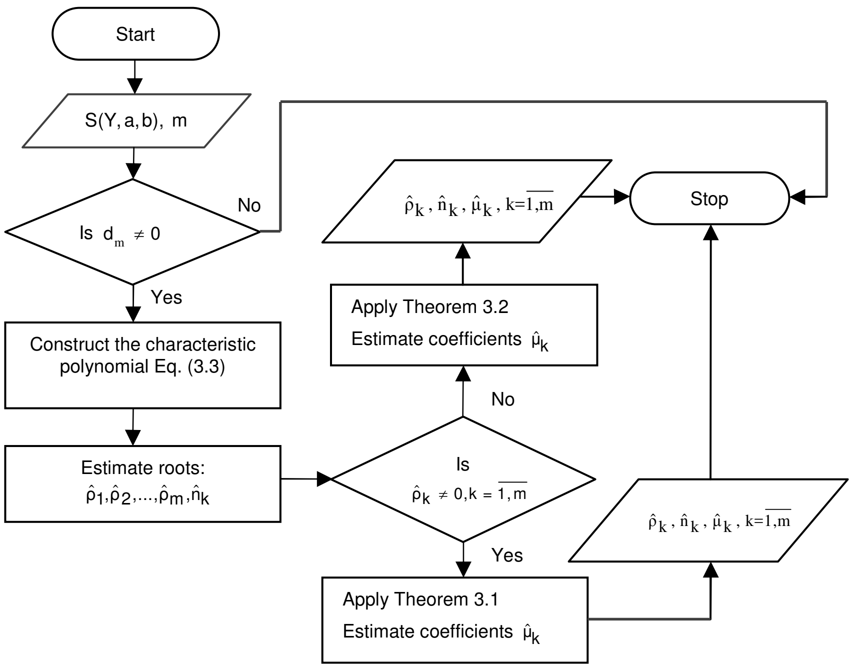

Figure 3.2. Fragment $S(Y, a, b)$ identification algorithm (FIM)

4. EXTENDED PRONY−TYPE INTERPOLATION METHOD

4.1. Algebraic Interpolation Scheme on an Equispaced Grid

It is well known that the *n*-point polynomial interpolants in equally spaced points do not necessarily converge to function *f* on [-1, 1] as $n \to \infty$, even if *f* is analytic. Instead, one may see wild oscillations near the endpoints, an effect known as the Runge phenomenon (Runge, 1901). Moreover, the interpolation process becomes exponentially ill-conditioned, as shown first by Turetskii (1940) and later independently by Schonhage (1961). This ill-conditioning means that even if the interpolants converge in theory, they will diverge in floating point arithmetic, at least for values of t near the endpoints because of the exponential amplification of rounding errors.

On the other hand, the polynomial interpolation in Chebyshev points is numerically stable since the associated Lebesgue constants are of size $O\left(\log\left(n\right)\right)$ (Ehlich and Zeller, 1996). Is is shown in the manuscript of Higham (2004) that the Chebyshev interpolant can be evaluated in floating point arithmetic by Salzer's (1972) barycentric formula. Moreover, Chebyshev interpolants are used in the Chebfun software where polynomials in degrees of tens of thousands are routinely used for practical computations (Trefethen, 2008; Platte, Trefethen and Kuijlaars, 2011).

In the following subsection, a strategy will be developed for finding the nearest algebraic interpolant to an analytic function. It will be demonstrated that an approach based on the nearest algebraic interpolant does not suppress the Runge phenomenon, but interpolation errors produced by this method are much lower if compared to the classic schemes on equispaced grids. Moreover, the proposed algebraic interpolation method can be effectively exploited for analytic interpolation of noisy and/or defected signals, for example, ECG, ultrasonic, etc.

4.2. Linear Recurring Functions and Their Properties

The definition of the linear recurring function and their properties will be presented in this subsection.

Definition 4.1. Linear recurring function (LRF) $f\left(t\right)$ is expressible in the following form:

$$f(t)=\sum_{r=0}^{n}Q_r(t)\exp\left(\lambda_r t\right); \tag{4.1}$$

where $Q_r(t)=\sum_{k_r=0}^{m_r-1}a_{rk_r}t^{k_r}$; $m_r\geq 1$; $a_{rk_r}\in C$; $a_{r,\left(m_r-1\right)}\neq 0$; $r=0,1,\ldots,n$ and $t,f\left(t\right)\in R.$ It is important to note that n is finite in Equation (4.1).

Theorem 4.1. Let $f\left(t\right)$ be an LRF. Then the sequence

$$y_j := f(p+jh);\ j\in 0,1,2,\ldots, \tag{4.2}$$

is a LRS ($p,h\in R$ are fixed parameters).

Proof. Let the function $y=f(t)$ be an LRF. Then, the following equalities hold for all $p,h\in R$:

$$y_j := f(p+jh) = \sum_{r=1}^{l}\left(\sum_{k=0}^{m_r-1} a_{rk}(p+jh)^k \exp\left(\lambda_r(p+jh)\right)\right)$$
$$= \sum_{r=1}^{l}\left(\sum_{k=0}^{m_r-1} b_{rk}\cdot j^k\right)\left(\exp\left(\lambda_r h\right)\right)^j; \tag{4.3}$$

where coefficients b_{rk} can be expressed in terms of coefficients $a_{r0}, a_{r1}, ..., a_{rm_r-1}$. It can be noted that index *k*, parameters p and h do not depend on *j*. The introduction of symbol

$$\rho_r = \exp\left(\lambda_r h\right) \tag{4.4}$$

reduces into the LRS:

$$y_j = \sum_{r=1}^{l}\left(\sum_{k=0}^{m_r-1} b_{rk}\cdot j^k\right)\rho_r^j;\ j = 0,1,2,\ldots. \tag{4.5}$$

Theorem 4.2. Let $(y_j; j \in Z_0)$ be an LRS. Then, the following inequalities hold true:

$$0 \le rank(y_j;\ j \in Z_0) \le m_1 + m_2 + ... + m_r.$$

Definition 4.2. Let $f(t)$ be a LRF. Then, the minimal order of $f(t)$ is denoted as $rank\left(f(t)\right)$ and is defined as follows:

$$rank\left(f(t)\right) := \max_{p,h} rank\left(f(p+jh; j \in Z_0)\right); \tag{4.6}$$

where $p, h \in R;\ h > 0$.

For example, $rank\left(\cos(t)\right) = 2$.

Theorem 4.3. Let $(y_j; j \in Z_0)$ be a representative LRS. Then, indexes $\lambda_1, \lambda_2, ..., \lambda_l$ read:

$$\lambda_r = \frac{1}{h}\left(\ln|\rho_r| + i(\arg\rho_r + 2\pi k_r)\right);\ k_r = 0, \pm1, \pm2, ...;\ r = \overline{1,l}. \tag{4.7}$$

Definition 4.3. Step *h* is sufficiently small if there exist such $h_0 > 0$ that the following relationship holds true for all $0 < h < h_0$:

$$\lambda_r = \frac{1}{h}\left(\ln|\rho_r| + i\arg\rho_r\right); \tag{4.8}$$

where $0 \le \left|\arg\rho_r\right| < \pi$ and $r = \overline{1,l}$.

Lemma 4.1. If $f(t)$ is LRF defined by Equation (4.1), then such $h_0 > 0$ exists that Equation (4.8) holds true.

4.3. The Extended Prony Interpolation Algorithm

Let us assume that function $f(t)$ is not necessarily LRF. Then, the reconstruction of the closest LRF to function $f(t)$ in the interval $a \le t \le b$ would be an important practical problem which is discussed in details in this subsection.

First of all, step h of the regular grid in the interval $[a;b]$ should be selected. The selection of step h is directly related to the order of the LRF $F(t)$ which will be used to mimic the original function $f(x)$. Let the order of $F(t)$ be m (we strive to mimic $f(t)$ by using an LRF with the order equal to m). Then, step h reads:

$$h = \frac{a-b}{2m-1}.$$

Now, function $f(t)$ can be sampled at the nodes of the grid:

$$y_0 = f(a); y_1 = f(a+h); y_2 = f(a+2h); \ldots; y_{2m-1} = f(b). \quad (4.9)$$

We have assumed that the minimal order of the LRF $F(t)$ is m. Therefore, according to Equation (3.1), the following equality holds true:

$$d_{m+1} = \det \begin{bmatrix} y_0 & y_1 & \cdots & y_{m-1} & y_m \\ y_1 & y_2 & \cdots & y_m & y_{m+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_m & y_{m+1} & \cdots & y_{2m-1} & F(b+h) \end{bmatrix} = 0. \quad (4.10)$$

We should note that it is easy to determine $F(b+h)$ from Equation (4.10) (Ragulskis *et al.*, 2011). However, we will not use $F(b+h)$ (nor $f(b+h)$) in further computations.

Now, the characteristic polynomial (Equation 3.3) takes the form:

$$\det \begin{bmatrix} f(a) & f(a+h) & \cdots & f(a+mh) \\ f(a+h) & f(a+2h) & \cdots & f(a+(m+1)h) \\ \cdots & \cdots & \cdots & \cdots \\ f(a+(m-1)h) & f(a+mh) & \cdots & f(b) \\ 1 & \rho & \cdots & \rho^m \end{bmatrix} = 0. \quad (4.11)$$

Let us assume that all roots $\rho_1, \rho_2, \ldots, \rho_m$ are different. Then, Equation (4.7) can be used to compute indexes λ_r; $r = 1, 2, \ldots, m$. Now, a linear system of equations is constructed by using Equation (4.2); its solution produces coefficients μ_r; $r = 1, 2, \ldots, m$. Finally, the mimicking LRF in the interval $a \le t \le b$ reads:

$$F(t)=\sum_{r=1}^{m}\mu_r \exp(\lambda_r t).$$

If some roots of Equation (4.11) are multiple, the algorithm of computations is similar, yet the expression of the mimicking algebraic interpolant becomes more complex (Equation 3.4 or Equation 3.5). This proposed method can be called the extended Prony's interpolation method. Its algorithm scheme is shown in Figure 4.1.

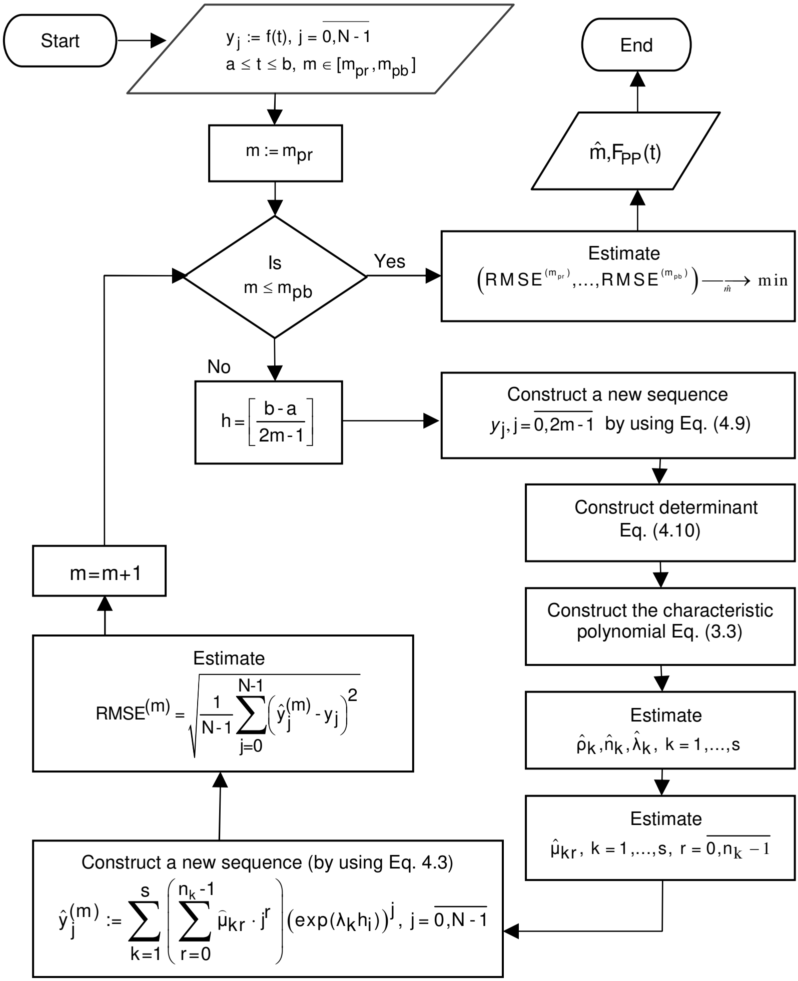

Figure 4.1. The extended Prony's interpolation (EPI) algorithm

Example 4.1. Algebraic interpolation of an LRF.

Let us consider the following LRF:

$$f_a(t) = 0.3t^2 \sin(2.14t)e^{-0.13t} + \cos(0.18t)e^{-0.31t}.$$

We will construct the algebraic interpolation of this function in the interval $0 \le t \le 10$. Let us assume that the order of LRS of values of $f_a(t)$ is m $\left(rank\left(y_j; j \in Z_0\right) = m\right)$; the linear recurrent function is denoted as $F_m(t)$. Then, $h = \dfrac{10}{2m-1}$ and $y_j = f_a\left(jh\right)$; $j = 0,1,\ldots,(2m-1)$. We perform a number of computational experiments for different values of m; $m = 1,2,\ldots,35$. The algorithm of algebraic interpolation produces 35 different algebraic functions $F_m(t)$ and 35 values of RMSE (root mean square errors) of the interpolation defined as

$$RMSE^{(m)} = \sqrt{\frac{1}{10}\int_0^{10}\left(f_a(t) - F_m(t)\right)^2 dt}.$$

The variation of RMSE from m is illustrated in Figure 4.2.

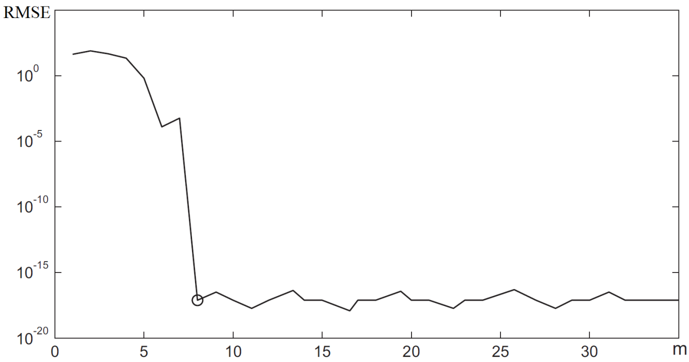

Figure 4.2. RMSE errors of the algebraic interpolation at different *m*; the circle denotes the best *m* (RMSE = 0 at *m* = 8)

It is clear that RMSE = 0 when $m = 8$ because the order of $f_a(t)$ is equal to 8. We will illustrate the EPI algorithm of algebraic interpolation for $m = 8$ in detail. Step h is equal to $2/3$; the characteristic polynomial has eight roots: $\rho_{1,2,3} = 0.1317 + 0.9075i$; $\rho_{4,5,6} = 0.1317 - 0.9075i$; $\rho_{7,8} = 0.8074 \mp 0.0974i$.

The values of λ_r; $r = \overline{1,8}$ (computed by using Equation (4.7) at $k_r = 0$; $r = \overline{1,8}$) read:

$\lambda_1^* = \lambda_{1,2,3} = -0.0867 + 1.4267i,\ \lambda_2^* = \lambda_{4,5,6} = -0.0867 - 1.4267i,\ \lambda_{3,4}^* = \lambda_{7,8} =$ $-0.2067 \mp 0.1200i.$ Now, Equation 3.4 yields the following equality:

$$(\mu_1 + \mu_2 j + \mu_3 j^2)e^{j\lambda_1^*} + (\mu_4 + \mu_5 j + \mu_6 j^2)e^{j\lambda_2^*} + \mu_7 e^{j\lambda_3^*} + \mu_8 e^{j\lambda_4^*} = f(jh),\ j = \overline{0,7}.$$

Solutions of the linear algebraic system of equation now read: $\mu_{1,2,3,4} = 0;\ \mu_{3,6} = \mp 0.0667i;\ \mu_{7,8} = 0.5.$

Finally, the expression of $F_8(t)$ reads:

$$\begin{aligned} F_8(t) = {} & \left(1.5\right)^2 (-0.0667i)t^2 e^{1.5(-0.0867+1.4267i)t} + \left(1.5\right)^2 0.0667i \cdot t^2 e^{1.5(-0.0867-1.4267i)t} + \\ & 0.5e^{1.5(-0.2067+0.12i)t} + 0.5e^{1.5(-0.2067-0.12i)t} = -0.15it^2 e^{-0.13t}(e^{2.14it} - e^{-2.14it}) + \\ & 0.5e^{-0.31t}(e^{0.18it} + e^{-0.18it}) = -0.15it^2 e^{-0.13t}(\cos(2.14t) + i\sin(2.14t) - \\ & \cos(-2.14t) - i\sin(-2.14t)) + 0.5e^{-0.31t}(\cos(0.18t) + i\sin(0.18t) + \\ & \cos(-0.18t) + i\sin(-0.18t)) = 0.3t^2 \sin(2.14t)e^{-0.13t} + \cos(0.18t)e^{-0.31t}. \end{aligned}$$

The algebraic interpolant is illustrated in Figure 4.3.

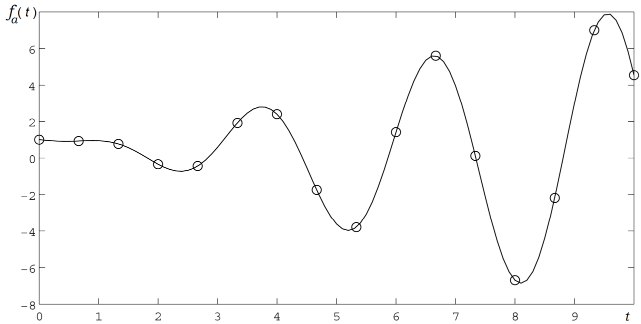

Figure 4.3. Algebraic interpolant of $f_a(t)$ in the interval $\left[0,10\right]$ (the interpolant coincides with the function; circles denote nodes of the equispaced grid)

As the following step, EPI algorithm application for the real time series identification will be presented .

4.3.1. Algebraic Interpolation of the Real Time Series

The possibility of algebraic interpolation on a regular grid (for the nearest algebraic interpolant) suggests interesting possibilities for the application of this approximation scheme for the real time series. We use a time series of monthly PMI Composite Index. A PMI reading above 50 percent indicates that the manufacturing economy is generally expanding whereas the value below 50 percent suggests that it is generally declining. The unfiltered PMI data S_k in interval $k = 1, 2, \ldots, 788$ is shown in Figure 4.4.

We will use the algebraic interpolation scheme in an equally-spaced grid in the interval [1; 788]. First, we preselect the order of the LRF which will be used to mimic the experimental data. For the order of the LRF equal to *m*, step $h = \frac{787}{2m-1}$ and $y_n = S_j$; $j = 1 + nh$; $n = 0,1,\ldots,2m-1$.

Once more we perform a number of computational experiments for different values of *m*; $m = 1,2,\ldots,90$. The algorithm of algebraic interpolation produces 90 different LRF $F_m(t)$. RMSE of algebraic interpolation is now computed as the square root of the sum of squared differences between the values of the given data and the values of the LRF $F_m(t)$ at all sampling points in the interval [1; 788]. Computational experiments show that the best result (the minimal RMSE = 3.17) is achieved at $m = 56$ (Figure 4.6). The graph of $F_{56}(t)$ is shown in Figure 4.4.

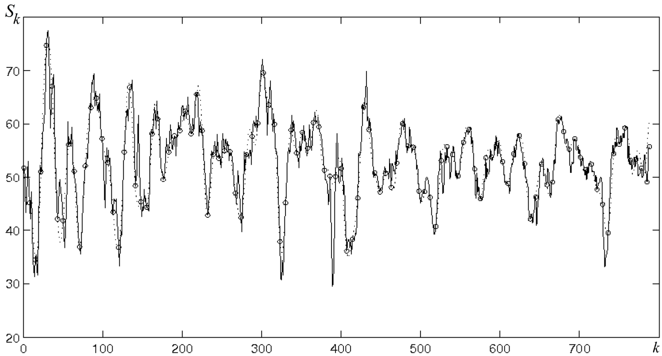

Figure 4.4. PMI Composite Index data (the solid line) and an algebraic interpolant (the dotted line) of the experimental data at *m* = 56.

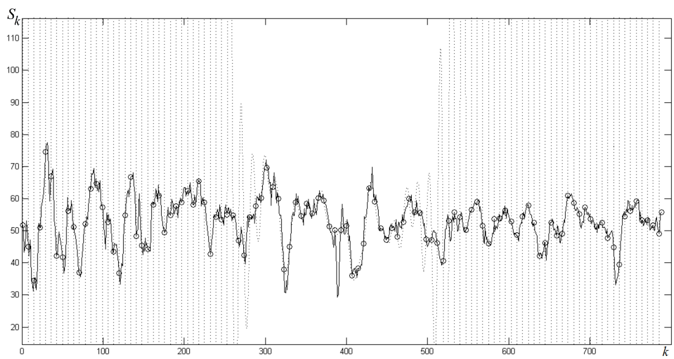

Figure 4.5. PMI Composite Index data (the solid line) and the Lagrangian interpolant (the dotted line). The circles denote nodes of the equispaced grid

As noted previously, the extended Prony-type interpolation scheme outperforms the Lagrange polynomial on equispaced grids. The Lagrangian interpolant is illustrated in Figure 4.5 (the nodes are the same as in Figure 4.4). Runge's effect prevents Lagrange interpolation from being a reasonable approximation (the maximum absolute value of the Lagrange interpolant in Figure 4.5 is equal to $3.1634 \cdot 10^{32}$), yet the proposed extended Prony-type interpolation does not face that problem.

The polynomial interpolation in Chebyshev points is numerically stable even for high polynomial degrees. However, real-world time series are usually recorded by using a constant sampling rate. Thus it would be complicated to find the values of this time series at Chebyshev points without transforming the scale (Chebyshev interpolation is straightforward for continuous functions of course). $F_{56}(t)$ in Figure 4.5 is a good example of such alternative interpolation.

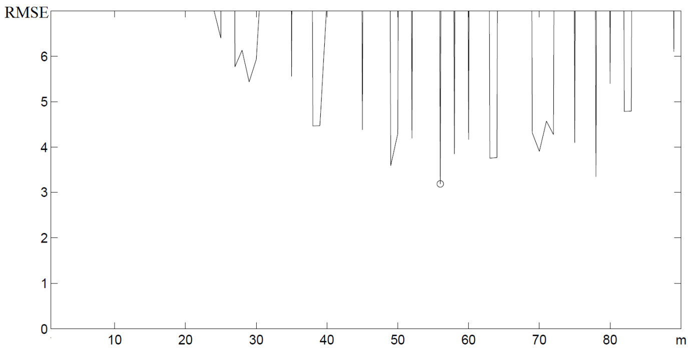

Figure 4.6. RMSE errors of the algebraic interpolation at different *m*; the circle denotes the best *m* (RMSE = 3.17 at m = 56)

Numerical experiments have shown that optimal interpolants produced by the proposed algebraic technique based on the order of LRS outperform classical Lagrange polynomial interpolants on equispaced grids. This effect can be explained by the fact that the functional base used to construct algebraic interpolants is wider if compared to polynomial interpolants. As a matter of fact, Equation (4.1) would represent a polynomial interpolant if all indexes $\lambda_r; r = 0, 1, \ldots, n$ were equal to zero.

The main drawback of the proposed interpolation scheme is that rather complex computations are required as the number of nodes becomes large. In the first place, this is associated with the necessity to find all the roots of the characteristic Hankel equation. Thus the proposed scheme of interpolation loses its aptitude when the number of nodes becomes higher than one hundred. Nevertheless, the scheme preserves interesting potential of practical application at a lower number of nodes. The explicit error bound of the interpolation and the possibility of using adaptive grids remains a definite objective of future research.

4.3.2. Algebraic Interpolation of Noisy Signals with Defects

The possibility of extended algebraic interpolation on a regular grid suggests interesting possibilities for the application of this approximation scheme for noisy signals with defects. We use a data set from the experiment with surface acoustic waves where a digital oscilloscope is used to register ultrasonic surface waves in a carbon composite. It must be noted that the digital oscilloscope used in the experiment is capable of registering the signal only within a number of discreet magnitudes (the sensitivity step is 0.1 mV); yet, the sampling rate is quite high (Figure 4.7). Unfiltered raw measurement signal S_k in interval $k = \overline{11100,13500}$ is shown in Figure 4.7.

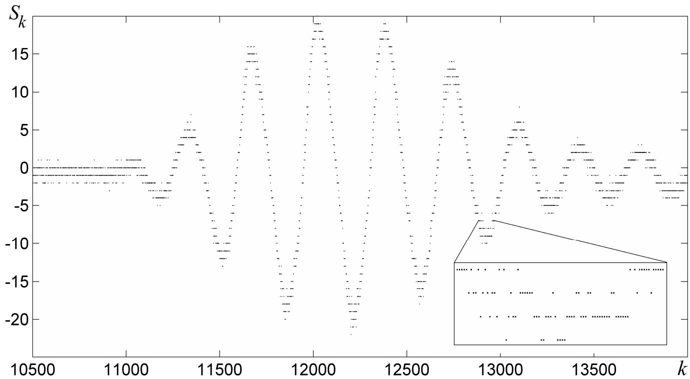

Figure 4.7. The experimental signal; the inset demonstrates the sensitivity step of the digital oscilloscope

We will use the algebraic interpolation scheme in an equally-spaced grid within the interval [11100; 13500]. First, we preselect the rank of the algebraic function which will be used to mimic the experimental signal. For the rank equal to *m*, the step will be $h = 2400/(2m-1)$ and $y_k = S_j$; $j = 11100 + kh$, $k = 0,1,..., (2m-1)$.

We again perform a number of computational experiments for different values of $m = 1,2,...,35$. The algorithm of algebraic interpolation produces 35 different algebraic functions $F_{PP}(t)$. The RMSE of algebraic interpolation is now computed as the square root of the sum of squared differences between the values of the measured signal and the values of algebraic function $F_{PP}(t)$ at all sampling points within the interval [11100; 13500]. Computational experiments show that the best result (the minimal RMSE=1.266) is achieved at m = 16. The graph of $F_{16}(t)$ is shown in Figure 4.8; the analytic representation of this algebraic function reads:

$$F_{16}(t) = 5.38 \cdot 10^{-5} e^{0.36t} \cos(0.76t) - 4.91 e^{-0.2t} \cos(0.76t) - 3.93 e^{-0.04t} \cdot \cos(1.19t) + 9.8 e^{-0.03t} \cdot \cos(1.41t) - 0.02 e^{0.08t} \cdot \cos(1.99t) - 0.29 e^{-0.03t} \cdot \cos(2.54t) + 0.97 e^{-0.19t} \cdot \cos(2.69t) + 0.15 e^{-0.01t} - 1.78 e^{-0.08t}.$$

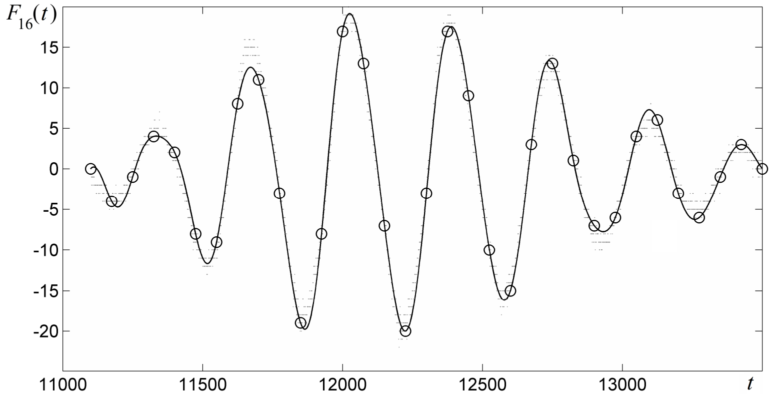

Figure 4.8. Algebraic interpolant (the thin solid line) of the experimental signal within the interval [11100, 13500] at m = 16. The circles denote nodes of the equispaced grid

Thus it is demonstrated that the proposed EPI algorithm can be used for the identification of the nearest algebraic interpolant for a given function. This computational effect can be explained by the fact that even though all the nodes are located inside the bounded interval, the interpolant is reconstructed in the global domain. Such an interpolation scheme can be extended to the extrapolation scheme what can be successfully exploited in time series prediction applications. On the other hand, this advantageous feature can be successfully exploited for the analytic approximation of noisy and/or defected real-world signals.

Finally, it can be noted that the proposed scheme can be considered as an effective numerical tool for the identification of the nearest algebraic 'skeleton' functions and extends the applicability of the classical interpolation schemes to the real-world data contaminated with the inevitable noise.

4.4. LRS versus Prony Decomposition

The composed algorithms were compared with the well-known Prony's interpolation method (Potts and Tasche, 2013) which is presented below.

Let us consider a sequence $y_j, j = 0, 1, \ldots, 2N$ where $K, N \in \mathrm{N}$, $3 \le K \le N$, $K -$ the upper bound of the number of exponentials, and bounds $\varepsilon_{0,1}$ are positive. An LRS of a given sequence could be found by using the Prony's interpolation method. The main steps of this algorithm (cf. Potts and Tasche, 2013) read:

1. Determine the smallest singular value of the rectangular Hankel matrix $H_{\breve{m}} = (y_{o+g})_{o,g=0}^{2N-K,K}$ and use singular value decomposition to find the related right singular vector $\mathbf{u} = (u_l)_{l=0}^{K}$;

2. Compute all the zeros of polynomial $\sum_{l=0}^{K} u_l \hat{\rho}^l$ and determine all the zeros $\hat{\rho}_k, k = 1,...,K$ for which $\left\| \hat{\rho}_k \right| - 1 \right| \le \varepsilon_0$, $k = 1,...,\overline{m}$. Note that $K \ge \overline{m}$.

3. For $\overline{e}_k = \hat{\rho}_k / |\hat{\rho}|, k = 1,...,\overline{m}$ compute $\overline{\mu}_k \in C, k = 1,...,\overline{m}$ as the least squares solution of the overdetermined linear Vandermonde-type system:

$$\sum_{k=1}^{\overline{m}} \overline{\mu}_k \overline{e}_k^j = y_j,\ j = 0,...,2N.$$

4. Delete all the $\overline{e}_l$, $l = 1,...,\overline{m}$ with $|\overline{\mu}_k| \le \varepsilon_1$ and denote the remaining entries by $\hat{e}_k$, $k = 1,...,\hat{m}$ with $\hat{m} \le \overline{m}$.

5. Repeat step 3 and compute $\hat{\mu}_k \in C$, $k = 1,...,\hat{m}$ as the least squares solution of the overdetermined linear Vandermonde-type system

$$\sum_{k=1}^{\overline{m}} \hat{\mu}_k \hat{e}_k^j = y_j,\ j = 0,...,2N.$$

in accordance to the new set $\hat{\mu}_k \in C$, $k = 1,...,\hat{m}$ again.

We must note that if ρ_k are multiple zeros of order n_k then coefficients $\mu_{k,r} \in C$ $(k = 1,...,\ddot{m},\ r = 0,...,\hat{n}_k)$ are obtained as the least squares solution of the overdetermined linear Vandermonde-type system:

$$\sum_{k=1}^{m} \left(\sum_{r=0}^{n_k} \mu_{k,r} j^r \right) e_k^j = y_j, j = 0,...,2N.$$

Example 4.2. Let us consider a sequence $y_j := j, j = 0,...,2N,\ N = 10.$ Let us find an LRS of a given sequence by using two alternative methods: a) an algebraic identification algorithm; b) the Prony's interpolation method (Potts and Tasche, 2013):

a) The minimal order of LRS of a given sequence is 2 because the sequence of determinants of Hankel matrices reads $(1, 0, -1, 0, 0, ..., 0)$. It can be found that the LRS of the given sequence reads:

$$\hat{y}_j^{(H)} = \mu_{10}\binom{j}{0}\rho_1^j + \mu_{11}\binom{j}{1}\rho_1^j = \binom{j}{1}1_1^j = j, j = 0,...,20.$$

b) Let $K = 9$, $\varepsilon_{0,1} = 10^{-10}$. According to the Prony's interpolation algorithm (Potts and Tasche, 2013), the LRS of the given sequence reads:

$$\hat{y}_j^{(P)} = 0.5j + 0.5j = j, j = 0,...,20.$$

Example 4.3. Let us consider the following function: $\tilde{t}_j = j$, $j =$ 0,1,2,..., $10, 11 + e$, $12, 13, ..., N-1$, $N = 101$, $e = -0.1$. Let us find an LRF of a given function by using the following methods: a) the extended Prony's (EPI) interpolation method; b) the Prony's interpolation method (Potts and Tasche, 2013).

a) Let the minimal order of the LRF be 14. Then the LRF of the given sequence reads:

$$\hat{Y}_j^{(H)} = \mu_{111}\binom{j}{11}\rho_1^{j-11} + \mu_{21}\binom{j}{1}\rho_2^j = j - 0.1\binom{j}{11}0^{j-11}, j = 0,...,100.$$

b) Let $K = [N/2-1] = 49$, $\varepsilon_0 = 10^{-4}$, $\varepsilon_1 = 10^{-4}$. The sequence of the singular values of Hankel matrix reads: $(264.7, 196.8, 0.1, ..., 5 \cdot 10^{-17})$. The characteristic polynomial $\sum_{l=0}^{49} u_l z^l$ has two multiple zeros (with property $\left\| \tilde{z}_t \right| - 1 \right| \le \varepsilon_1$): $\tilde{z}_1 = \tilde{z}_2 = 1$; Then, the constructed overdetermined linear Vandermonde-type system yields: $\tilde{c}_{11} = \tilde{c}_{21} = 0.5$, $\tilde{c}_{10} = \tilde{c}_{20} = -0.0026$, $\tilde{c}_{12} =$ $\tilde{c}_{22} = 0$ and the LRF of the given sequence reads:

$$\hat{Y}_j^{(P)} = -0.0026 + 0.5j - 0.0026 - 0.5j = j - 0.0052, j = 0,..., N-1.$$

Example 4.4. Now let us consider functions

$$\overline{y}_j = j, j = 0,1,2,...,10,11+e,12,13,...,N-1, N = 101;$$

where $e \in [-0.1; 0.1]$. Let us find an LRF of the each given function by using the extended Prony's and Prony's interpolation methods. The differences between the computed LRF functions are shown in Figure 4.9.

The thick solid line (see Figure 4.9) stands for $e = 0$; $\hat{Y}_j^{(P)}$ and $\hat{Y}_j^{(H)}$ coincide for all j then. However, even slight perturbation e results in computational errors in $\hat{Y}_j^{(P)}$. Moreover, these errors do not concentrate only around the 12-th element of the sequence – they are actually distributed throughout the whole domain (see Figure 4.9).

This example shows that the LRS theory (especially when the roots of the characteristic polynomial are multiple) enables formal manipulation with such algebraic expressions as 0^0 – which becomes a very important issue concerning practical problems of interpolation.

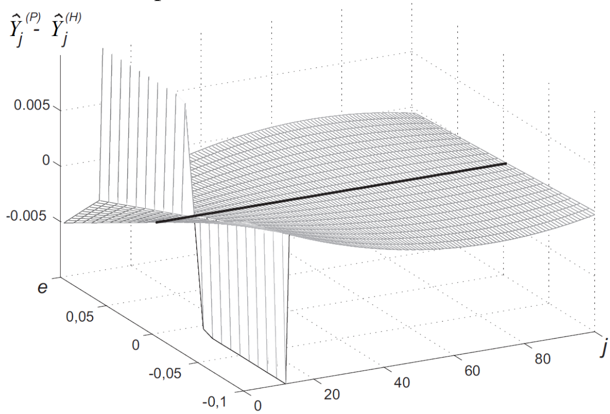

Figure 4.9. The difference between $\hat{Y}_j^{(P)} - \hat{Y}_j^{(H)}$ at different values of perturbation parameter *e*. The thick solid line stands for $e = 0$

5. ANALYSIS OF ALGEBRAIC ESTIMATES OF ELECTROCARDIOGRAPHIC SIGNALS

In this dissertation, the extended Prony's interpolation algorithm was used for the complexity analysis of the electrocardiographic (ECG) signal. The proposed technique was integrated in the prototype for the ECG complexity analysis system which was applied for the monitoring and analysis of physiological processes during the bicycle ergometry test.

5.1. Complexity Analysis of ECG Signal Based on the Concept of the Minimal Order of LRS

Studies of the complexity in human body functioning have recently proven to be an important area of research (Costa *et al.*, 2006). Physiological output signals such as the heart rate, the blood pressure and others are denoted by fluctuating dynamics. We can treat the human body as a complex system in which the organism adapts to the ever-changing environment. An integral evaluation model based on the combination of the three main functional holistic (Berskienė, Navickas and Vainoras, 2006) systems of the human body – the skeletal and muscle system (the performing system), the cardiovascular system (the supplying system) and the central nervous system (the regulatory system) was developed several decades ago. Thus all the three systems always react together during the adaptation processes in the human body; the general reaction of the body is always a combination of the responses of these systems. Reactions of the cardiovascular system to a constant load test (or to a gradually increasing load test) can reveal the peculiarities of the functioning of all the human body (Vainoras, 2002). At the onset of an exercise, the cardiovascular system adapts to the variations of loads with a series of integrated responses in order to meet the metabolic demands of the exercising muscles. Interconnections and fluctuations in the cardiovascular system output can provide valuable information not only towards the evaluation of the adaptation processes but also towards the improvement of the understanding of recovery processes. The period of recovery is most definitely influenced by the intensity of the exercise.

ECG parameters may have a different duration (larger structures could be associated with longer time scales) and could show the complexity in different fractal levels (Bikulčienė, Venskaitytė and Jaruševicius, 2014).

A number of techniques have been used for the analysis of ECG complexity: spectral analysis, entropy-based algorithms (as, for example, approximate entropy, sample entropy, multiscale entropy), chaos-based algorithms (as, for example, Lyapunov exponent, permutation entropy, Hankel matrix), algorithms for Kolmogorov estimates (as, for example, Lempel-Ziv, hidden Markov chains) and other methods (Talbi *et al.*, 2012; Costa, Peng and Goldberger, 2008; Milanesi *et al.*, 2009; Rickards, Ryan and Convertino, 2010; Conte, 2014). These methods evaluate the global features of processes and are unable to detect the local features of dynamical processes.

The created extended Prony interpolation algorithm was applied for the analysis of physiological processes during the bicycle ergometry test. The ECG parameters (RR, QRS and JT) of different durations were used for the investigation of the dynamics of different physiological processes in the human heart during the load. It is necessary to remind that the ECG parameters which were examined actually reveal different complexity levels, e.g. RR interval helps to characterize the state of organism at the regulatory level, JT interval represents the metabolic reactions of the systems, and QRS reflects the intrinsic regulatory state of an organ.

The proposed analysis consists of the following main steps shown in Figure 5.1.

1. *Data preparation.* Let a time series of the ECG parameters of one patient be given. Let us consider that the time series of the ECG parameters, for example, RR, is fragmented manually into k non-overlapping contiguous fragments (Figure 5.2):

$$S = \bigcup_{l} S_l, l = \overline{1, M};$$

where $S_l = S_l(Y, a_l, b_l) = (y_{u_l}, y_{u_l+1}, ..., y_{v_l}), a_l \le b_l$, a_l -is the start and b_l is the end position of each fragment $S_l, l = \overline{1, M}$.

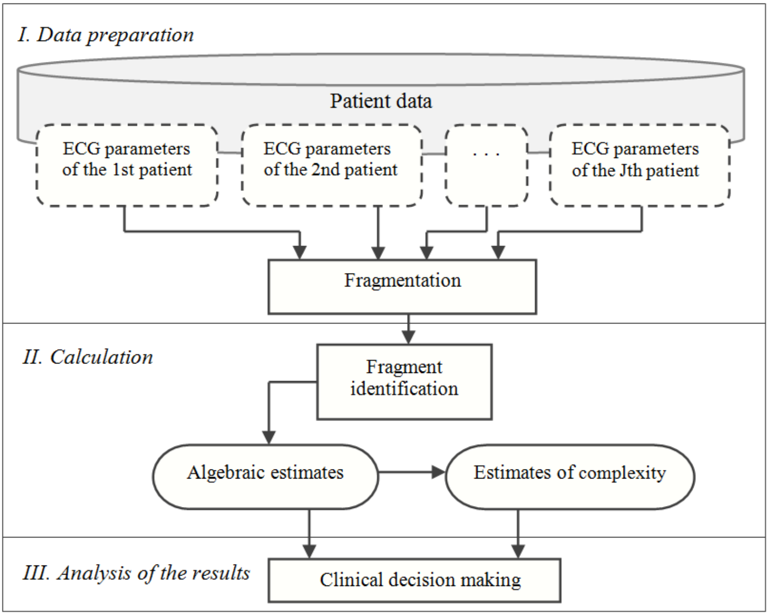

Figure 5.1. Algebraic method for ECG complexity analysis

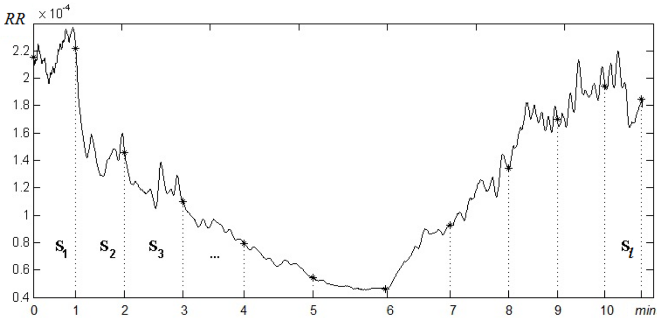

Figure 5.2. ECG signal fragmentation into *M* non-overlapping contiguous fragments

2. *Calculation.* This step includes fragments $S_l = S_l(Y, a_l, b_l)$ identification and calculation of the complexity estimates. First of all, by using the EPI algorithm, we find the LRS expressions of each fragment S_l, $l = \overline{1, M}$ and estimates: $m^{(l)}, \hat{\mu}_k^{(l)}, \hat{\rho}_k^{(l)}, k = \overline{1, s^{(l)}}$, $l = \overline{1, M}$. Then, according to Corollary 3.2,

we calculate stationary, stimulant and inhibitory components for each parameter fragment. These components are used for the calculation of the complexity estimates.

3. *Analysis of the results*. According to the calculated complexity estimates, it is possible to make the clinical decision.

The developed algebraic analysis method was integrated into the prototype for ECG parameters complexity analysis system.

5.2. A Prototype for ECG Complexity Analysis System

A prototype for ECG complexity analysis system developed for doing complexity analysis and making clinical decisions for a patient's data during the velorgometric test is as follows (Figure 5.3).

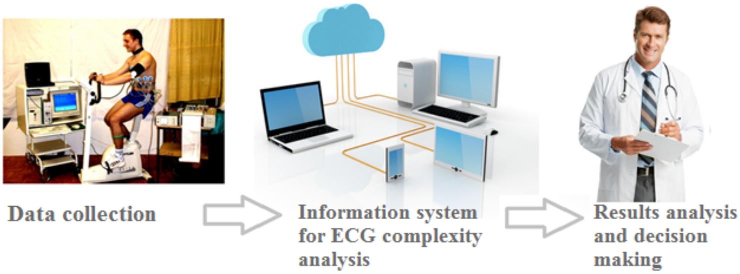

Figure 5.3. Process for the analysis of algebraic estimates of ECG parameters by using the program system

By using the computer cloud technology and developing a software system, we could access the patient's data locally or through a network via which users could import and save data, perform data analysis, and process the results. In our dissertation, we prepare a prototype of this system where data can be accessed from a local database.

The prototype users are thus specialists in the field of medicine, scientists, students and other researchers. The computerized system workload diagram is shown in Figure 5.4.

The system users must have the opportunity to create a new project, save a project or open a previously saved project. The users must have the option for importing data. The data import must proceed by using a predetermined file structure and format. The imported data must be saved in the project catalogue.

The imported ECG signal data must show up on the screen. The user must be able to do data manipulation which is composed of normalization and anti-alias filtering actions as well as ECG signal separation division into separate fragments. After preparing the data fragments, the user must be able to review the calculated values. Before doing calculations, the user must have an opportunity to identify and/or change calculation constants. The user must be able to see every selected and imported ECG parameter and the calculated results. The results must be presented in the graphical form. The final results must be saved in the project catalogue.

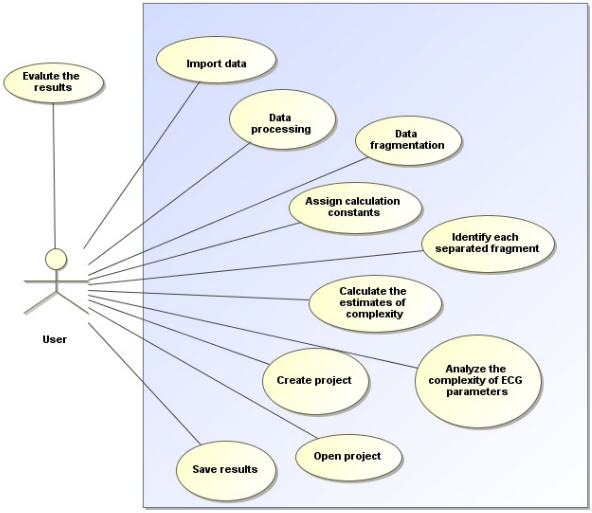

Figure 5.4. Computerized system workload diagram

For the described prototype realization, *MatLab* software package was selected for several reasons: it has a powerful mathematical function library; it supports vector and matrix actions; it can create graphical user interfaces; it features graphical data imaging tools; it can import data from MS Excel, ASCII, XML and other formats, other programs, databases, and other external equipment. Additionally, COM objects can be created, which can be used in all the COM-based programs (*Visual Studio.Net*, *Visual Basic* and others).

The implemented main graphic user interface of the prototype window is shown in Figure 5.5.

The developed prototype was used for the experimental patient's data during the velorgometric test. ECG parameters of different duration were used in order to investigate the dynamics of different physiological processes in the heart during the load. By using the proposed analysis in the reactions to the physical load, different behaviour could be seen in the fluctuations of the ECG parameters at different fractal levels (*RR*, *JT*, *QRS* intervals). In the case of muscles starting to monopolize some organism functions, especially during the maximum loads, the suppressive processes became dominant. An increase in the stationary processes could reveal the end of the recovery processes after the load.

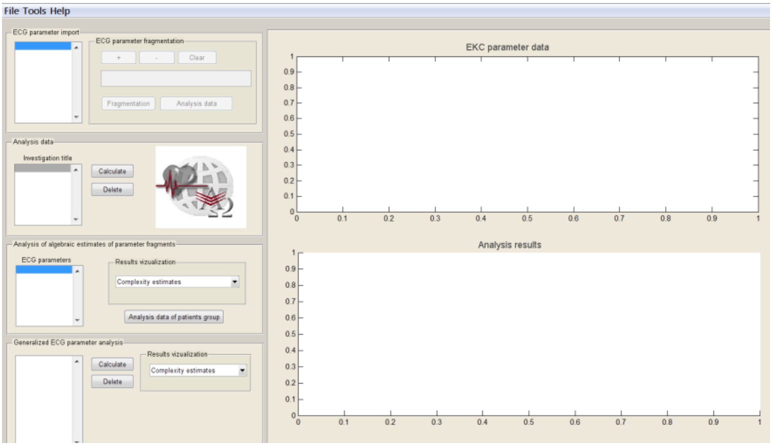

Figure 5.5. Graphical user interface of the prototype of the ECG algebraic analysis program system

The submitted veloergometric test samples showed that the developed program system can identify the artifacts of the ECG parameter and the analysis of the obtained algebraic estimates (the identified eigenvalues), which enables to monitor the ECG complexity changes and determine the cardiac function quality as well as to notice different degrees of the heart failure.

6. ANALYSIS OF ALGEBRAIC ESTIMATES OF ULTRASOUND SIGNALS

In our dissertation, we present a prototype of ultrasound signal identification and an algebraic estimates analysis system based on the concept of the minimal order of a sequence. In the prototype, the signal algebraic analysis method is integrated which is presented below.

6.1. Algebraic Analysis Method of Ultrasound Signal

It was shown that the developed extended Prony's model and the interpolation algorithm can be applied to the identification and analysis of the real-world time series. It has been noted that for accurate signal identification results, there exists the problem of the time of arrival (TOA) of the signal estimation. In this dissertation, we present a new method for TOA estimation. The developed ultrasound signal identification and analysis process is shown in Figure 6.1.

It can be seen that the process consists of three main steps: 1) signal fragment preparation for the identification; 2) identification of the prepared fragment and the calculation of meaningful parameters; 3) analysis of the obtained results.

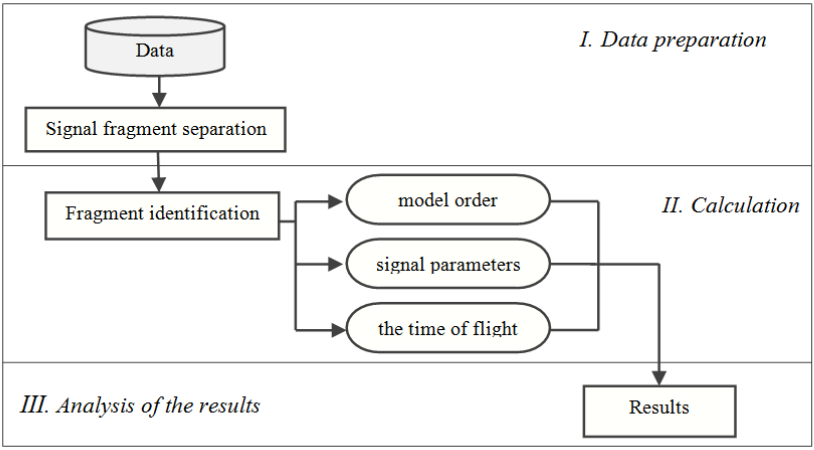

Figure 6.1. Algebraic analysis process scheme of the ultrasound signal

6.2. A Prototype of Program System for the Analysis of the Algebraic Estimates of Ultrasound Signal

In this subsection, we shall present a prototype of the program system for ultrasound signal identification and analysis of the obtained algebraic estimates. The prototype is created on the grounds of a predetermined algebraic analysis process of the ultrasound signal.

The intended system users are engineers, scientists, students and other ultrasound signal researchers who analyze material control issues combined with other tasks. Figure 6.2 presents a program system usage plan.

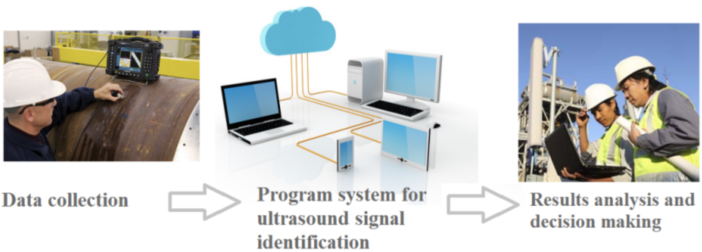

Figure 6.2. Process of the analysis of algebraic estimates of the ultrasound signal by using the program system

Those users who perform ultrasound research can undertake the final expert data analysis in the program. By using cloud computer technologies, the system and data could be accessed by the user from any computer or phone with an internet connection.

The computerized system workloads diagram is presented in Figure 6.3. The main system requirements are that the user must be able to create a new project catalog which would store the processed data analysis and results; the system must contain a function for opening a previously saved project; the user must be able to import data files and perform actions with the imported signal data: to separate and analyze signal fragments, to change the predetermined calculation constants, and to select a signal fragment; the user must be able to select the desired signal fragment, several or all the identified signal components and to separately perform the desired analysis.

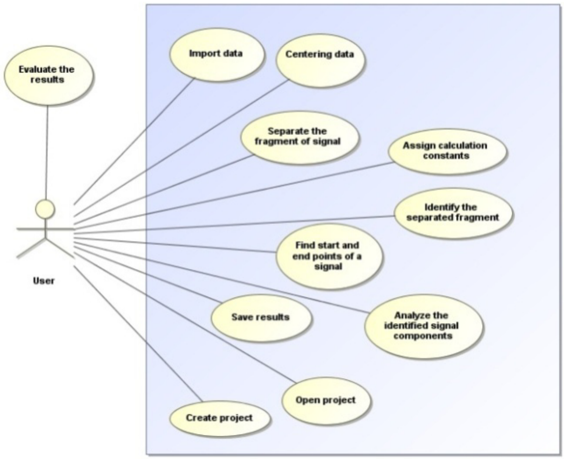

Figure 6.3. The computerized system workloads diagram

For the presented program prototype function implementation, *Matlab* software package is selected because of the advantages outlined above. The developed system prototype user interface main window is presented in Figure 6.4.

The developed program system can be used in order to identify and compare the form of input (standard) and output signals, to detect the time of arrival (TOA) and the time of flight (TOF) of signals and to extract any other characteristics of the signal, for example, the model order, the amplitude, the frequency, the damped factors, or the signal envelope.

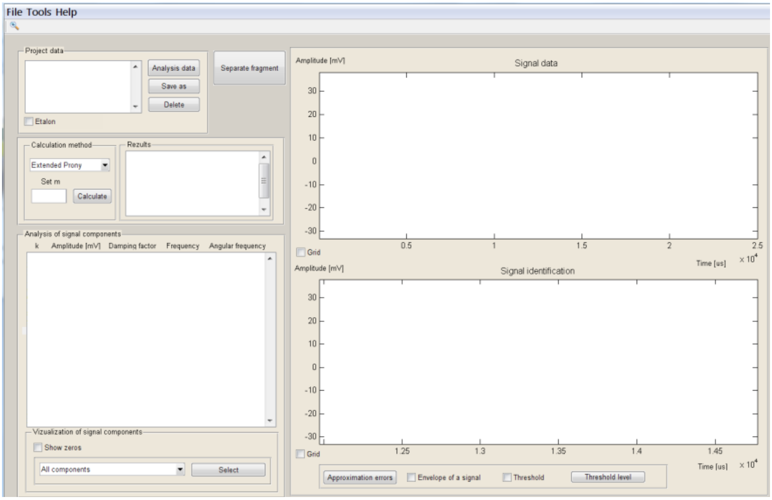

Figure 6.4. The graphical user interface of the prototype of the ultrasound signal algebraic analysis program system

In this thesis, we presented study examples of the prototype of the program system applications for:

- ultrasonic nondestructive testing for inhomogeneous materials with differing structures. The main aim of the ultrasonic pulse-echo technique application for quality control is to characterize the internal structure of the object under investigation. It is possible to analyze and control the material structure by comparing the ultrasonic pulse propagation in the material input and output signals. The experimental research was done with two inhomogeneous materials: composite with carbon fiber and composite with fiberglass. For getting the input signal, we use the reference material – polystyrene – in which, the speed of sound is similar to the speed of sound inside the analyzed composite. By carrying out the identified signal component parameters analysis, we can see the signal complexity changes (such as the decline or increase in the amplitude, changes of the model order or the damping factor), and we can determine the degree of homogeneity of the investigated material.
- longitudinal surface acoustic waves (LSAW) propagation testing on the cylindrical convex surface. Specific propagation properties of the LSAW offer new use opportunities in the field of ultrasonic non-destructive control. Our analysis was conducted with the physical seismic shock wave model from duralumin. Testing parameters were: phase velocity of LSAW propagation on a cylindrical convex surface, variation dependences of signal amplitude, and testing surface curvature radius R. The samples with different curvature radiuses (R=600 mm; 400 mm, 200 mm) were made for testing LSAW propagation

characteristics It was experimentally observed that LSAW are the fastest propagated acoustic waves with the speed and, especially, strength increasing when they are propagating on the cylindrical convex surface. LSAW are the specific whispering gallery effect manifestation form in the solid. It was found that the convexity of the cylindrical surface has a major influence on signal damping. Hence the exhibition of the whispering gallery effect in solids on cylindrical convex surfaces is the main reason of LSAW intensification. This LSAW characteristic could be extremely important for seismic LSAW propagation as a result of the Earth's seismic shocks.

CONCLUSIONS

In this dissertation, we have studied signal identification techniques in order to modify the Prony's method for the identification of signals that can be approximated by the sum of exponential functions with polynomial coefficients by using the optimal number of components based on approximation errors and convergence speed. In order to reach these goals, we have accomplished following tasks:

1. Academic literature review showed that the existing methods are not sufficient for the discussion of the analyzed problems.
2. The development of algebraic identification algorithms for the time series which are capable of identifying the LRS expressions of a time series fragment and, also, fragmenting a time series consisting of several LRS. It is notable that the fragmentation algorithm possesses the capacity to distinguish similar LRS.
3. By using the classical Hankel matrix properties, LRS and LRF theory, this dissertation expanded the Prony's model for time series approximation and developed an extended Prony interpolation algorithm (EPI). When using the EPI algorithm, it is possible:
   - to identify the optimal model order and obtain parameters which characterize the analyzed signal and the associated dynamic system;
   - to identify (by using the algebraic property that $0^0 := 1$) the 'defected' signal components.
4. In our dissertation, the presented examples showed that the EPI (compared with the Prony-type and Chebychev interpolation algorithms) can accurately approximate the given time lines while using a smaller number of components; in addition, it was observed that the EPI algorithm 'picks up' the Runge phenomenon.
5. The developed ECG parameter identification and complexity analysis method. This method was integrated into the prototype of a program system for the analysis of the algebraic estimates of ECG parameters. The submitted veloergometric test samples showed that the analysis of the obtained estimates enables to monitor the ECG complexity changes

and determines the quality of the cardiac function as well as allows to identify the different degrees of heart failure.

6. The developed ultrasonic signal identification and analysis method was integrated into the prototype of a program system for the analysis of ultrasonic signal algebraic estimates. By using the prototype, it is possible to identify and analyze the following signal parameters: the amplitude, the damping factor, the frequency, the time of arrival and the time of flight.

REFERENCES

1. Costa, M., Cygankiewicz, I., Zareba, W., Bayes de Luna, A., Goldberger, A.L. and Lobodzinski, S. (2006). Multiscale complexity analysis of heart rate dynamics in heart failure: Preliminary findings from the music study. In: *Computers in Cardiology*. Valencia, 2006, pp. 101-103.
2. Badeau, R., David, B. (2006). High-resolution spectral analysis of mixtures of complex exponentials modulated by polynomials, In: *IEEE Trans. Signal Process.* 2006, 54, pp. 1341–1350.
3. Badeau, R., Richard, G., David, B. (2008). Performance of ESPRIT for estimating mixtures of complex exponentials modulated by polynomials. In: *IEEE Trans. Signal Process.* 2008, 56, pp. 492–504.
4. Berskienė, K., Navickas, Z., Vainoras, A. (2006). Modeling of Long Lasting Functional State of Healthiness Dynamics. In: *Electronics and electrical engineering*. 2006, 71, 7, pp. 73–76.
5. Bikulčienė, L., Venskaitytė, E., Jaruševicius, G. (2014). The estimation of human vital signs complexity. In: *International Journal of Medical, Pharmaceutical Science and Engineering*. 2014, 8, 1, pp. 39–44.
6. Ehlich, H., Zeller, K. (1996). Auswertung der Normen von Interpolationsoperatoren. In: *Math. Ann.* 1996, 164, pp. 105–112.
7. Higham, N.J. (2004). The numerical stability of barycentric Lagrange interpolation. In: *IMA J. Numer. Anal.* 2004, 24, pp. 547–556.
8. Navickas, Z., Bikulčienė, L. (2006). Expressions of solutions of ordinary differential equations by standard functions. In: *Mathematical Modeling and Analysis*. 2006, 11, pp. 399–412.
9. Peter, T., Plonaka, G. (2013). A generalized Prony method for reconstruction of sparse sums of eigenfunctions of linear operators. In: *Inverse Problems*. 2013, 29, 025001.
10. Platte, R.B., Trefethen, L.N., Kuijlaars, A.B.J. (2011). Impossibility of fast stable approximation of analytic functions from equispaced samples. In: *SIAM Review*. 2011, 53, pp. 308–314.

11. Runge, C. (1901). Uber empirische Funktionen and die Interpolation zwischen aquidistanten Ordinaten. In: *Z. Math. Phys*. 1901, 46, pp. 224–243.
12. Salzer, H.E. (1972). Lagrangian interpolation at the Chebyshev points x = cos(/n), = 0(1)n; some unnoted advantages. In: *Computer J*. 1972, 15, pp. 156–159.
13. Schonhage, A. (1961). *Fehlerfortpflanzung bei Interpolation*. In: Numer. Math. 1961, 3, pp. 62–71.
14. Trefethen, L.N., Pachon, R., Platte, R.B., Driscoll, T.A. (2008). *Chebfun Version 2*. http://www.comlab.ox.ac.uk/chebfun/, Oxford University.
15. Turetskii, A.H. (1940). The bounding of polynomials prescribed at equally distributed points. In: *Proc. Pedag. Inst.* Vitebsk. 1940, 3, pp. 117–127.
16. Osborne, M.R., Smyth, G.K. (1995). A Modified Prony Algorithm For Exponential Function Fitting. In: *SIAM Journal of Scientific Computing*. 1995, 16, pp. 119–138.
17. Martin, C., Miller, J., Pearce, K. (1989). Numerical solution of positive sum exponential equations. In: *Applied Mathematics and Computation*. 1989, 34, pp. 89–93.
18. Fuite, J., Marsh, R.E., Tuszynski, J.A. (2007). An application of Prony's sum of exponentials method to pharmacokinetic data analysis. In: *Commun. Comput. Phys*. 2007, 2, pp. 87–98.
19. Giesbrecht, M., Labahn, G., Wen-shin, Lee. (2009). Symbolic-numeric sparse interpolation of multivariate polynomials. In: *Journal of Symbolic Computation*. 2009, 44, pp. 943–959.
20. Steedly, W., Ying, C.J., Moses, O.L. (1992). A modified TLS-Prony method using data decimation. In: *IEEE Transactions on Signal Processing*. 1992, 42, pp. 2292–2303.
21. Kurakin, V.L., Kuzmin, A.S., Mikhalev, A.V., Nechavev, A.A. (1995). Linear recurring sequneces over rings and modules. In: *Journal of Mathematical Sciences*. 1995, 76, pp. 2793–2915.
22. Kurakin, V. (2001). Linear complexity of polinear sequences. In: *Disctrete Math. Appl*. 2001, 11, pp. 1–51.
23. Vainoras, A. (2002). Functional model of human organism reaction to load – evaluation of sportsman training effect. In: *Education, Physical Training, Sport*. 2002, pp. 88–93.
24. Potts, D., Tasche, M. (2013). Parameter estimation for multivariate exponential sums. In: *Electron. Trans. Numer. Anal.* 2013, 40, pp. 204–224.

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LIST OF PUBLICATIONS

Papers in Master List Journals of the Institute of Scientific Information (ISI)

1. Karalienė, Dovilė; Navickas, Zenonas; Slapšinskaitė, Agnė; Vainoras, Alfonsas. (2013). Investigation of the stability of fluctuations in electrocardiography data. In: *Journal of Vibroengineering/Vibromechanika*. Lithuanian Academy of Sciences, Kaunas University of Technology, Vilnius Gediminas Technical University. Kaunas: Vibroengineering. ISSN 1392-8716. 2013, Vol. 15, no. 1, pp. 291–301.
2. Karalienė, Dovilė; Navickas, Zenonas; Čiegis, Raimondas; Ragulskis, Minvydas. (2015). An extended Prony's interpolation scheme on an equispaced grid. In: *Open Mathematics*. ISSN 1392-8716. 13(1). Retrieved 26 Jun. 2015, from doi:10.1515/math-2015-0031.
3. Piper, Ian; Vainoras, Alfonsas; Berškienė, Kristina; Ruseckas, Rimtautas; Jurkonis, Vidmantas; Bikulčienė, Liepa; Navickas, Zenonas; Karalienė, Dovilė; Ragauskas, Arminas; Deimantavičius, Mantas. (2016). Hypotension investigation, prospective clinical study // In: *Elektronika ir elektrotechnika*. Kaunas: KTU. ISSN 1392-1215. 2016, vol. 22, iss. 2, pp. 33–37.

Papers in Proceedings List of the Institute of Scientific Information (ISI)

1. Karalienė, Dovilė; Navickas, Zenonas; Vainoras, Alfonsas. (2012). Segmentation algorithm for algebraic progressions. In: *Information and software technologies: 18th International Conference, ICIST 2012*. Kaunas, Lithuania, September 13–14, 2012: proceedings / [edited by] Tomas Skersys, Rimantas Butleris, Rita Butkienė. Berlin, Heidelberg: Springer, 2012. ISBN 9783642333071, pp. 149–161.

Papers in Journals Referred in the Databases Included in the List Approved by the Science Council of Lithuania

1. Karalienė, Dovilė; Navickas Zenonas; Sajauskas, Stanislovas. (2010). Computer analysis of ultrasonic pulse signals. In: *Mechanika 2010*: proceedings of the 15th international conference, April 8–9, 2010, Kaunas, Lithuania/ Kaunas University of Technology, Lithuanian Academy of Sciences, IFTOMM National Committee of Lithuania, Baltic Association of Mechanical Engineering. Kaunas: Technologija. ISSN 1822-2951. 2010, pp. 223–226.
2. Sajauskas, Stanislovas; Navickas, Zenonas; Karalienė, Dovilė. (2010). Propagation properties of longitudinal surface acoustic waves (creeping waves) on the cylindrical convex surface. In: *Ultragarsas = Ultrasound*/ Kauno technologijos universitetas. Kaunas: Technologija. ISSN 1392-2114. 2010, Vol. 65, No. 4, pp. 35–39.
3. Karalienė, Dovilė; Navickas, Zenonas; Slapšinskaitė, Agnė. (2013). A Mathematical Information Algorithm for the Analysis of ECG Complexity. In: *Athens: ATINER'S Conference Paper Series*, No: COM2013-0596.

SANTRAUKA

Tyrimo objektas–signalų, aproksimuojamų tiesinėmis rekurentinėmis sekomis, identifikavimo algoritmai.

Temos aktualumas. Skaičiavimo technikos bei technologijų tobulėjimas suteikia galimybę kaupti vis didesnius informacijos kiekius ir atlikti sudėtingesnę bei informatyvesnę signalų analizę. Antra vertus, didėjant žinių bei informacijos kiekiui, įvairių diagnostikos aparatų pasiūlai, tobulėjant įvairiarūšių signalų bei duomenų kaupimo elektroninėms priemonėms, vis sudėtingiau atlikti informacijos apimčiai adekvačią sukauptų duomenų analizę ir apibendrinimus. Kyla naujos signalų ir juose slypinčios informacijos apdorojimo problemos, ypač opios tose srityse, kur signalų – pavyzdžiui, biomedicininių, seisminių, ultragarsinių ir kitų – šaltiniai yra labai sudėtingi.

Darbe analizuojami elektrokardiografiniai ir ultragarsiniai impulsiniai signalai. Elektrokardiografiniai signalai – tai fiziologinės sistemos (žmogaus širdies) generuojami signalai. Pagrindinis fiziologinių sistemų požymis – jų kompleksiškumas, „paslėptas“ biomedicininiuose signaluose. Iš kompleksinių sistemų pozicijų apdorojant šiuos signalus, atsiveria galimybės suvokti juos generuojančios sistemos komponentus ir dinamines sąsajas. Šitas atradimas pastūmėjo kompleksinių sistemų teorijos taikymus nuo molekulinio iki organizmo lygio. Literatūroje tokie signalai vadinami kompleksiniais.

Širdies generuojamų netiesiškų ir nestacionarių signalų analizei nepakanka tradicinių metodų. Elektrokardiografinių signalų analizei taikomi netiesinės dinamikos metodai, pagrįsti deterministinio chaoso ir kompleksinių sistemų teorijomis. Šios teorijos leidžia iš esmės praplėsti ir pagilinti signalų analizės galimybes tiek kokybiškai, tiek kiekybiškai. Be to, kompleksinį signalą sąlygojančios dinamikos pažinimas turi lemiamą reikšmę atitinkamo proceso suvokimui ir kartu atitinkamo tyrimo metodo parinkimui. Ši tyrimų sritis pastaruoju metu plėtojama daugelyje mokslo krypčių.

Kiti darbe analizuojami – ultragarsiniai impulsiniai signalai. Šie signalai taikomi medicinoje tiriant žmogaus vidaus organus bei diagnozuojant ligas, taip pat neardomojoje medžiagų kontrolėje tiriant įvairių medžiagų savybes, šiurkščius (pavyzdžiui, vamzdžių, traukinių bėgių, branduolinių reaktorių bei kitų objektų) vidinius paviršius ir kitose srityse.

Ultragarsinių signalų analizei sukurta daug įvairių spektrinės ir laikinės srities analizės metodų. Metodai parenkami pagal analizės tikslus, juos generuojančių procesų charakteristikas ir signalo savybes. Disertacijoje tiriami ultragarsinio signalo identifikavimo, t.y. jo matematinio modelio sudarymo metodai. Signalo identifikavimas suteikia galimybę filtruoti signalus, nustatyti tokius parametrus, kaip signalo trukmė, galia, energija, ir kitas signalo savybes. Vieni populiariausių literatūroje sutinkami signalų identifikavimo metodai – Furjė ir Prony. Pagal Prony metodą signalai aprašomi fiksuoto skaičiaus eksponentinių funkcijų tiesiniais dariniais su pastoviaisiais koeficientais. Kitaip

nei Furjė, Prony metodas suteikia galimybę neperiodinius gęstančius baigtinio ilgio signalus identifikuoti neprarandant informacijos apie fazę. Šita metodo savybė ypač svarbi, kai metodas taikomas signalo vėlinimo trukmei nustatyti. Tačiau bene pagrindinis Prony metodo trūkumas – nežinomas signalą identifikuojančio modelio komponenčių skaičius, be to, jis yra apribotas dėl eksponentinių funkcijų koeficientų pastovumo, todėl tam tikrais atvejais padidėja signalų aproksimavimo paklaida. Nepaisant to, Prony metodas taikomas daugelyje sričių: biomedicinos, neardomosios medžiagų kontrolės, genetikos, finansų ir kt., todėl jų analizei kuriamos bei taikomos įvairios Prony metodo modifikacijos.

Tyrimo tikslas ir uždaviniai. Patobulinti Prony signalo identifikavimo metodą, pagal kurį signalas būtų identifikuotas eksponentinių funkcijų su polinominiais koeficientais tiesiniu dariniu, siekant rasti optimalų komponenčių skaičių, įvertinti aproksimavimo paklaidas ir padidinti konvergavimo greitį.

Darbo tikslui įgyvendinti formuluojami šie uždaviniai.

1. Atlikti signalų identifikavimo metodų lyginamąją analizę.
2. Modifikuoti Prony identifikavimo metodą signalams, kuriuos galima aproksimuoti eksponentinių funkcijų su polinominiais koeficientais tiesiniais dariniais.
3. Pritaikant modifikuotą Prony metodą, sukurti signalų algebrinį interpoliavimo algoritmą, kurį panaudojant signalas būtų optimaliai identifikuojamas eksponentinių funkcijų tiesiniu dariniu, kai koeficientai yra daugianariai.
4. Sukurti programų sistemų prototipus:
   - elektrokardiografinių signalų parametrų fragmentams identifikuoti bei kompleksinei analizei atlikti;
   - ultragarsiniam signalui identifikuoti, pradžios taškui bei sklidimo trukmei nustatyti ir signalo komponenčių analizei atlikti.

Tyrimo metodai ir priemonės

1. Tyrimuose panaudota algebrinės analizės teorija ir jos pritaikymas signalų identifikavimo metodams sukurti.
2. Sukurto signalo identifikavimo algoritmui tirti ir palyginti naudoti modeliai ir taikyti identifikavimo metodai, pagal kuriuos signalai aprašomi tiesiniais eksponentinių funkcijų dariniais.
3. Kompiuteriniams skaičiavimams atlikti, algoritmams realizuoti, eksperimentiniams tyrimams vykdyti ir programos sistemos prototipui kurti panaudotas *Matlab v. R2009a* programos paketas.
4. Sukurtų programų sistemų prototipų praktiniam taikymui demonstruoti buvo naudojami osciliatoriumi gauti ultragarsinių impulsinių bei elektrokardiografinių signalų duomenys.

Mokslinis naujumas ir praktinė svarba

1. Pagal praplėstą (modifikuotą) Prony metodą signalai identifikuojami eksponentinių funkcijų su polinominiais koeficientais tiesiniais dariniais. Sukurtas metodas suteikia galimybę atitinkamais atvejais signalus tiksliau identifikuoti lyginant su kitais algoritmais, pagrįstais Prony metodu bei signalus aprašančiais eksponentinių funkcijų su pastoviaisiais koeficientais modeliais (t.y. apibendrinamas Prony metodas vietoj pastoviųjų koeficientų panaudojant algebrinius daugianarius). Tai leidžia atlikti dar tikslesnes analizes daugelyje Prony rūšies metodų taikymo sričių.

2. Sukurtas algebrinis signalo interpoliavimo algoritmas leidžia nustatyti optimalų signalą aprašančių eksponentinių funkcijų komponenčių skaičių; ši savybė Prony metodą daro patrauklesnį signalų apdorojimų taikymams.

3. Sukurtas algebrinis interpoliavimo algoritmas taip pat gali būti taikomas signalo nežinomų reikšmių interpoliavimui bei ekstrapoliavimui. Ši savybė gali būti taikoma laiko eilučių prognozavimo uždaviniuose.

4. Sukurti nauji elektrokardiografinių bei ultragarsinių impulsinių signalų identifikavimo ir analizės metodai.

Ginamieji teiginiai

1. Darbe pasiūlytas naujas Prony signalo (laiko eilutės) identifikavimo modelis užtikrina tiriamo signalo (laiko eilutės) aproksimaciją eksponentinių funkcijų su polinominiais koeficientais tiesiniu dariniu.

2. Darbe pasiūlytas praplėstas Prony interpoliacijos algoritmas užtikrina optimalų tiriamo signalo (kuris gali būti aprašytas tiesine rekurentine seka) matematinio modelio parametrų (algebrinių įverčių) identifikavimą.

3. Darbe pasiūlyti programų sistemų prototipai suteikia galimybę atlikti elektrokardiografinių ir ultragarsinių signalų algebrinių įverčių tyrimus.

Darbo rezultatų aprobavimas

Darbo tema paskelbtos 7 mokslinės publikacijos, iš jų 3 mokslinės informacijos instituto (ISI) pagrindinio sąrašo leidinyje su citavimo indeksu, 1 – Lietuvos pripažintame periodiniame leidinyje, 3 – tarptautinių konferencijų pranešimų medžiagoje.

Darbo rezultatai buvo pateikti ir aptarti 4 mokslinėse konferencijose (1 respublikinėje, 3 tarptautinėse, iš kurių 1 – užsienyje).

Darbo apimtis ir struktūra

Disertaciją sudaro įvadas, 5 pagrindiniai skyriai, išvados, literatūros bei publikacijų sąrašai ir priedai. Disertacijos apimtis – 138 puslapiai, 66 paveikslai, 13 lentelių ir 117 šaltinių cituojamos literatūros sąrašas.

Įvadiniame skyriuje pateikiamas darbo temos aktualumas, nurodomas tyrimo tikslas bei uždaviniai. Trumpai išdėstomi pagrindiniai disertacijos rezultatai, jų praktinė reikšmė ir mokslinis naujumas.

Pirmajame skyriuje trumpai aptariamos signalų savybės ir identifikavimo problemos. Pateikiama signalų identifikavimo metodų apžvalga bei disertacijoje signalų identifikavimui taikoma tiesinių rekurentinių sekų teorija. Skyriaus pabaigoje pateikiami elektrokardiografinių bei ultragarsinių signalų tyrimų tikslai bei problemos ir jų sprendimui taikomi matematiniai metodai.

Antrajame skyriuje pateikiamos tiesinės rekurentinės sekos sąvokos ir jos koncepcijos taikymas sekų fragmentų identifikavimo algoritmams. Pateikiamas sukurtų algoritmų taikymas sekos, sudarytos iš kelių tiesinių rekurentinių sekų, fragmentavimui.

Trečiajame skyriuje supažindinama su tiesinių rekurentinių funkcijų teorija bei jos taikymu. Pateikiamas naujas praplėstas Prony interpoliacijos metodas ir jo palyginimas su kitais interpoliacijos metodais.

Ketvirtajame skyriuje pristatomas programos sistemos prototipas, skirtas elektrokardiografinių signalų parametrų kompleksiškumui tirti. Prototipas parengtas vadovaujantis skyriuje pateiktu signalo fragmentų algebrinės analizės metodu, pagal kurį naudojamas praplėstas Prony interpoliacijos algoritmas. Pabaigoje pateikiamas programos sistemos prototipo praktinis taikymas veloergometriniam tyrimui atlikti.

Penktajame skyriuje pateikiamas praplėsto Prony modelio ir metodo taikymas ultragarsiniam signalui identifikuoti, pradžios bei pabaigos taškui nustatyti ir analizei atlikti. Pristatomas signalo identifikavimo programos sistemos prototipas ir jo praktinis taikymas eksperimentiniams ultragarsinių signalų duomenims.

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