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Moments of the MRC and EGC Combiner Output

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Introduction

Fading, which appears as the result of spreading multichannel signals in the wireless telecommunication systems, is the main cause of performance degradation of digital wireless communication systems. Many statistical distributions are available in the literature to model the fading process in order to predict the system performance. One of these is the Nakagami-distribution, which has attracted considerable interest as a channel-fading model. This model is very flexible, because it can be used to account for both severe and weak fading conditions, and includes Rayleigh fading as a special case. The Nakagami-distribution provides a very good fit for measured data in a variety of fading environments [1].

Diversity systems are an effective and often used techniques for mitigating the fading appeared as the results of spreading multichannel signals in the wireless telecommunication systems (for example: mobile phone system). In this work, authors discuss the results of EGC (Equal Gain Combiner) and MRC (Maximal-Ratio Combiner) diversity systems. Also, these systems are used in exceptional cases with shadow effect and in special cases with simultaneous fading and shadow effect.

The performance of MRC in a Nakagami-fading environment has been studied extensively [1-3]. It is well known that MRC, which provides the best system performance (the highest average output signal-to-noise ratio – SNR), is difficult to implement in practice. EGC, however, is easier to implement, but incurs a performance penalty.

In this study, we calculated the moments of output combiner signals for EGC and MRC combiner with two or more branches in presence of Nakagami and Rician fading. Mathematical relations who were formed in this paper calculate the average signal, the square average signal and the signal variance. The moments of the combiner output signals can be used for determining optimal parameter values of diversity systems.

EGC model system with L inputs (diversity channels)

We consider a diversity system with L uncorrelated branches (L diversity channels) and a predetection EGC combiner. This architecture is shown in Fig. 1.

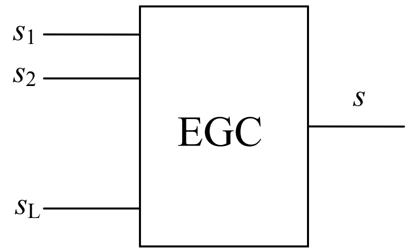

Fig. 1. EGC diversity system model with L inputs

Output signal of EGC diversity system is equal to the sum of combiner input signal amplitudes and we can write:

$$s = \sum_{k=1}^{L} s_k \ , \tag{1}$$

where s_k denotes envelope of the k-th diversity channel.

Probability density function for input signal s_k, in case of Nakagami fading is:

$$p_{s_k}(s_k) = \frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)^{m_k} s_k^{2m_k-1} e^{-\frac{m_k}{\Omega_k}s_k^2} \ , \ s_k \geq 0 \ , \tag{2}$$

where $\Gamma[\cdot]$ is the Gamma function, $\Omega_k = E\left[s_k^2\right]$, k=1; L – the average power; $m_k \geq 0.5$ – the fading severity parameter of the k-th channel; $E[\cdot]$ – denoting statistical expectation. Note that $m_k = 0.5$ describes a one-sided Gaussian fading; $m_k = 1$ represents the Rayleigh fading model; $0.5 \le m_k < 1$ models fading conditions more severe than Rayleigh, whereas $m_k > 1$ models fading conditions less severe than Rayleigh.

N-order moment of the output signal is expressed as:

$$M_n = \overline{s^n} = \overline{\left(\sum_{k=1}^{L} s_k\right)^n} \; . \tag{3}$$

If we use multinomial formula, we obtain:

$$M_n = \sum_{i_1=0}^{n}\sum_{i_2=0}^{n-i_1}\sum_{i_3=0}^{n-i_1-i_2}\cdots\sum_{i_{L-1}=0}^{n-\sum_{k=1}^{L-2} i_k} \frac{n!}{i_1!i_2!..i_L!}\overline{s_1^{i_1}s_2^{i_2}...s_L^{i_L}} \; . \tag{4}$$

Since input signals are independent, the joint probability density function can be expressed as a product of input probability density functions. Thus, the average value for product in (4) can be written as:

$$\overline{\prod_{k=1}^{L} s_k^{i_k}} = \int_0^{\infty}\prod_{k=1}^{L} s_k^{i_k}\, p_{sk}(s_k)\,ds_k \; . \tag{5}$$

After substituting (1) in (5) and integrating, we obtain:

$$\overline{\prod_{k=1}^{L} s_k^{i_k}} = \prod_{k=1}^{L}\frac{1}{\Gamma(m_k)}\left(\frac{\Omega_k}{m_k}\right)^{i_k/2}\Gamma(m_k + i_k/2) \, . \tag{6}$$

Putting $\rho_k = m_k / \Omega_k$, expression (6) becomes:

$$\overline{\prod_{k=1}^{L} s_k^{i_k}} = \prod_{k=1}^{L}\frac{1}{\Gamma(m_k)}\left(\frac{1}{\rho_k}\right)^{i_k/2}\Gamma(m_k + i_k/2) \, . \tag{7}$$

Substituting (7) in (4) we obtain final expression for N-order moment:

$$M_n = \sum_{i_1=0}^{n}\sum_{i_2=0}^{n-i_1}\cdots\sum_{i_{L-1}=0}^{n-\sum_{k=0}^{L-2} i_k}\frac{n!}{i_1!i_2!\cdots i_L!}\times$$

$$\times \prod_{k=1}^{L}\left[\frac{1}{\Gamma(m_k)}\left(\frac{1}{\rho_k}\right)^{i_k/2}\Gamma(m_k + i_k/2)\right]; \tag{8}$$

where $i_L = n - \sum_{k=1}^{L-1} i_k$.

As a special case, we consider EGC diversity system with two channels, which often appears in practical telecommunications systems. By using binomial formula and (7), we obtain N-order moment as:

$$M_n = \sum_{i=0}^{n}\binom{n}{i}\frac{1}{\Gamma(m_1)\Gamma(m_2)}\left(\frac{1}{\rho_1}\right)^{(n-i)/2}\left(\frac{1}{\rho_2}\right)^{i/2}\times$$

$$\times\Gamma\left(m_1 + \frac{n-i}{2}\right)\Gamma\left(m_2 + \frac{i}{2}\right). \tag{9}$$

The average value of signal from output of EGC combiner with two branches and the signal variance can be calculated by:

$$M_1 = \sqrt{\frac{1}{\rho_1}}\frac{\Gamma(m_1 + 1/2)}{\Gamma(m_1)} + \sqrt{\frac{1}{\rho_2}}\frac{\Gamma(m_2 + 1/2)}{\Gamma(m_2)}; \tag{10}$$

$$\sigma^2 = M_2 - M_1^2 = \frac{1}{\rho_1}\left(\frac{\Gamma(m_1+1)}{\Gamma(m_1)} - \frac{\Gamma^2(m_1+1/2)}{\Gamma^2(m_1)}\right) +$$

$$+\frac{1}{\rho_2}\left(\frac{\Gamma(m_2+1)}{\Gamma(m_2)} - \frac{\Gamma^2(m_2+1/2)}{\Gamma^2(m_2)}\right). \tag{11}$$

We consider only the case of equal fading severities on all the branches. Putting $m_1 = m_2 = m$ and $\rho_1 = \rho_2 = \rho$, we obtain:

$$M_1 = \frac{2}{\sqrt{\rho}}\frac{\Gamma(m+1/2)}{\Gamma(m)}; \tag{12}$$

$$\sigma^2 = M_2 - M_1^2 = \frac{2}{\rho}\left[\frac{\Gamma(m+1)}{\Gamma(m)} - \frac{\Gamma^2(m+1/2)}{\Gamma^2(m)}\right]. \tag{13}$$

Fig. 2 and Fig. 3 show the average value and the variance of the EGC diversity system output with two inputs as functions of ratio ρ, respectively.

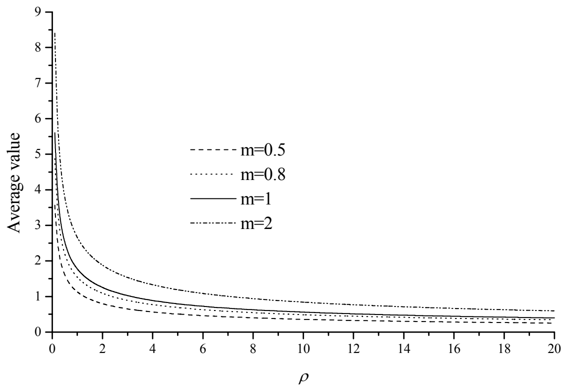

Fig. 2. Average value of EGC diversity system output versus ratio ρ in case L=2 and m=0.5, 0.8, 1, 2

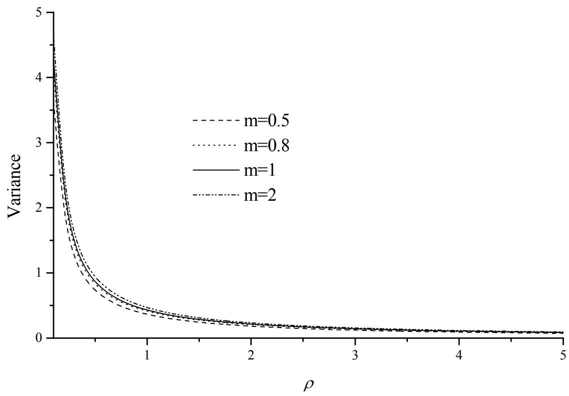

Fig. 3. Variance of EGC diversity system output versus ratio ρ in case L=2 and m=0.5, 0.8, 1, 2

Now, we consider a case when the input probability densities are Rician with mean A_k and variance σ_k^2

$$p_{s_k}(s_k)=\frac{s_k}{\sigma_k^2}e^{-\frac{s_k^2+A_k^2}{2\sigma_k^2}}I_0\left(\frac{s_k A_k}{\sigma_k^2}\right),\quad s_k\ge 0, \tag{14}$$

where $I_0[.]$ denotes zero-order modified Bessel function of the first kind.

If we put (14) into (5) and use integral relation

$$\int_0^\infty t^{\mu-1}I_\nu(\alpha t)e^{-\beta^2t^2}dt=\frac{\Gamma\left(\frac{\mu+\nu}{2}\right)\left(\frac{\alpha}{2\beta}\right)^\nu}{2\beta^\mu\Gamma(\nu+1)}e^{\frac{\alpha^2}{4\beta^2}}\times$$

$$\times{}_1F_1\left(\frac{\nu-\mu}{2}+1,\nu+1,-\frac{\alpha^2}{4\beta^2}\right), \tag{15}$$

where ${}_1F_1[.]$ denotes Kummer confluent hypergeometric function, we obtain an expression for average value of product in form

$$\overline{\prod_{k=1}^{L}s_k^{i_k}}=\prod_{k=1}^{L}2^{i_k/2}\sigma_k^{i_k}\Gamma\left(\frac{i_k+2}{2}\right){}_1F_1\left(\frac{-i_k}{2},1,-\frac{A_k^2}{2\sigma_k^2}\right). \tag{16}$$

By using binomial formula and (16), we obtain N-order moment for EGC diversity system with two channels as:

$$M_n=\sum_{i=0}^{n}\binom{n}{i}2^{n/2}\sigma_1^{n-i}\sigma_2^{i}\Gamma\left(\frac{n-i+2}{2}\right)\Gamma\left(\frac{i+2}{2}\right)\times$$

$$\times{}_1F_1\left(\frac{-(n-i)}{2},1,-\frac{A_1^2}{2\sigma_1^2}\right){}_1F_1\left(\frac{-i}{2},1,-\frac{A_2^2}{2\sigma_2^2}\right). \tag{17}$$

MRC model system with L inputs (diversity channels)

We consider MRC architecture such as shown in Fig. 4. This is the diversity system with signals in L separate branches with the Nakagami fading.

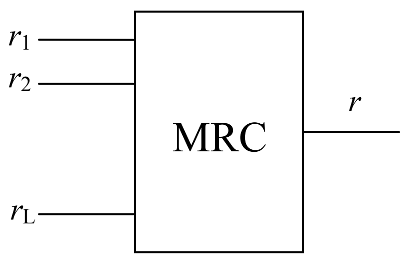

Fig. 4. MRC diversity system model with L inputs

The probability density function of r_k is mathematically expressed as:

$$p_{rk}(r_k)=\frac{1}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)^{m_k}r_k^{m_k-1}e^{-\frac{m_k}{\Omega_k}r_k},\quad r_k\ge 0, \tag{18}$$

where $\Gamma[\cdot]$ – the Gamma function; $\Omega_k=E[r_k]$, k=1; L - the average power; $m_k\ge 0.5$ – the fading severity parameter of the k-th channel.

Output signal of MRC diversity system is equal the sum of square combiner input signals:

$$r=\sum_{k=1}^{L}r_k\ ; \tag{19}$$

where r_k denotes the square envelope of the k-th diversity channel. Note, that formula (19) has the same form as (1), with difference that r_k denotes square envelope while s_k is the envelope of the k-th diversity channel. Thus, we can use (4) to obtain a final expression for N-order moment of the output of MRC combiner.

Putting (18) in (5), we obtain an expression for average value of product in form:

$$\overline{\prod_{k=1}^{L}r_k^{i_k}}=\prod_{k=1}^{L}\frac{1}{\Gamma(m_k)}\left(\frac{1}{\rho_k}\right)^{i_k}\Gamma(m_k+i_k) \tag{20}$$

and the final expression for N-order moment is

$$M_n=\sum_{i_1=0}^{n}\sum_{i_2=0}^{n-i_1}\cdots\sum_{i_{L-1}=0}^{n-\sum_{k=1}^{L-2}i_k}\frac{n!}{i_1!i_2!\cdots i_L!}\times$$

$$\times\prod_{k=1}^{L}\left[\frac{1}{\Gamma(m_k)}\left(\frac{1}{\rho_k}\right)^{i_k}\Gamma(m_k+i_k)\right], \tag{21}$$

where $i_L=n-\sum_{k=1}^{L-1}i_k$.

N-order moment for MRC diversity system with two branches is given with:

$$M_n=\sum_{i=0}^{n}\binom{n}{i}\frac{1}{\Gamma(m_1)\Gamma(m_2)}\left(\frac{1}{\rho_1}\right)^{n-i}\left(\frac{1}{\rho_2}\right)^{i}\times$$

$$\times\Gamma(m_1+n-i)\Gamma(m_2+i). \tag{22}$$

Now, from (22), we can determinate average value and variance for the output of MRC combiner:

$$M_1=\frac{1}{\rho_1}\frac{\Gamma(m_1+1)}{\Gamma(m_1)}+\frac{1}{\rho_2}\frac{\Gamma(m_2+1)}{\Gamma(m_2)}; \tag{23}$$

$$\sigma^2=M_2-M_1^2=\frac{1}{\rho_1^2}\left[\frac{\Gamma(m_1+2)}{\Gamma(m_1)}-\frac{\Gamma^2(m_1+1)}{\Gamma^2(m_1)}\right]+$$

$$+\frac{1}{\rho_2^2}\left[\frac{\Gamma(m_2+2)}{\Gamma(m_2)}-\frac{\Gamma^2(m_2+1)}{\Gamma^2(m_2)}\right]. \tag{24}$$

For $m_1=m_2=m$ and $\rho_1=\rho_2=\rho$ we have:

$$M_1=\frac{2}{\rho}\frac{\Gamma(m+1)}{\Gamma(m)}, \tag{25}$$

$$\sigma^2 = M_2 - M_1^2 = \frac{2}{\rho^2}\left[\frac{\Gamma(m+2)}{\Gamma(m)} - \frac{\Gamma^2(m+1)}{\Gamma^2(m)}\right]. \quad (26)$$

The graphs of the average value and variance of the MRC diversity system output with two inputs as functions of ratio ρ are shown in Fig. 5 and Fig. 6, respectively.

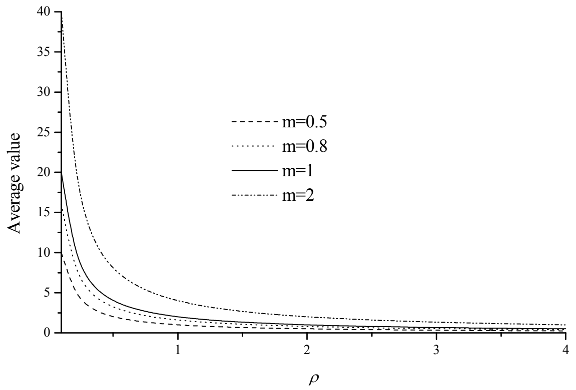

Fig. 5. Average value of MRC diversity system output versus ratio ρ in case L=2 and m=0.5, 0.8, 1, 2

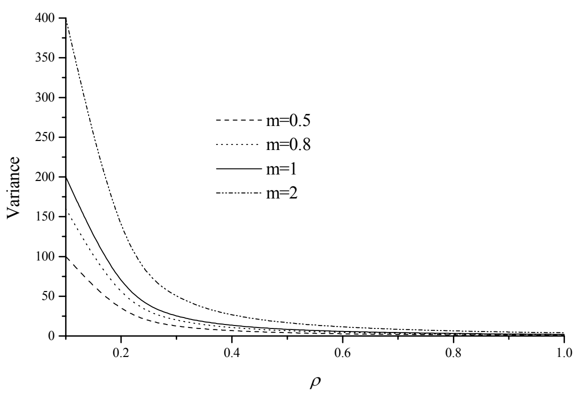

Fig. 6. Variance of MRC diversity system output versus ratio ρ in case L=2 and m=0.5, 0.8, 1, 2

Conclusions

In this paper, we consider output signals of EGC and MRC combiner in presence of Nakagami fading which is the general and the most useful form of fading besides Rician and Rayleigh fading. The output combiner signal is simply the sum of combiner input signals in case of EGC or it is the sum of square combiner input signals in case of MRC combiner. Also, we determinate N-order moments of output signals for these combiners. The moments of the combiner output signals can be used for determining optimal parameter values of diversity systems. It was needed to use the multinomial formula for calculating N-order moment. The average signal from output of EGC and MRC combiner with two or more branches, the square average output signal and the signal variance can be calculated by obtain formulas. Also, the signal probability density can be calculated by, on that way, obtain output signal moments and some assembly of orthogonal functions.

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The output signals of MRC and EGC combiner in presence of Nakagami and Rician fading were considered in this paper. Also, the expression for N-order moment of output signals for these combiners was obtained. The moments of the combiner output signals can be used for determining optimal parameter values of diversity systems. The average signal from output of EGC and MRC combiner with two branches and the signal variance was calculated by obtained formulas, too. Ill. 6, bibl. 3 (in English; summaries in English, Russian and Lithuanian).

М. С. Стефанович, Н. Д. Капацинович, М. В. Бандьюр. Моменты выходных сигналов MRC и EGC сумматора // Электроника и электротехника. – Каунас: Технология, 2007. – № 1(73). – С. 59–62.

Рассмотрены MRC и EGC выходные сигналы при наличии Nakagami и Rician угасания сигналов. Было получено выражение для момента N-порядка выходных сигналов этих сумматоров. Моменты выходных сигналов сумматора могут быть использованы для определения оптимальных параметрических значений систем диверсификации. Средний выходной сигнал EGC и MRC сумматора с двумя ветвями и отклонение сигнала были вычислены с помощью приведенных формул. Ил. 6, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

M. C. Stefanovič, N. D. Kapacinovič, M. V. Bandjur. MRC ir EGC sumatoriaus išėjimo signalų momentai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 1(73). – P. 59–62.

Straipsnyje nagrinėjami MRC ir EGC sumatoriaus išėjimo signalai esant Nakagami ir Rician pobūdžio slopimui. Taip pat išvesta šio sumatoriaus išėjimo signalų N-osios eilės momento išraiška. Sumatoriaus išėjimo signalų momentai gali būti panaudojami įvairių diversifikavimo sistemų parametrų vertėms nustatyti. Pagal gautas formules taip pat buvo apskaičiuotas vidutinis EGC ir MRC sumatoriaus su dvejomis atšakomis išėjimo signalas ir signalo nuokrypis. Il. 6, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).