

KAUNAS UNIVERSITY OF TECHNOLOGY DEPARTMENT OF MULTIMEDIA ENGINEERING FACULTY OF INFORMATICS

PRATIBHA CHANDRAPPA KADASUR

INVESTIGATION ON IMAGE ZOOMING INTERPOLATION

Master's Degree Final Project

KAUNAS 2017



KAUNAS UNIVERSITY OF TECHNOLOGY DEPARTMENT OF MULTIMEDIA ENGINEERING FACULTY OF INFORMATICS

INVESTIGATION ON IMAGE ZOOMING INTERPOLATION

Master's Degree Final Project

Project made by	Title of programme (code IFMU-5)		
Name: Pratibha Chandrappa kadasur			
	Supervisor		
Date:	Name and surname: Asst. Prof. Armantas Ostreika		
	Date and Signature:		
Signature:	Grade		
	Reviewer		
	Name and surname: Prof. Dr. Alfonsas Misevicius		
	Date and Signature:		
	Grade:		

KAUNAS,2017



KAUNAS UNIVERSITY OF TECHNOLOGY DEPARTMENT OF MULTIMEDIA ENGINEERING FACULTY OF INFORMATICS

PRATIBHA CHANDRAPPA KADASUR

Final Thesis P000M106

INVESTIGATION ON IMAGE ZOOMING INTERPOLATION

DECLARATION OF ACADEMIC INTEGRITY

I confirm that the final project of mine, **PRATIBHA CHANDRAPPA KADASUR**, on the subject INVESTIGATION ON IMAGE ZOOMING INTERPOLATION is written completely by myself; all the provided data and research results are correct and have been obtained honestly. None of the parts of this thesis have been plagiarized from any printed, Internet-based or otherwise recorded sources. All direct and indirect quotations from external resources are indicated in the list of references. No monetary funds (unless required by law) have been paid to anyone for any contribution to this thesis.

I fully and completely understand that any discovery of any manifestations/case/facts of dishonesty inevitably results in me incurring a penalty according to the procedure(s) effective at Kaunas University of Technology.

(name and surname filled in by hand)

(signature)

ASSESSMENT OF THE CORRECTNESS OF LANGUAGE IN THE MASTER'S THESIS

Faculty of Informatics

INVESTIGATION ON IMAGE ZOOMING INTERPOLATION

Types of errors	Number of errors, p.	Examples
Typographical errors		
Punctuation errors*		
Errors of literary language		
Formal issues**(correction		
errors, errors of the text typed by		
computer)		
Total number of errors		
Literacy level:		

PRATIBHA CHANDRAPPA KADASUR

*One punctuation error is considered to be a 0,5 error.

**Not included into the total number of errors, however, in case there are many such violations, the thesis cannot be considered as having a high level of literacy.

Assessor's name, surname

SUMMARY

The purpose of the study was to conduct a research on image zooming interpolation, where investigation of different image interpolation is carried out and a new method is implemented to obtain a good image quality after zooming. When an object or a scene is photographed, user often tries to enhance its quality by changing the brightness level, sharpening, blurring, rotating, zooming, shrinking or by changing color accuracy.

In the entire process of modifying an image upon one's desire leads to the loss of many information from the image. The report gives the detailed information about the different methods of interpolations and work review of various authors who could achieve the good image quality after applying the interpolation algorithm. The implementation of existing work was executed and analyzed based upon which the new algorithm was implemented to improve the quality of a zoomed image. The results were compared using the PSNR, SNR, Contrast, energy, homogeneity, correlation and process time of an image.

The Peak Signal to Noise ratio of proposed hybrid method was better than the bilinear, nearest neighbor and Pixel Interpolation method. The SNR ratio and structural similarity were merely equal to the Bilinear, Nearest neighbor and Pixel interpolation method although a better improvement could not be achieved. The image quality factors energy, homogeneity, contrast and correlation were computed for all 4 methods. Where the Homogeneity, contrast and correlation obtained better results when compared to 3 existing interpolation methods.

The aim of the thesis was achieved by providing the improvement in image quality factors when they are zoomed to a zooming factor.

Table of Contents

1. Analysis of Raster images:	12
1.1. Raster image	12
1.1. How raster image files are stored?	12
1.2. Image Resolution:	12
1.3. Image Quality Factors:	12
1.4. Image Interpolation:	13
2. Existing interpolation methods:	15
2.1. Non-adaptive algorithms:	
2.1.1. Nearest neighbor interpolation:	15
2.1.2. Bilinear interpolation:	16
2.1.3. Bicubic interpolation:	17
2.2. Adaptive interpolation	
2.2.1. New Edge-Directed Interpolation	
2.2.2. Data Dependent Triangulation	20
2.2.3. Iterative Curvature-based Interpolation (ICBI)	21
2.3. Summary of existing interpolation methods	21
2.4. Existing work review	23
2.5. Implementation of existing codes	25
2.5.1. Bilinear zoom interpolation	25
2.5.2. Image enhancement through DCT scaling method	27
3. Design of proposed method	29
3.1. Tools required	
3.2. Calculation parameters	31
3.3. Executed snapshots of Bilinear, Nearest neighbor, Pixel replacement and proposed	d hybrid interpolation
method for a zoom factor = 2.	35
3.4. Comparison Table for results obtained	41
3.5. The comparison analysis,	43
4. Conclusion	47
References	48

List of Figures

Figure 1 Basic idea of Interpolation [5]	14
Figure 2 Basic flow of Interpolation [10]	14
Figure 3 Nearest neighbour interpolation	15
Figure 4 Bilinear Interpolation	17
Figure 5 Bicubic Interpolation	18
Figure 6 Original image [3]	19
Figure 7 NEDI image [3]	19
Figure 8 Formation of an Image mesh [5]	20
Figure 9 DDT image [5]	20
Figure 10 ICBI image [3]	21
Figure 11 All in one images of non-adaptive interpolation methods	22
Figure 12 Original image [5]	22
Figure 13 NEDI technique, DDT technique and ICBI technique respectively [5]	23
Figure 14 Original image to be zoomed	26
Figure 15 Bilinear zoomed image	27
Figure 16 Input image to be enhanced	27
Figure 17 Enhanced image	28
Figure 18 Flow diagram of proposed Hybrid Image interpolation	29
Figure 19 Flow chart of the process of obtaining comparison image quality factors.	30
Figure 20 Original Image	35
Figure 21 Enter zooming factor as 2	36
Figure 22 Nearest neighbour interpolation	36
Figure 23 Bilinear zooming interpolation	37
Figure 24 Pixel Replacement Interpolation	37
Figure 25 Hybrid interpolation of an image	
Figure 26 Original image	
Figure 27 Enter Zooming factor as 4	
Figure 28 Bilinear zoom for a zoom factor 3	
Figure 29 Nearest neighbour zoom interpolation	40
Figure 30 Pixel Replacement interpolation for zooming	40
Figure 31 Hybrid Zooming interpolation	41
Figure 32 The Average Peak Signal to Noise ratio of interpolations	44
Figure 33 Histogram error rate	44
Figure 34 The average process time required for zooming the image by all the interpolation	45
Figure 35 The Average storage file size required by all the interpolations	46

List of tables

Table 1 Comparison parameter for zooming factor 2	41
Table 2 Comparison parameter of various interpolation for zoom factor 2	42
Table 3 Comparison parameters for zoom factor 3	42
Table 4 Comparison parameter of various interpolation for zoom factor 3	42
Table 5 Comparison parameters for zoom factor 4	43
Table 6 Comparison parameter of various interpolation for zoom factor 4	43

List of abbreviations

Terms	Expansion
1. DPI	Dots Per Inch
2. GIF	Graphic Interchange Format
3. JPEG	Joint Photographic Expert Group
4. PNG	Portable Graphics Network
5. TIFF	Tagged Image File Format
6. BMP	Bitmap File Format
7. PPI	Pixel Per Inch
8. NEDI	New -Edge Directed Interpolation
9. DDT	Data Dependent Triangulation
10. ICBI	Iterative Curvature based Interpolation
11. FCBI	Fast curvature Based Interpolation
12. PSNR	Peak Signal to Noise Ratio
13. DIC	Digital Image Correlation
14. IC-GN	Inverse Composition Gauss Newton
15. MLS	Moving Least Square
16. NNV	Nearest Neighbor Value
17. DCT	Discrete Cosine Transformation

INTRODUCTION

Nowadays photography is gaining an immense attention of the people leading to obtain a highresolution image with much clarity, sharpness, blur free, artifacts free and detailed image. But sometimes when user tries to enhance image by scaling, rotating, increasing the color depth, trying to modify the facial changes and many more modifications leading to the changes in an original image ending up with certain loss of original image information. To maintain the same image resolution or enhancement of original image to certain extent we use various interpolation algorithm to obtain better results.

A Raster image is a digital photograph that can be images from satellite, scanned maps or digital pictures. Raster image or graphics also known as bitmap images which uses the color pixels to form an image. When we zoom in closely to a raster image we can find how these small square pixels come together to make up the picture. But sometimes we may find it necessary to resample or rectify the image if we want it to be resized, rotated or get it distorted intentionally. So, this leads to the addition or deletion of bitmap pixels where the calculation of intermediate values to compute the colors of the pixels in the transformed image becomes necessary leading to the interpolation.

Generally, resolution of an image means the number of pixels in an image which has a remarkable influence on the detail of an image. Resolution refers to the quality of an image, as the resolution increases, the image becomes more clear, sharper, well defined, and more detailed too. If we compress or enlarge an image it will result in pixel manipulation which is called as image interpolation.

Here in this project report the main idea is to carry out a research analysis and implement a better solution on raster image zooming with a fine interpolation method for a better image resolution enhancement.

THE AIM

To carry out a research analysis and implement a better solution on raster image zooming with a fine interpolation method for a better image resolution enhancement.

THE OBJECTIVES

The main objectives of the thesis work are:

- to investigate different raster image file formats.
- to research on different image zooming interpolation method.
- analyze the existing codes for various image interpolation.
- implement a new zooming interpolation method with good image enhancement.

1. Analysis of Raster images:

1.1. Raster image

A raster image is defined by a cell of matrix where pixels are arranged in rows and columns which represent the information about colors at every single point in an image.

A raster graphic which is also called as bitmap graphic is an image using tiny squares of pixels having capacity of storing the color value to make up an image. Raster graphic depends upon the resolution of an image which is counted in terms of dpi, i.e., dots per inch (DPI).

1.1. How raster image files are stored?

Raster image files are dependent upon the amount of information possessed by an image. Higher the resolution of an image, larger will be the file size and vice versa. Low resolution raster graphic can be set to 72 dpi, which are normally used for websites and presentations that are viewed on a computer screen. High resolution raster graphics are mostly set to 300 dpi which are most likely to be used for printing images.

There are certain number of raster file formats to store raster images where the file size is dependent upon the data present in an image which impacts on both color and quality of it. Some of image file formats are as given below [1].

- a) Graphic Interchange Format (GIF)
- b) Joint Photographic Expert Group (JPEG)
- c) Portable Network Graphics (PNG)
- d) Tagged Image File Format (TIFF)
- e) Raw Image Format
- f) Bitmap File Format (BMP)

1.2. Image Resolution:

The resolution of an image is the density of pixels in an image. It is evaluated in terms of pixels per inch (PPI) or dots per inch (DPI). Resolution does not give us the estimation of number of pixels or dots in an image, instead it measures the density of pixels or dots in an inch. If we keep the image size constant and increase the resolution the image becomes more sharp and detailed.

1.3. Image Quality Factors:

There are many quality factors which affect the appearance of an image after applying the interpolation for its enhancement.

Sharpness is one such main quality of an image which determines the amount of information an image can provide. Sharpness is measured in terms of Spatial Frequency Response(SFR) which is also called as Modulation transfer function (MTF). MTF is derived from sine pattern contrast *c*(*f*). The modulation of sine pattern is used to calculate the MTF.

$$c(f) = (V_{max} - V_{min}) / (V_{max} + V_{min});$$
(1)

Where V is Luminance (modulation), c(f) is sine pattern contrast.

$$MTF(f) = 100\% c(f)/c(0);$$
 (2)

Which normalizes the MTF to 100% at low spatial frequencies. [2,3]

- Noise is a random disturbance present in an image captured. It can be generated due to variation in color information, change of brightness, sensors used or may be because of the digital device used to capture it. There are few types of noises which are more often seen in images are Gaussian noise, Salt and pepper noise, Quantization noise, Periodic noise and Anisotropic noise. We can use some linear smoothing filters, anisotropic diffusion, chroma and luminance noise separation, Non-linear filters, wavelet transforms, etc.
- **Distortion** is generally some kind of deformation of an image which can be very much noticeable or very light to be observed in detail. It can be an optical distortion or perspective distortion. Where the optical distortion lies when design or manufacturing of lens goes wrong, also called as lens distortion. Perspective is due to the position of a digital device like camera, mobile phones etc. with respect to the subject or vice versa.
- **Tone Reproduction** is the method of reproducing the realistic features of a capture scene or image with certain limitations given by the printers for printing them on photographic papers.

There are few more image quality factors which come under considerations are dynamic range, contrast, color and exposure [2].

1.4. Image Interpolation:

Interpolation of an image is the process of finding the values at unknown points using known data points. Interpolation happens whenever we resize or scale to zoom or shrink it, to rotate an image and even to enhance an image resolution, which leads to addition or removal of pixels in an image. Sometimes interpolation becomes necessary to get more detailed, sharpen and high resolution quality of an image. [4]



Figure 1 Basic idea of Interpolation [5]

A general interpolation follows the step as given in the Figure 2. First, we target the position of pixel to be interpolated and then we access the neighboring pixel to find an average pixel value. It is done in both horizontal and vertical direction and finally the interpolated pixel value will be obtained in an image.



Figure 2 Basic flow of Interpolation [10]

2. Existing interpolation methods:

Generally, interpolation algorithms can be grouped into two types: adaptive and non- adaptive. Adaptive methods change depending on what they are interpolating. It can be sharp edges vs. smooth texture, whereas non- adaptive methods treat all pixels equally.

2.1. Non-adaptive algorithms:

The interpolation techniques under non-adaptive methods include direct manipulation of existing pixels instead of considering any other written content or feature of an image. It undertakes one on one interpolation of pixels. The following are the non-adaptive interpolation methods.

2.1.1. Nearest neighbor interpolation:

In nearest neighbor interpolation method, the pixels are replaced by the nearest pixel in an image. It is a kind of linear interpolation where the new pixels are formed within the range of a set of known pixel points. It is very easy and simple to implement. Good results can be obtained when image has high resolution. An example of nearest neighbour interpolation image is been shown in Figure 3.

$$d(x,y) = ||x - y|| = \sqrt{(x - y)(x - y)} = \left(\sum_{i} (x_i - y_i)^2\right)^{\frac{1}{2}};$$
(3)

Where, d(x, y) = defines the distance between interpolated point and grid point



 $x_{i,}y_{i}$ = the known points [5,6,].

Figure 3 Nearest neighbour interpolation

2.1.2. Bilinear interpolation:

Bilinear interpolation is the weighted average of the 4 nearest neighbour pixels to estimate it is final interpolated value. Here we calculate linear interpolation 2 times. One in horizontal direction and other in vertical direction. If all the known pixel distances are equal then the final interpolated pixel is simply their sum divided by 4. The outcome of this method gives smoother image than the original image. The computational time is also less. A bilinear interpolated image is being shown in the following Figure 4.

The algorithm to find a bilinear interpolation is given below.

If the value of the unknown function 'f 'at the point (x, y) has to be found out, then assume the values of 'f' at the four points $Q_{11} = (x_1, y_1)$, $Q_{12} = (x_1, y_2)$, $Q_{11} = (x_2, y_1)$, and $Q_{11} = (x_2, y_2)$

First the linear interpolation is done along *x*- axis,

$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}); \qquad (4)$$

$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22});$$
(5)

Where, $f(x, y_1) \& f(x, y_2) =$ unknown functions at (x, y)

 $Q_{11}, Q_{12}, Q_{21}, Q_{22}$ = values assumed at points x, y

$$f(x,y) \approx \frac{y_2 - y}{y_2 - y_1} f(x,y_1) + \frac{y - y_1}{y_2 - y_1} f(x,y_2)$$
(6)
$$f(x,y) \approx \frac{1}{(x_2 - x_1)(y_2 - y_1)} \begin{bmatrix} x_2 - x_1 & x - x_1 \end{bmatrix} \begin{bmatrix} f(Q_{11}) & f(Q_{12}) \\ f(Q_{21}) & f(Q_{22}) \end{bmatrix} \begin{bmatrix} y_2 - y_1 \\ y - y_1 \end{bmatrix}$$
(7)

The equation obtained for x and y-coordinates are used to interpolate the image [5,7,9].



Figure 4 Bilinear Interpolation

2.1.3. Bicubic interpolation:

This interpolation method takes a weighted average of the 16 pixels to form the final interpolated value. These known 16 pixels are at various distances from unknown pixel so the pixels which are closer are given more importance for calculation. This method consumes more time for computation when compared to other non-adaptive methods but gives out the much more detailed sharper image.

A bicubic interpolation is been shown in the Fig 5.

Bicubic interpolation is computed by using the algorithm below,

The function values f and the derivatives f_x , f_y and f_{xy} are known at the four corners of a unit square (0, 0), (0,1), (1,1) and (1,0) then the surface interpolation p(x, y) can be calculated as,

$$p(x,y) = \sum_{i=0}^{3} \sum_{j=3}^{3} a_{ij} x^{i} y^{j}; \qquad (8)$$

Where, a_{ij} = determines the 16 coefficients of the surface.

Matching values of p(x, y) yields 4 derivatives, 8 derivatives are obtained along *x*-direction and *y*-direction and final 4 are obtained through cross derivatives *xy* [8,9,12].



Figure 5 Bicubic Interpolation

2.2. Adaptive interpolation

Adaptive methods consider the image qualities like intensity, texture, sharpness, edge information and many more. (But they also have a drawback of artifacts, blurred edges and storage of low frequency components of original images.). These are introduced to obtain a better image quality with high frequency component. It takes more computational time when compared to the non-adaptive methods. The following are the various adaptive interpolation methods.

2.2.1. New Edge-Directed Interpolation

It is an incorporated approach of bilinear interpolation and covariance based adaptive interpolation method introduced for two main purpose:

a) to produce a better image quality than non-adaptive interpolation method.

b) to reduce the computational complexity of covariance with respect to adaptive interpolation methods.

It includes the following steps:

1. The linear interpolation coefficients are calculated according to the classical wiener filtering theory.

$$\vec{\alpha} = R^{-1}\vec{r}; \tag{9}$$

Where, $\vec{\alpha}$ = optimal prediction coefficient.

R and r = the local covariance at the high resolution.

$$\hat{R} = \frac{1}{M^2} C^T C; \tag{10}$$

$$\vec{r} = \frac{1}{M^2} C^T \vec{y}; \tag{11}$$

$$\vec{\alpha} = (\mathcal{C}^T \mathcal{C})^{-1} (\mathcal{C}^T \vec{y}); \tag{12}$$

Where, $\vec{y} = [y1 \dots yk \dots yM^2]^T$ is the data vector containing the M*M pixels in the local window.

C = is a 4* M^2 which is a sa data matrix whose K^{th} column vector is the four nearest neighbors of *yk* along the diagonal direction.

2. The high-resolution covariance is calculated from low resolution image [11,13].



Figure 6 Original image [3]



Figure 7 NEDI image [3]

These steps are involved to calculate only the edge pixels interpolation which reduces the computational complexity where as non-edge pixels are interpolated by bilinear interpolation.

2.2.2. Data Dependent Triangulation

It was developed to produce a good visual quality of image and to reduce the time complexity. But mainly to overcome the disadvantages of bilinear interpolation. Data dependent triangulation outputs a better visual appearance and less computational complexity. It has certain features like

- 1. It is as simple as bilinear interpolation
- 2. It can be used for arbitrary scaling and arbitrary enhancement while other techniques are for magnifying [5].



Figure 8 Formation of an Image mesh [5]



Figure 9 DDT image [5]

In DDT, the triangles from four neighbouring pixels are formed. Then diagonal pixel divides the four pixels into two triangles. Then we must choose which triangle is the best for new pixel generation. The common steps involved in DDT are:

Step 1: For every set of four pixels calculate the edge as diving pixel to form two triangles.

Step 2: Then create a mesh which stores the direction of every edge.

Step 3: Finally apply linear interpolation within the triangles [5].

2.2.3. Iterative Curvature-based Interpolation (ICBI)

The motive of the ICBI is to minimize the artifacts in an image compared to other linear, non-linear and NEDI techniques. It uses low cost and computational time. ICBI uses a combination of two techniques where in first technique, new pixels are produced by interpolating along the direction (Fast Curvature Based Interpolation, FCBI) whereas second technique uses iterative method to interpolate the pixels with energy for edge preservation [14].



Figure 10 ICBI image [3]

2.3. Summary of existing interpolation methods

From non-adaptive methods as shown in Figure 11 we can find the various difference in terms of sharpness, clarity, detail, blurriness and depth of color.

- From the analysis, we see that although nearest neighbour interpolation is the fastest method but it is the worst interpolation scheme which induces some artifacts leading to a loss of information in an image.
- Bilinear interpolation is slower than nearest neighbour but better than it as some artifacts are reduced leading to a smoother image than the original.
- Bicubic interpolation gives out a very good result among all other non-adaptive interpolation methods, with almost no artifacts. It takes more time for calculation as it includes lot of complex calculation.



Figure 11 All in one images of non-adaptive interpolation methods

Figure 12 given below has an edge based interpolation methods.

- In Figure 12 and 13, the NEDI technique provides better result with high complex calculation but still has some artifacts because of the edge discontinuities present in the window used to estimate the covariance.
- The DDT method gives a good visual appearance and has less computing time.
- Whereas the image obtained through ICBI technique does not produce much artifacts.



Figure 12 Original image [5]



Figure 13 NEDI technique, DDT technique and ICBI technique respectively [5]

2.4. Existing work review

Vaishali Patel and Prof. Kinjal Mistree in 'A Review on Different Image Interpolation Techniques for Image Enhancement' conducted an overview about the general Non-adaptive and Adaptive interpolation techniques. They conducted an analysis of different interpolation techniques with respect to PSNR ratio and conclude that adaptive techniques like NEDI, ICBI and DDT are better than nonadaptive in terms of Computational time and Visual appearance. In future work, they aim to combine both adaptive and non-adaptive methods when no time constraints are considered to obtain a good image [5].

Dianyuan Han in 'Comparison of Commonly Used Image Interpolation Methods' discusses how the image magnification algorithm affects the quality of an image. Based on the principles of image interpolation algorithm, features of the nearest neighbor, bilinear, bicubic and cubic B spline interpolation are analyzed in terms of subjective and objective aspects. And provide the similar advantages and disadvantages as discussed in above non-adaptive algorithms part, letting the users to choose the better interpolation for magnifying an image [11].

Win-shan Tam, Chi-wah Kok and Wan-chi Slu presented 'Modified Edge Directed Interpolation for images' which is a modified version of new edge directed interpolation which eliminates the prediction error accumulation problem by adopting a modified training window and extending the covariance matching into multiple directions to suppress the covariance mismatch problem. The presented work results in achieving the sharpness and edge smoothness [13].

Andrea Giachetti and Nicola Asuni present 'Real time artifact-free image upscaling', a new upscaling method (ICBI) based on two step grid filling and an iterative correction for the interpolated pixels images which are obtained by minimizing the objective functions depending upon the second order

directional derivatives of the image intensity of lower resolution original data. They obtained efficient results for the ability of removing artifacts without generating an artificial detail [14].

ZhiweiPan, WeiChen, ZhenyuJiang, LiqunTang, YipingLiu, ZejiaLiu in 'Performance of global lookup table strategy in digital image correlation with cubic B-spline interpolation and bicubic interpolation' proposed a global look up table strategy to accelerate the interpolation techniques which consumes more time in iterative sub-pixel digital image correlation method algorithm(DIC).

In this paper, a global look-up table strategy with cubic B-spline interpolation is been developed for the DIC method based on the inverse compositional Gauss–Newton (IC-GN) algorithm. They carried out experiments with bicubic and cubic B spline interpolation method in terms of accuracy, efficiency and precision. The proposed method was proven to have better result in obtaining a more accurate result [15].

Ahmadreza baghaie and Zeyun Yu in 'Structure tensor based image interpolation method' put forward a new edge directed image super resolution algorithm based on structure tensor. By using an isotropic Gaussian filter, the structure tensor at each pixel of the input image was computed. Comparing to previously propose edge directed image interpolation, this method outperforms in both objective and subjective aspects achieving higher quality in terms of nose and JPEG compression file images. It achieved higher speed process. This does not use any optimization procedure and being robust in case of noisy images make this proposed method to be more suitable for day to day used electronic device [16].

David Darian Muresan, and David muresan proposed a 'fast Edge Directed Polynomial Interpolation' for image interpolation which produces high quality outputs although it's computational time is lightly comparable to cubic interpolation. In this they majorly focused on obtaining a sharp image instead of jagged free edges in the image [17].

Ms. Shabana Parveen and Ms. Rajbala Tokas proposed 'Faster image Zooming Using Cubic Spline Interpolation Method' to obtain an efficient image interpolated. This method is carried out in two steps, at first the area of the image is being enlarged by inserting zeros in between every two rows and columns according to the input given by the user for zooming intensity. Second step includes the estimation of correct values of those zero values inserted. This method works in a way that the zoomed image does not rupture with absurd values of the pixel [18].

Yeon Ju Lee and Jungho Yoon put forth an 'Image zooming method using edge-directed moving least squares interpolation based on exponential polynomials' which is a non-linear interpolation algorithm.

The implemented algorithm is based on Moving least squares (MLS) projection technique. When up sampled images are considered it has a limitation in producing sharp edges resulting in blurred image at the end. So, they used a novel MLS to preserve the edge features [19].

Ammar, E.M. Saad, I. Ashour and M. Elzorkany in 'Image zooming and multiplexing techniques based on K-Space Transformation' introduce a new technique based on K-space transformation. It is a concept of representing a digital image in K-space domain. This method provides a new image zooming technique with little image artifacts, problems and high zooming factors. This has a feature of zooming in and zooming out. The analysis indicated that this method is efficient for zooming a full image and interested region of an image providing a good subjective quality [20].

Rukundo Olivier and Cao Hanqiang 'Nearest Neighbor Value Interpolation' present a nearest neighbor value algorithm (NNV) to obtain a high resolution interpolated image. The difference between the proposed and existing nearest neighbor method is that the concept includes finding the missing pixel value guided by the nearest value than the distance value. This analysis provides a good interpolated result compared to the conventional interpolation algorithms [21].

P. Priya in 'A Method for High Resolution Color Image Zooming using Curvature Interpolation' proposed a Curvature interpolation method(CIM) algorithm based on linear equation to produce a high-resolution image by solving a linearized curvature equation. CIM first evaluates the curvature of the low resolution, after interpolating the curvature to the high-resolution image domain to minimize the artifacts like blur and the checkerboard effect. The result of the proposed system provides an image with high quality and sharp edges [22].

In the work review many methods and constraints have been used to get an enhanced interpolated image. In this thesis work, aim is to obtain an image interpolation where the zoomed image has the better effects of the following,

- Good PSNR and SNR ratio
- short computational time
- Good contrast and homogeneity
- To have less complexity

2.5. Implementation of existing codes

2.5.1. Bilinear zoom interpolation

• In this code the image is been read by imread(.jpg)

- Then a signed integer 16 with a syntax int16 is used to manipulate arrays of pixels without changing elements, for example resize, and then unsigned 8bit integer is used with a syntax unit8 because images cannot have pixel more than 2^8-1.
- Then a user defined variable 'fac' is used so that if user want to zoom it by 2 then factor value will be 2.
- Then bilinear interpolation is applied with rows and column being multiplied by factor 'fac' and pixels are calculated along x and y coordinates of the image with bilinear interpolation formula.
- The given output is zoomed by a factor of 3 [23].



Figure 14 Original image to be zoomed



Figure 15 Bilinear zoomed image

2.5.2. Image enhancement through DCT scaling method

- The algorithm contains the Discret Cosine Transform (DCT) scaling method to enhance an image.
- The DCT parameters are defined by block size, then the original image is been converted to Y,Cb,Cr format because the original color image obtained cannot be processed directly.
- After conversion of image, it is converted into block size where it has AC and DC components. First component is DC and rest of the blocks are AC coefficients. Code for changing the AC and DC values are written [24].



Figure 16 Input image to be enhanced



Figure 17 Enhanced image

3. Design of proposed method

The basic flow diagram for the implementation of image interpolation is given below



Figure 18 Flow diagram of proposed Hybrid Image interpolation

The image is loaded and the user can enter the zoom factor between 1 to 10. The next step is to extract the information about the size, format of the image, width & height and bit depth. For every pixel 4 nearest pixel values are extracted and then the pixel coordinates are localized. The bilinear interpolation equations are computed for the input image as shown in the equation (4,5,6,7) in section 2.1.2.

Then the image is split up in the RGB planes and Median filter is applied to each component. The Median filter is a non-linear filter, which is often used to remove the noise from the image. It is more likely to be used before processing an image into some modification. It is used in this method to remove the noise and preserve the edges in an image.

Then the image is enhanced by using the histogram profile. The Histogram where the first step is to get 'bin' values that is the entire range of values in each component is divided into a series of intervals. The 'bin' values must be adjacent and of equal size. Then the profile projection is calculated along vertical (*y-axis*) and horizontal direction (*x-axis*). It is given by

$$VPP(y) = \sum_{1 \le x \le m} f(x, y); \qquad (13)$$

$$HPP(x) = \sum_{1 \le x \le n} f(x, y); \qquad (14)$$

Where, *VPP*= vertical projection profile for histogram,

HPP= Horizontal projection profile for histogram,

f(x,y) = functional known data at point (x,y)

m & n = the rows and columns

After histogram profiling, all 3 color planes are concatenated to obtain an enhanced zoom image [25].

The flow chart for the comparison of image quality is given below,



Figure 19 Flow chart of the process of obtaining comparison image quality factors.

In Figure 19 the flow chart explains how the process of comparing the image qualities between existing and proposed method. A user interface is provided so that user can choose the desired interpolation to execute. The image is loaded and zoom factor between 1 to 10 is entered by the user. Then the user clicks on Bilinear interpolation then it the bilinear zoom factors are applied and zoomed image is obtained and image quality parameters are calculated. It is the same for other Nearest Neighbour, Pixel replacement and proposed hybrid interpolation where the image is zoomed with respect to their interpolation method and after obtaining the zoomed image the quality factors are calculated for all methods and displayed on the workspace screen.

3.1. Tools required

Matlab v7.12.0(R2017a) software in a system configuration of 64 bit, core *i5*, Windows 10 Operating System.

Coding languages:

Matlab code

3.2. Calculation parameters

• Calculation for PSNR ratio in decibel

The PSNR ratio of two images, where the quality of an images is measured between an original and reconstructed or modified image. The higher the PSNR value, the better the obtained or reconstructed image's quality.

It first calculates the Mean Squared Error (MSE) by using the equation below

$$MSE = \frac{\sum_{M,N} [I_1(m,n) - I_2(m,n)]^2}{M*N};$$
(15)

Where m, n = height and width of the image

 I_1 = Original image

- I_2 = Modified or Reconstructed image
- M, N = Rows and column of an image

The PSNR is a ratio of maximum possible power of a signal to the power of interrupting noise which affects the accuracy of image representation. PSNR is given by

$$PSNR = 10 \log_{10}(\frac{R^2}{MSE}); \tag{16}$$

Where, R = the maximum variation in the input image data.

If the input image data type has a double precision floating point data type then the value of R is 1. If it has 8-bit unsigned integer data type then the value will be R=255 [26].

• Calculation of Signal to Noise Ratio (SNR)

SNR is used in image processing to measure signal strength with respect to the background noise. It is generally a physical measure of the image sensitivity. In image processing the Root Mean Square (RMS) is considered as the background region so, that SNR is calculated as ratio of net signal to the RMS noise. SNR is given by,

$$SNR = \frac{\mu_{sig}}{\sigma_{bg}}; \tag{17}$$

Where, μ_{sig} = the average signal value

 σ_{bg} = the standard deviation of the background

A second order polynomial was used to the array of data and subtracted from the original array line data to measure the average signal value and background values. The difference of average signal and background signal values was used to calculate the net signal value.

The second order polynomial is calculated by double summation,

$$f_i = \sum_{j=0}^m \sum_{i=1}^n a_j x_i^j;$$
(18)

Where, f =Output sequence

x = the input array values from the signal region or background region

 a_i = the polynomial fit coefficient

m= the polynomial order

n = the number of lines

This second order polynomial is subtracted from the original signal value to remove any distortion and then averaged to a signal value and background value.

$$\mu_{sig} = \frac{\sum_{i=1}^{n} (X_i - f_i)}{n},$$
 (19)

$$\mu_{bkg} = \frac{\sum_{i=1}^{n} (X_i - f_i)}{n};$$
 (20)

Where, μ_{sig} = average signal value

 μ_{bkg} = average background value

 X_i = value of the *i*th line in the signal region or background region

 f_i = value of the i^{th} output of the second order polynomial

n = number of lines in background or signal region

the net signal value is obtained by

$$signal = \mu_{sig} - \mu_{bkg}; \qquad (21)$$

The RMS noise is calculated by

RMS noise =
$$\sqrt{\frac{\sum_{i=1}^{n} (X_i - \frac{\sum_{i=1}^{n} X_i}{n})^2}{n}}$$
 (22)

The SNR is given by,

$$SNR = \frac{signal}{RMS \ noise} \tag{23}$$

After using the 20-log rule,

$$SNR = 20 \log_{10} \frac{Signal}{RMS \ noise} \ dB \tag{24}$$

The SNR value is positive then the image quality is accepted to be a good image. According to industry standard measure if SNR= 32.04 dB, the image quality is excellent. SNR=20 dB, then the image quality is not the best but in a range acceptability [27].

• Histogram error with respect to the original image

It is the representation of an image quality in a graph with the number of pixels for each tonal value. In this analysis, the histogram error is calculated between the original image and the zoomed image. The mathematical function of histogram for an image is given by,

$$n = \sum_{i=1}^{k} m_i; \qquad (25)$$

Where, n = total number of observation that fall into 'bin'

K= total number of bins

 $m_i = histogram$

Then the cumulative histogram M_i is calculated for histogram m_i ,

$$M_i = \sum_{j=1}^k m_j \tag{26}$$

This is calculated for both original image and the zoomed image. The difference is calculated between them and the error is notified [28].

• Homogeneity of the images

Homogeneity of an image is the level of uniform composition of an image characteristics like shape, color, texture, contrast, tone, brightness, height, width, temperature etc. It expresses the similarities of all the qualities of an image. The condition for homogeneous function is,

$$f(\alpha x, \alpha y) = \alpha^k f(x, y); \qquad (27)$$

Where, x and y= real value functions,

 α = all real number

k = constant

$$f(\alpha v) = \alpha^k f(v); \tag{28}$$

Where, v = function between two vector spaces in a field F.

k = integer value

f = the homogeneous function of degree k [29].

• Digital Image Correlation

It is method of comparing the digital photographs at different level of modification. It works effectively by tracking the pixel blocks and surface displacement with unique range of intensity levels and contrast in an image. The correlation relation of an image is found by using the correlation coefficient,

$$\rho(A,B) = \frac{1}{N-1} \sum_{i=1}^{N} \overline{\left(\frac{A_i - \mu_A}{\sigma_A}\right)} \left(\frac{B_i - \mu_B}{\sigma_B}\right);$$
(29)

Where, σ_A and σ_B = standard deviation of *A* and *B*.

 A_i and B_i = mean of A and B,

N = number of scalar observation for the variables A and B

 $\rho(A, B) = \text{correlation coefficient [30]}.$

• Contrast of an image

The contrast of an image is the difference in the color intensity or the luminance. It is also distinguished in terms of difference in brightness of an image and the original image on the same scenic field.

$$Contrast = \frac{Luminancedifference}{Average Luminance};$$
(30)

It is also defined as Weber contrast,

Weber Contrast
$$=$$
 $\frac{I-I_b}{I_b}$; (31)

Where, I = Luminance of features in an image

 I_b = Luminance of the background of an image [31].

• Energy

It is some information present in an image. When the image undergoes some modification or compression it tends to lose some information.

3.3. Executed snapshots of Bilinear, Nearest neighbor, Pixel replacement and proposed hybrid interpolation method for a zoom factor = 2.



Figure 20 Original Image



Figure 21 Enter zooming factor as 2



Figure 22 Nearest neighbour interpolation



Figure 23 Bilinear zooming interpolation



Figure 24 Pixel Replacement Interpolation



Figure 25 Hybrid interpolation of an image

Executed snapshots of Bilinear, Nearest neighbor, Pixel replacement and proposed hybrid interpolation method for a zoom factor = 4.



Figure 26 Original image



Figure 27 Enter Zooming factor as 4



Figure 28 Bilinear zoom for a zoom factor 3



Figure 29 Nearest neighbour zoom interpolation



Figure 30 Pixel Replacement interpolation for zooming



Figure 31 Hybrid Zooming interpolation

3.4. Comparison Table for results obtained

For zooming factor = 2

 Table 7 Comparison parameter for zooming factor 2

Image	File size (Bytes)	File Format	Height×Width	Bit depth
			(Pixels)	
Original image	10,278	.jpg	200*200	24
Bilinear zoomed image	23,059	.jpg	399*399	24
Nearest neighbor zoomed image	24,769	.jpg	400*400	24
Pixel replaced zoomed image	23,940	.jpg	400*400	24
Hybrid zoomed image	23,538	.jpg	399*399	24

Table 8 Comparison parameter of various interpolation for zoom factor 2

Image1(mushroom)	Bilinear	Nearest	Pixel	Hybrid
	Interpolation	Neighbour	Replacement	Interpolation
		Interpolation	Interpolation	
PSNR in dB	73.2115	68.6984	68.694	73.2544
SNR in dB	4.2322	45.5288	45.528	45.5288
Histogram Error	0.0001	0.7908	0.7908	0.0007
Structural similarity	0.2027	0.0031	0.0031	0.0031
Energy	[0.1534 0.1504]	[0.1405 0.1377]	[0.1488 0.1463]	[0.1324 0.1294]
Homogeneity	[0.8973 0.8894]	$[0.8778\ 0.8678]$	[0.8853 0.8776]	[0.8925 0.8833]
Contrast	[0.2629 0.3105]	[0.3500 0.3966]	[0.3135 0.3521]	[0.2775 0.3308]
Correlation	[0.9438 0.9336]	[0.9340 0.9251]	[0.9344 0.9262]	[0.9465 0.9362]
Process	56.9814	0.869512	5.2829	24.810347
time(seconds)				

For zooming factor = 3

Lubic <i>y</i> comparison parameters for zoom factor <i>y</i>
--

Image	File size (Bytes)	File Format	Height×Width	Bit depth
			(Pixels)	
Original image	10,278	.jpg	200*200	24
Bilinear zoomed image	39,526	.jpg	598*598	24
Nearest neighbor zoomed image	50,822	.jpg	600*600	24
Pixel replaced zoomed image	48,786	.jpg	600*600	24
Hybrid zoomed image	40,424	.jpg	598*598	24

 Table 10 Comparison parameter of various interpolation for zoom factor 3

Image1(mushroom)	Bilinear	Nearest Neighbour	Pixel Replacement	Hybrid
	Interpolation	Interpolation	Interpolation	Interpolation
PSNR in dB	76.6079	68.7002	68.7002	77.129
SNR in dB	6.5788	46.2904	45.2904	46.2904
Histogram Error	0.0000	0.7847	0.7947	0.0007
Structural similarity	0.2916	0.0018	0.0018	0.0018
Energy	[0.1686 0.1660]	[0.1541 0.1516]	[0.1624 0.1601]	[0.1466 0.1438]

Homogeneity	[0.9269 0.9209]	[0.9088 0.9020]	[0.9143 0.9086]	[0.9228 0.9157]
Contrast	[0.1765 0.2055]	[0.2498 0.2816]	[0.2261 0.2519]	[0.1889 0.2219]
Correlation	[0.9622 0.9559]	[0.9529 0.9468]	[0.9527 0.9472]	[0.9635 0.9571]
Process	80.57013	0.940110	5.3358	29.442491
time(seconds)				

For zooming factor = 4

Table 11 Comparison parameters for zoom factor 4

Image	File size (Bytes)	File Format	Height×Width	Bit depth
			(Pixels)	
Original image	8018	.jpg	200*200	24
Bilinear zoomed image	46,899	.jpg	797*797	24
Nearest neighbor zoomed image	51,080	.jpg	800*800	24
Pixel replaced zoomed image	50,596	.jpg	800*800	24
Hybrid zoomed image	47,572	.jpg	797*797	24

Table 12 Comparison parameter of various interpolation for zoom factor 4

Image1(mushroom)	Bilinear	Nearest	Pixel	Hybrid
	Interpolation	Neighbour	Replacement	Interpolation
		Interpolation	Interpolation	
PSNR in dB	77.089	68.0086	68.0086	77.158
SNR in dB	11.6170	42.0179	42.0179	42.0179
Histogram Error	0.0001	0.5524	0.5524	0.0003
Structural similarity	0.8275	0.0144	0.0144	0.0144
Energy	[0.1249 0.1216]	[0.1209 0.1178]	[0.1225 0.1194]	[0.1466 0.1438]
Homogeneity	[0.9554 0.9486]	[0.9502 0.9422]	[0.9508 0.9429]	[0.9228 0.9157]
Contrast	[0.1594 0.1266]	[0.1374 0.1762]	[0.1355 0.1730]	[0.1889 0.2219]
Correlation	[0.9812 0.9851]	[0.9842 0.9798]	[0.9842 0.9799]	[0.9635 0.9571]
Process	111.8427	1.2588	5.40855	34.20259
time(seconds)				

3.5. The comparison analysis,

• When the Peak Signal to Noise ratio (PSNR) is considered, the proposed method outperforms among the existing system. The average PSNR obtained for hybrid interpolation was 75.84 dB,

which is nearly equal to the Bilinear interpolation. It can be observed in the graph given below in Figure 32



Figure 32 The Average Peak Signal to Noise ratio of interpolations

• The histogram error of a zoomed image with respect to original image is slightly higher when compared to the bilinear interpolation. Histogram gives the stats of differences of the image quality between an original image and modified or zoomed image.

As the hybrid zoom interpolated image performs better in quality of contrast, homogeneity and correlation, it is obvious to get an acceptable change from the original image. Lower the histogram error rate better the zoomed image. The histogram error decreases as the zooming factor increases. Means the zoomed image has retained most of the information with respect to original image.



Figure 33 Histogram error rate

• The file size and Processing time parameter where the better quality image occupies less space but consumes more process time and the poor quality image occupies more storage space but less process time. In both the cases the hybrid interpolation method performs better in consideration to process time and storage space. It is a better method in presence of no time restrictions. The average values of process time and File sizes in Bytes are depicted in Figure 33 and Figure 34 respectively.



Figure 34 The average process time required for zooming the image by all the interpolation



Figure 35 The Average storage file size required by all the interpolations

4. Conclusion

- The proposed method provides a good performance in terms of PSNR, Process time, Homogeneity, Contrast, Correlation and Histogram Error rate with respect to the 3 existing methods which are considered here for the comparison.
- The Process time and the File size obtained were better if the image quality parameters are considered. Which have a better result. Although the process time for nearest and pixel replacement was very short, their PSNR ratio, histogram error rate and contrast values were poor comparatively to the proposed method.
- The SNR value obtained has the same outcome as compared to nearest and pixel interpolation.
- In future work the improvement can be done on obtaining a better SNR value, lesser process time for zooming and lesser file storage space.

References

1. Sakshica, Dr. Kusum Gupta 'Various Raster and Vector Image File Formats' published by International Journal of Advanced Research in Computer and Communication Engineering Vol. 4, Issue 3, March 2015.

2. <u>http://www.imatest.com/docs/sharpness/</u> accessed date 20th May 2017

3. <u>http://www.imatest.com/docs/iqfactors/</u> accessed date 20th May 2017

4. Saeed V. Vaseghi 'Interpolation' Advanced Digital Signal Processing and Noise Reduction, Second Edition, Copyright © 2000 John Wiley & Sons Ltd ISBNs: 0-471-62692-9 (Hardback): 0-470-84162-1 (Electronic).

5. Vaishali Patel, Prof. Kinjal Mistree 'A Review on Different Image Interpolation Techniques for Image Enhancement' International Journal of Emerging Technology and Advanced Engineering Website: www.ijetae.com (ISSN 2250-2459, ISO 9001:2008 Certified Journal, Volume 3, Issue 12, December 2013).

6. Paul Daniel Dumitru, Marin Plopeanu, Dragos Badea in 'Comparative study regarding the methods of interpolation' Department of Geodesy and Photogrammetry Faculty of Geodesy Technical University for Civil Engineering Bucharest 122-124, Bvd. Lacul Tei, Sector 2, Bucharest ROMANIA.

7. Bilinear interpolation – Wikipedia <u>https://en.wikipedia.org/wiki/Bilinear_interpolation</u> visited on 20th May, 2017.

8. Bicubic interpolation Wikipedia <u>https://en.wikipedia.org/wiki/Bicubic_interpolation</u> visited on 20th May 2017.

9. Panzy, Preeti Gulati 'Review of Adaptive and Non-Adaptive Image Interpolation Techniques' International journal of Science Technology & Management (IJSTM), ISSN: 2229-6646.

10. C. John Moses, D. Selvathi 'A Survey on non-adaptive image interpolation algorithms based on quantitative measures' Moses et al., International Journal of Advanced Engineering Technology. E-ISSN 0976-3945.

11. Dianyuan Han 'Comparison of Commonly Used Image Interpolation Methods' Proceedings of the 2nd International Conference on Computer Science and Electronics Engineering (ICCSEE 2013).

12. Ankit Prajapati, Sapan Naik, Sheethal Mehta 'Evaluation of Different Image Interpolation Algorithms' International Journal of Computer Applications (0975 – 8887) Volume 58–No.12, November 2012.

13. Wing-Shan Tam, Chi-Wah Kok, Wan Chi Siu 'Modified Edge Directed Interpolation for Images', Journal of Electronic Imaging 19(1), 013011 (Jan–Mar 2010).

14. Andrea Giachetti and Nicola Asuni 'Real time artifact-free image upscaling', submitted to ieee transactions on image processing.

15. ZhiweiPan, WeiChen, ZhenyuJiang, LiqunTang, YipingLiu, ZejiaLiu in 'Performance of global look-up table strategy in digital image correlation with cubic B-spline interpolation and bicubic interpolation'.

16. Ahmadreza baghaie and Zeyun Yu in 'Structure tensor based image interpolation method' International Journal of Electronics and Communications (AEU). Int.J. Electron.Commun. (AEU)69 (2015) 515-522.

17. David Darian Muresan, and David muresan proposed a 'fast Edge Directed Polynomial Interpolation' patent no. US 7,212,689 B2, date of patent: - May 1, 2007.

18. Ms. Shabana Parveen and Ms. Rajbala Tokas proposed 'Faster image Zooming Using Cubic Spline Interpolation Method' International Journal on Recent and Innovation Trends in Computing and Communication, Volume: 3 Issue: 1, 22 – 26, ISSN: 2321-8169.

19. Yeon Ju Lee and Jungho Yoon put forth an 'Image zooming method using edge-directed moving least squares interpolation based on exponential polynomials', Applied Mathematics and computation. <u>http://dx.doi.org/10.1016/j.amc.2015.07.086</u>.

20. Ammar, E.M. Saad, I. Ashour and M. Elzorkany in 'Image zooming and multiplexing techniques based on K-Space Transformation' International Journal of Signal Processing, Image Processing and Pattern Recognition Vol. 5, No. 4, December, 2012.

21. Rukundo Olivier and Cao Hanqiang 'Nearest Neighbor Value Interpolation' (IJACSA) International Journal of Advanced Computer Science and Applications, Vol. 3, No. 4, 2012.

22. P.Priya in 'A Method for High Resolution Color Image Zooming using Curvature Interpolation' IJCSEC-International Journal of Computer Science and Engineering Communications, Vol.2 Issue.3, May 2014. ISSN: 2347–8586.

23. Image zooming with bilinear interpolation with matlab code in matlab simulink, http://www.theengineeringprojects.com/2016/02/image-zooming-bilinear-

interpolationmatlab.html Accessed date - 6 June 2016

24. Image enhancement through DCT scaling method, http://www.mathworks.com/matlabcentral/fileexchange/28877-color-

imageenhancement/content/ColorEnhance/SampleUsage.m Accessed date: 6 June 2016

25. Mohammed Javed, P. Nagabhushan, B.B. Chaudhuri in 'Extraction of Projection Profile, Run-Histogram and Entropy Features Straight from Run-Length Compressed Text-Documents'from Computer Vision and Pattern Recognition Unit Indian Statistical Institute, Kolkata-700108, India 3bbc@isical.ac.in. 26. D. Poobathy and Dr. R. Manicka Chezian in 'Edge Detection Operators: Peak Signal to Noise Ratio Based Comparison' I.J. Image, Graphics and Signal Processing, 2014, 10, 55-61 Published Online September 2014 in MECS (http://www.mecs-press.org/) DOI: 10.5815/ijigsp.2014.10.07.

27. Signal to noise ratio (imaging)- Wikipedia <u>https://en.wikipedia.org/wiki/Signal-to-noise_ratio (imaging)</u> accessed on 20th May 2017.

28. <u>https://en.wikipedia.org/wiki/Histogram#Mathematical_definition</u> Histogram - wikipedia visited on 20th May 2017.

29. <u>https://en.wikipedia.org/wiki/Homogeneous_function</u> Homogeneous function - wikipedia visited on 20th May 2017.

30. <u>https://se.mathworks.com/help/matlab/ref/corrcoef.html</u> visited on 21st May 2017.

31. <u>https://en.wikipedia.org/wiki/Contrast_(vision)</u> visited on 20th May 2017.