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Linearisation Method for Two-dimensional Memoryless Laplass Source

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Introduction

The use of digital representation for audio, speech, images and video is rapidly growing with the extended use of computers and multimedia computer applications. To provide a more efficient representation of data, many compression algorithms have been developed, and the quantization is the basis of all these algorithms. The superiority of vector or multidimensional quantization over scalar quantization is evident from the increased freedom in choosing the partition geometry for vector quantization (VQ) compared to the very restrictive geometry in the case where each vector component is scalar quantized [1] and the resulting quantization cells are rectangles. We can say that scalar quantization is simply a restricted special case of VQ. Furthermore, VQ can yield smaller average mean squared error per dimension than scalar quantization for the case of fine quantization.

The principal goal in design of vector quantizers is to find a codebook, specifying the decoder, and a partition of the vector space or encoding rule, specifying the encoder, which will maximize an overall performance. As a measure of performance, we use statistical average of a quantization error called distortion. Hence, in designing an optimal vector quantizer we must minimize a predefined distortion measure between input and output. Na and Neuhoff [2] consider the problem of finding the optimal maximum amplitude, so-called, support region for scalar quantizers by minimization of the total distortion *D*, which is a combination of granular (D_g) and overload (D_o) distortion, $D = D_g + D_o$.

In paper [3], the expressions for the optimum number of output points are derived; however the proposed partitioning of the multidimensional space for memoryless Laplacian source does not consider the geometry of the multidimensional source.

In this work we will use this minimum distortion criterion to design a piecewise nonuniform vector quantizer, but our focus will be the analysis of the distortion for method for linearization of the nonuniform characteristic of the quantizer [4]. Memoryless Laplacian source was analyzed.

Description vector quantizer

Joint probability density function (pdf) function of two independent, identically distributed Laplace random variables (x_1 , x_2) with zero mean and unit variance is given with the following expression:

$$f_{1,2}(x_1, x_2) = \frac{1}{2} e^{-\sqrt{2}(|x_1|+|x_2|)}. \tag{1}$$

After applying the Helmert transformation [4]:

$$r = \frac{1}{\sqrt{2}}(|x_1| + |x_2|),\; u = \frac{1}{\sqrt{2}}(|x_1| - |x_2|) \tag{2}$$

we get probability density function:

$$f(r,u) = \frac{1}{2} e^{-2r}. \tag{3}$$

In two-dimensional *ru* system the pdf function given by equation (3) represents square line. This square surface representing dynamic range of a two dimensional quantizer, can be partitioned into *L* concentric domains as shown in Fig. 1. In the case of nonuniform vector quantization, these concentric domains are of unequal width.

The number of output points in each domain is denoted by N_i, where $N = \sum_{i=1}^{L} N_i$ represents the total number of output points. Every concentric domain can be further partitioned into L_i concentric subdomains of equal width. Every subdomain is divided into four regions each containing $p_{i,j}$ rectangular cells. An output point is placed in the centre of each cell. Coordinates of the *k*th output point in *j*th subregion of the *i*th region in *ru* coordinate system are $(m_{i,j}, \hat{u}_{i,j,k})$.

The initial expression for granular distortion is:

$$D_g = 4\sum_{i=1}^{L}\sum_{j=1}^{L_i}\sum_{k=1}^{p_{i,j}} \int_{r_{i,j}}^{r_{i,j+1}} \int_{u_{i,j,k}}^{u_{i,j,k+1}} \left[(r - m_{i,j})^2 + (u - \hat{u}_{i,j,k})^2\right]\frac{1}{2}e^{-2r}\,dr\,du. \tag{4}$$

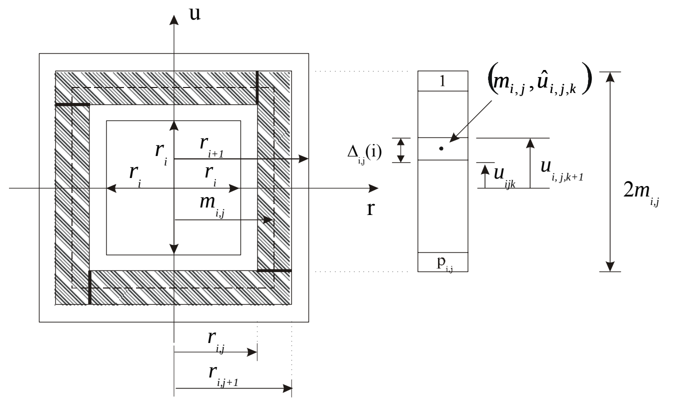

Fig. 1. Two-dimensional space partitioning

The output point coordinates are given by the equations:

$$m_{i,j} = \frac{r_{i,j+1} + r_{i,j}}{2} \quad \text{and} \quad \hat{u}_{i,j,k} = \frac{u_{i,j,k} + u_{i,j,k+1}}{2} \,. \quad (5)$$

Rectangular cell dimensions are:

$$\Delta_i = r_{i+1} - r_i \,,\ \Delta_i' = \frac{\Delta_i}{L_i} \,,\ \Delta_{ij} = \frac{r_{i,j} + r_{i,j+1}}{p_{i,j}} \,; \quad (6)$$

$$r_{i,j} = r_i + j \cdot \Delta_i \,,\ i = 0,\ldots,L \,,\ j = 0,\ldots,L_i \,. \quad (7)$$

The range of the quantizer is $r_{\max}$. To determine the boundary values of every concentric domain, denoted as r_i, for the case of nonuniform vector quantization we will use three different methods for linearization of the compress function, which are described in the next section.

Before we describe these methods, we must introduce the optimal compression function used in two-dimensional vector quantization:

$$h(r) = r_{\max} \frac{\int_0^r e^{-\frac{\sqrt{2}}{4}x} dx}{\int_0^{r_{\max}} e^{-\frac{\sqrt{2}}{4}x} dx} = r_{\max} \frac{e^{-\frac{\sqrt{2}}{4}r} - 1}{e^{-\frac{\sqrt{2}}{4}r_{\max}} - 1} \,. \quad (8)$$

Method for linearization

In this method we have approached the problem of segmentation from mathematical point of view. The best way to linearize a function is to do a uniform segmentation of its first derivate. This is shown in the Fig. 2, and the explanation follows below.

The first derivate of the compress function is:

$$h'(r) = -\frac{\sqrt{2}}{4} r_{\max} \frac{e^{-\frac{\sqrt{2}}{4}r}}{e^{-\frac{\sqrt{2}}{4}r_{\max}} - 1} \,. \quad (9)$$

The main idea is to divide the range from $h'(0)$ to $h'(r_{\max})$ into L equal segments:

$$\Delta' = \frac{h'(r_{\max}) - h'(0)}{L} \,,\quad h_i' = h'(0) + i\Delta_i \,,\ i = 0,\ldots,L \quad (10)$$

in our case, we have

$$\Delta' = \frac{r_{\max}}{2\sqrt{2}L} \,. \quad (11)$$

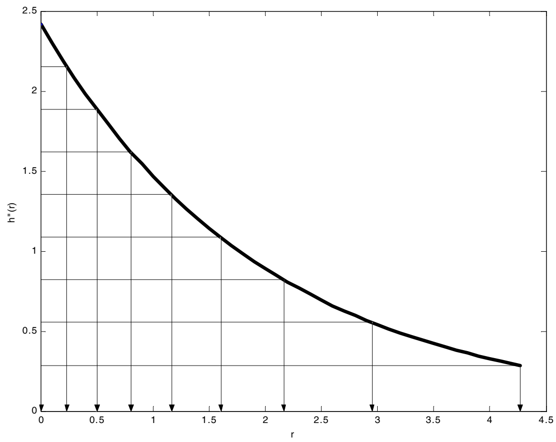

Fig. 2. Segmentation of the first derivate

When substituted into the formula for the inverse function of $h'(r)$ denoted as $h'^{-1}(h)$, this values for h_i' will give us the wanted values for r_i:

$$r_i = h'^{-1}(h_i') = -2\sqrt{2} \ln\left(-2\sqrt{2} h_i' \frac{e^{-\frac{\sqrt{2}}{4}r_{\max}} - 1}{r_{\max}} \right) . \quad (12)$$

The illustration of this method is given in Fig. 3. In Table 1 we have presented the obtained values for r_i for described method; values for $r_{\max}$ are calculated for the case of minimal distortion and $L = 8$. The results of our calculating we can check as we compare values for r_8 and r_{max}. Those values must be identical.

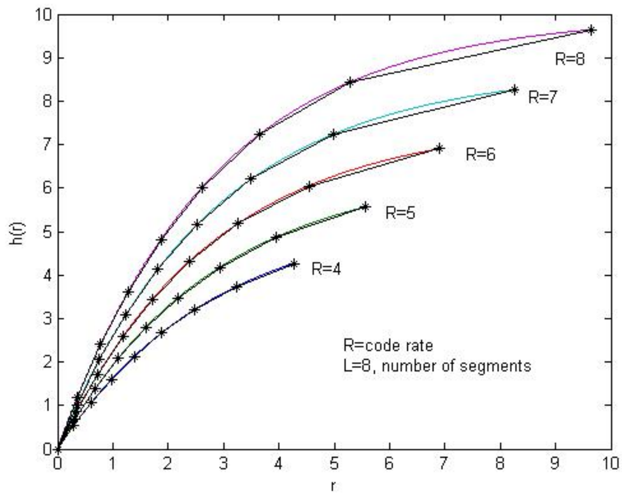

Fig. 3. Segmentation with first derivate segmentation method

Table 1. Numerical results

R	4	5	6	7	8
r_{max}	4.2662	5.5629	6.9084	8.2634	9.6418
Δ'	0.18854	0.24585	0.30531	0.36519	0.42611
$h'(0)$	1.93693	2.28671	2.67507	3.08783	3.52551
$h'(r_{max})$	0.4286	0.31993	0.23258	0.16628	0.11662
h_1'	1.74839	2.04086	2.36976	2.75564	3.0994
h_2'	1.55985	1.79501	2.06445	2.35745	2.67329
h_3'	1.37131	1.54916	1.75914	1.99226	2.24718
h_4'	1.18277	1.30331	1.45383	1.62707	1.82107
h_5'	0.99423	1.05746	1.4852	1.26188	1.39496
h_6'	0.80569	0.81161	0.84321	0.89669	0.96885
h_7'	0.61715	0.56576	0.5379	0.5315	0.54274
h_8'	0.42860	0.31991	0.23259	0.16631	0.11663
r_1	0.28966	0.32171	0.34277	0.356	0.3643
r_2	0.6124	0.68477	0.7329	0.7634	0.7827
r_3	0.9768	1.10139	1.1855	1.2394	1.2738
r_4	1.39511	1.59016	1.7247	1.8121	1.8685
r_5	1.88626	2.1814	2.3914	2.53107	2.6224
r_6	2.481	2.9298	3.2655	3.4974	3.6534
r_7	3.235	3.9505	4.537	4.9767	5.2924
$r_8=r_{max}$	4.2662	5.5629	6.9084	8.2634	9.6418

We derive L_{iopt} from [4]

$$L_{iopt} = (r_{i+1} - r_i)\sqrt[4]{\frac{I_0(i)N_i^2}{64 I(i)^3}}\ . \tag{13}$$

The functions $I_0(i)$ and $I(i)$ are defined as [5]:

$$I_0(i) = \int_{r_i}^{r_{i+1}} r \cdot g(r) dr \qquad I(i) = \int_{r_i}^{r_{i+1}} r \cdot \sqrt[3]{g(r)}\, dr \tag{14}$$

and function $g(r)$ is defined as

$$g(r) = e^{-2r}\ . \tag{15}$$

The function N_i we define as [5]

$$N_i = N \frac{\left[I(i)^3 I_0(i)\right]^{1/4}}{\sum_{k=1}^{L}\left[I(k)^3 I_0(k)\right]^{1/4}}\ , \tag{16}$$

where

$$N = 2^{2R}\ .$$

Performance

For a comparison of these methods, we will show performance of nonuniform two-dimensional vector quantization, given with total distortion. The expression for total distortion for one dimension is given by

$$D_{tot} = \frac{1}{2}\left(D_g + D_0\right), \tag{17}$$

where are D_g granular distortion and D_o overload distortion.

Granular distortion is given by [5]:

$$D_g = \frac{16}{3N}\left(\sum_{i=1}^{L}\left[I(i)^3 I_0(i)\right]^{\frac{1}{4}}\right)^2, \tag{18}$$

where $I(i)$ and $I_0(i)$ are defined in (14).

We can calculate the overload distortion as

$$D_0 = m_{L,L_L} e^{-2r_{\max}}[(2r_{\max}^2 + 2r_{\max} + 1 - 4m_{L,L_L} r_{\max} - 2m_{L,L_L} + 2m_{L,L_L}^2) + \frac{2m_{L,L_L}^2}{3p_{L,L_L}^2}], \tag{19}$$

where m_{L,L_L} is the output point coordinates of last region and it is given by

$$m_{L,l_L} = r_{\max} - \Delta_L \tag{20}$$

and $\Delta_{L_L} = \frac{r_{\max} - r_{L-1}}{L_L}$, where is $L_L = L_{Lopt}$, and represents number of subdomains in the last Lth domain. The optimum number of cells $p_{i,j}$ is defined in [5] and optimum number of cells in the last region p_{L,L_L} is given by

$$p_{L,L_L} = \frac{N_i}{4}\frac{m_{L,L_L}\sqrt[3]{g(m_{L,L_L})}\Delta_i}{I(L)}\ . \tag{21}$$

From calculated values for both methods, we obtain their distortions. In Table 2 we have presented the obtained values for total distortion.

Table 2. Numerical results for distortion

R	4	5	6	7	8
Linearisation Method (LM)					
D_g	0.01803	0.00502	0.001306	0.0003312	8.965*10^-5
D_0	0.00214	0.0001	0.000012	8.072*10^-7	5.003*10^-8
D_{tot}	0.01009	0.00256	0.000659	0.000166	4.485*10^-5

In Table 3 we have presented the obtained values for SQNR for linear method (*L.M*) and nonlinear method (*N.M*); values for $r_{\max}$ are calculated for the case of minimal total distortion.

Table 3. Numerical results for *SQNR*

R		4	5	6	7	8
r_{max}		4.266	5.563	6.908	8.263	9.642
SQNR [dB]	*L.M*	19.961	25.918	31.811	37.799	43.730
	N.M	19.8227	25.8433	31.8639	37.8845	43.9051

Theoretical limit for the ideal case ($N.M$) calculated from the following approximated expression [4]:

$$D_{tot}\big|_{ideal} = \frac{8}{3N}. \quad (22)$$

SQNR is a signal-to-quantization noise ratio, given with:

$$SQNR[dB] = 10\log\frac{1}{D_{tot}}. \quad (23)$$

Conclusions

Piecewise linear quantizer has complexity implementation between optimal nonlinear quantizer and linear quantizer. These quantizers, although not optimal, may have asymptotic performance arbitrarily close to the optimum.

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In this work we consider piecewise linear vector quantizer of two-dimensional memoryless Laplacian source. Linearization method of the compression function is analyzed, then the effects of this linearization methods of the compression function on total distortion (granular and overload) are presented an this method is also compared with nonlinear method. The vector space is partitioned using rectangular cells. Ill. 3, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

З. Перич, М. Новкович, В. Деспотович. Метод линеаризации для двумерных незапоминающих источников Лапласа // Электроника и электротехника. – Каунас: Технология, 2007. – № 1(73). – С. 41–44.

Анализируется линейный векторный квантизатор двумерных незапоминающих источников Лапласа. Рассматривается метод линеаризации функции сжатия, представлены результаты этих методов линеаризации функции сжатия, при полном искажении (деформации). Этот метод был сравнен с нелинейным методом. Векторное пространство было сегментировано с помощью прямоугольных ячеек. Ил. 3, библ. 5 (на английском языке, рефераты на английском, русском и литовском яз.).

Z. Perič, M. Novkovič, V. Despotovič. Dvimačių neįsimenančiųjų Laplaso šaltinių linearizacijos metodas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 1(73). – P. 41–44.

Nagrinėjamas dvimačio neįsimenančiojo Laplaso šaltinio tiesinis intervalinis vektorinis kvantorius. Analizuojamas suglaudinimo funkcijos linearizacijos metodas, aptariamas suglaudinimo funkcijos linearizavimo metodų poveikis bendrajam iškraipymui (nelygumui ir perkrovos efektui). Šis metodas palyginamas su netiesiniu metodu. Vektorių erdvė segmentuojama naudojant stačiakampius elementus. Il. 3, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).