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Digital Measurement of Frequency with Linear Interpolation in Dynamic States

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Introduction

Frequency modulated pulse signals are often used in measuring channels [1] as they can be converted into digital form in easy way and are more resistant against disturbances especially in case of long distance transmission [2]. A counter converts the analogue signal to digital form therefore the resolution depends on the capacity of the counter and the interval of measurement.
A counter with a great capacity, by selecting suitable gating time, makes in easy way a transducer of 16 bits or more. It is difficult to obtain transducers with the same resolution in the case of voltage signal. The advantage of f/C conversion in comparison with A/C conversion is the possibility of easy access to standards of high accuracy.

Digital frequency measurements of small values of frequency are based on the measurement of the period of the investigated signal. It can be determined by the counting of pulses of the standard frequency generator in the time window defined by the measured period or its multiple [3]. The result of measurement corresponds to the mean value of frequency in the defined time window. In these measurements the instant which starts the measurement depends on the position of the slope of the measured signal. For this reason it is impossible to compare the results of measurements in different measuring channels especially frequency and voltage channels. When the frequency of the signal changes in time of measurement the results are delivered in different intervals and therefore it is impossible to calculate Fourier transform. In this case it is recommended to use the linear interpolation. In the paper are presented two such methods.

On-line mode

The first method is used in an on-line mode [4]. When the value of frequency is determined in instant t_k one has only information of instants pulses preceding instant t_k.

The value of frequency f_k in the instant t_k, can be calculated as f'_k (formula 1) by extrapolation the periods preceding instant t_k (Fig. 1).

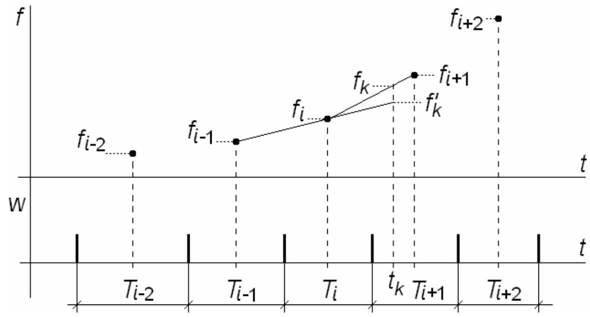

Fig. 1. Frequency measurement from two last periods

$$f_k' = f_{i-1} + \frac{\left(f_i - f_{i-1}\right)\left(t_k - \left(\sum_{j=1}^{i-2} T_j + \frac{1}{2} T_{i-1}\right)\right)}{\left(\sum_{j=1}^{i-1} T_j + \frac{1}{2} T_i\right) - \left(\sum_{j=1}^{i-2} T_j + \frac{1}{2} T_{i-1}\right)} =$$

$$= \frac{1}{T_{i-1}} + \frac{2\left(\frac{1}{T_i} - \frac{1}{T_{i-1}}\right)\left(t_k - \sum_{j=1}^{i-2} T_j - \frac{1}{2} T_{i-1}\right)}{T_{i-1} + T_i}. \quad (1)$$

The formula (1) is valid in assumption that the frequency varies in linear mode i.e. the computed value of frequency is equal to the value of frequency in the centre of examined period.

Practically two counters are used for the measurement. The first one works in a buffer mode and measures intervals between the pulses of the same signal. Therefore for arbitrary instant in the memory are registered all previous intervals between pulses of measured signal. The second counter registered the intervals of pulses of different signals. It measures interval T_k between the last pulse of pulse signal and the pulse determining the instant of measurement t_k. The value of frequency in the instant of

t_k is calculated basing on the values of last periods T_{i-1} and T_i and interval T_k according to the formula

$$f_k' = \frac{1}{T_{i-1}} + \frac{2\left(\frac{1}{T_i} - \frac{1}{T_{i-1}}\right)\left(\frac{1}{2}T_{i-1} + T_i + T_k\right)}{T_{i-1} + T_i} = \\ = \frac{T_{i-1}\cdot\left(T_{i-1} + 2T_i + 2T_k\right) - T_i\cdot\left(T_i + T_k\right)}{T_{i-1}\cdot T_k\cdot\left(T_{i-1} + T_i\right)}. \quad (2)$$

Off-line mode

The second method is used in an off-line mode [5]. In the first step one defines and registers the positions of pulses of measured signal. Then the value of frequency is computed for chosen instants. For the arbitrary chosen instant of frequency the position of pulses preceding and following this instant is well known.

The value of frequency f_k in the instant t_k can be calculated from two successive periods in assumption that the frequency has changed in linear mode. Depending on the instant of t_k these periods are T_i and T_{i+1} for calculation frequency f_k or T_{i+1} and T_{i+2} for frequency f'_k (Fig. 2).

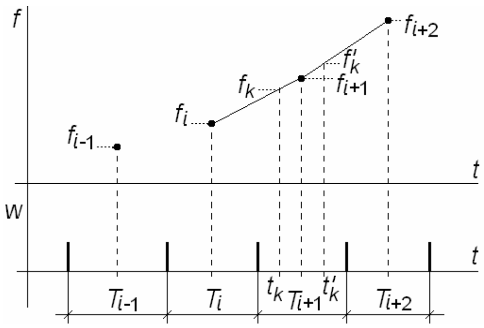

Fig. 2. Frequency measurement from two periods

In the case $\sum_{j=1}^{i} T_j \le t_k \le \sum_{j=1}^{i} T_j + \frac{1}{2}T_{i+1}$ frequency f_k (Fig. 2) is calculate according to the formula

$$f_k = f_i + \frac{\left(f_{i+1} - f_i\right)\left(t_k - \left(\sum_{j=1}^{i-1} Tj + \frac{1}{2}T_i\right)\right)}{\left(\sum_{j=1}^{i} Tj + \frac{1}{2}T_{i+1}\right) - \left(\sum_{j=1}^{i-1} Tj + \frac{1}{2}T_i\right)} = \\ = f_i + \frac{2\left(f_{i+1} - f_i\right)\left(t_k - \sum_{j=1}^{i-1} Tj - \frac{1}{2}T_i\right)}{T_i + T_{i+1}}, \quad (3)$$

while in the case $\sum_{j=1}^{i} T_j + \frac{1}{2}T_{i+1} < t_k < \sum_{j=1}^{i+1} T_j$ frequency f'_k (Fig. 2) is calculated according to the formula

$$f'_k = f_{i+1} + \frac{\left(f_{i+2} - f_{i+1}\right)\left(t_k - \left(\sum_{j=1}^{i} Tj + \frac{1}{2}T_{i+1}\right)\right)}{\left(\sum_{j=1}^{i+1} Tj + \frac{1}{2}T_{i+2}\right) - \left(\sum_{j=1}^{i} Tj + \frac{1}{2}T_{i+1}\right)} =$$

$$= f_{i+1} + \frac{2\left(f_{i+2} - f_{i+1}\right)\left(t_k - \sum_{j=1}^{i} Tj - \frac{1}{2}T_{i+1}\right)}{T_{i+1} + T_{i+2}}. \quad (4)$$

For one measuring channel with frequency carrier complete measuring cycle is realised by two counters working in continuous buffered mode. The first one counts pulses of standard signal in meantime of consecutive periods of pulse signal, and loads them to the memory. The second counter counts the pulses of standard signal in the instants between the pulse determining the instant of sampling and the next pulse of measured signal.

In the instant of measurement the number of last measured period is registered in the memory (the number of measured periods up to the instant of given sampling pulse).

After the end of measuring cycle in the memory are registered three one dimensional table for simple measuring channel:

- numbers of pulse signal in instants of strobe pulses: a_1, a_2, …, a_m,
- length of periods of the measured signal: T_1, T_2, …, T_n,
- intervals between the sampling pulse and the nearest pulse of measured signal: τ_1, τ_2, …, τ_p.

The first two tables have the same numbers of elements m. The third table has the same number of elements as the two first tables ($p = m$) in the case when the longest period of pulse signal is equal or less than the periods of sampling (max $T_i \le T_s$) as presented in Fig. 3.

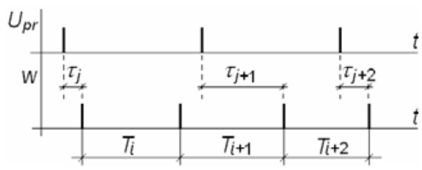

Fig. 3. Sampling for max $T_i \le T_s$

Whereas, if the longest period of pulse signal is greater than the sampling period (max $T_i > T_s$), the third table could have less elements than the two first ($p \le m$). It is the result of working mode of counters measuring intervals between pulses of different signals. After the pulse of the first signal initiating the measurement the measuring cycle is finished in the instant of the first pulse of the second signal regardless of the successive pulses that can appear in the first signal (Fig. 4).

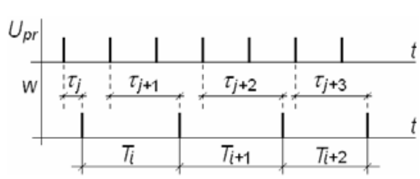

Fig. 4. Sampling for max $T_i > T_s$

Omitted intervals τ_j between both the signals must be taken into consideration, therefore in the third table in the place of individual element τ_j for $\tau_j > T_{pr}$ w elements τ'_1, τ'_2, …, τ'_w are inserted (Fig. 5), where:

$$\tau'_1 = \tau_j \;,\; \tau'_2 = \tau_j - T_s ,\; \tau'_w = \tau_j - \left(w-1\right)T_w , \quad (5)$$

$$w = trunc\left[\frac{\tau_j}{T_s}\right] + 1, \qquad (6)$$

and $trunc[x]$ is integer of number x.

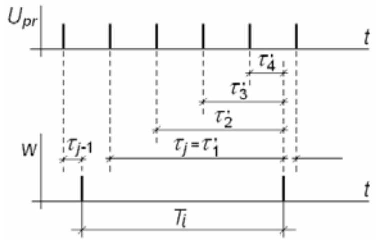

Fig. 5. Conversion of table τ_p

In that way table τ_1, τ_2, … , is created τ_m with m elements, where successive values represent interval between every sampling pulse and the nearest pulse of pulse signal.

The value of frequency f_k of pulse signal in the instant of k^{th} pulse of sampling signal is computed from formulae (3) and (4) taking into consideration two periods (T_{a_k} i T_{a_k+1} or T_{a_k+1} i T_{a_k+2}) and interval τ_{a_k}, where a_k is index of period which precedes sampling signal.

For $\tau_{a_k} \geq \frac{1}{2}T_{a_k+1}$ frequency f_k is calculated from formula

$$f_k = \frac{1}{T_{a_k}} + \frac{\left(\frac{1}{T_{a_k+1}} - \frac{1}{T_{a_k}}\right)\left(\frac{1}{2}T_{a_k} + T_{a_k+1} + \tau_{a_k}\right)}{\frac{1}{2}T_{a_k} + \frac{1}{2}T_{a_k+1}} =$$

$$= \frac{T_{a_k}{}^2 - T_{a_k+1}{}^2 + 2\cdot T_{a_k}\cdot T_{a_k+1} - 2\cdot\tau_{a_k}\cdot\left(T_{a_k} + T_{a_k+1}\right)}{\left(T_{a_k} + T_{a_k+1}\right)\cdot T_{a_k}\cdot T_{a_k+1}}. \qquad (7)$$

While for $\tau_{a_k} < \frac{1}{2}T_{a_k+1}$ frequency f_k is used formula

$$f'_k = \frac{1}{T_{a_k+1}} + \frac{\left(\frac{1}{T_{a_k+2}} - \frac{1}{T_{a_k+1}}\right)\left(\frac{1}{2}T_{a_k+1} - \tau_{a_k}\right)}{\frac{1}{2}T_{a_k+1} + \frac{1}{2}T_{a_k+2}} =$$

$$= \frac{T_{a_k+1}{}^2 - T_{a_k+2}{}^2 - 2\cdot\tau_{a_k}\cdot\left(T_{a_k+1} - T_{a_k+2}\right)}{\left(T_{a_k+1} + T_{a_k+2}\right)\cdot T_{a_k+1}\cdot T_{a_k+2}}. \qquad (8)$$

In this way one dimensional table is created for the specific channel with the values of frequency in the instants of sampling: $f_1, f_2, \ldots, f_m$.

The numbers of investigated channels can be greater than one and is only restricted by the numbers of counters (there must be two counters for individual channel).

Simulation of frequency measurements

Consider a measured signal f expressed by formula (9) that fluctuates in sinusoidal way with a frequency f_{sin}, immersed in a *noise* in the time of measurement [4]

$$f = f_0 + f_m \cdot \sin(2\pi f_{\sin} t) + noise, \qquad (9)$$

where f_0 – constant component of frequency of pulse signal f, f_m – amplitude of deviation of pulse signal frequency f.

For the instant $t_i = \sum_{j=1}^{i-1} T_j + \frac{1}{2}T_i$ the value of frequency is expressed as

$$f_i = f_0 + f_m \cdot \sin\left[2\pi f_{\sin}\left(\sum_{j=1}^{i-1} T_j + \frac{1}{2}T_i\right)\right] + noise_i \cdot \qquad (10)$$

After transformation equation (11) is obtained in form

$$f_0 + f_m \cdot \sin\left[2\pi f_{\sin}\left(\sum_{j=1}^{i-1} T_j + \frac{T_i}{2}\right)\right] + noise_i - \frac{1}{T_i} = 0, \qquad (11)$$

From this equation it is possible to compute the values of successive intervals between the pulses T_i of measured signal.

In the Fig. 6 distribution of errors is presented for sinusoidal fluctuation of the pulse signal in the case of $f_0 = 20$ Hz, $f_m = 2$ Hz, $f_{sin} = 0{,}3$ Hz, $noise_{\max} = 0{,}001$ Hz and sampling frequency $f_s = 50$ Hz.

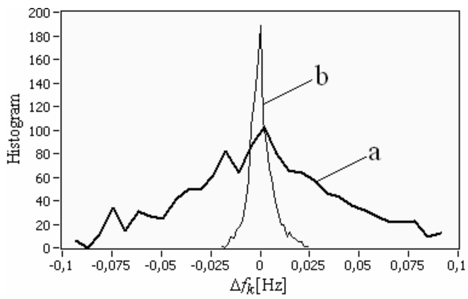

Fig. 6. Errors Δf_k distribution for $f_0 = 20$ Hz, $f_m = 2$ Hz, $f_{sin} = 0{,}3$ Hz, $noise_{\max} = 0{,}001$ Hz and $f_s = 50$ Hz; a – measurement from last time interval, b – measurement from two last intervals

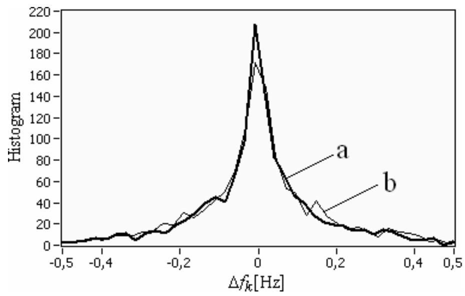

Fig. 7. Errors Δf_k distribution for $f_0 = 20$ Hz, $f_m = 2$ Hz, $f_{sin} = 0{,}3$ Hz, $noise_{\max} = 0{,}1$ Hz and $f_s = 50$ Hz; a – measurement from last time interval, b – measurement from two last intervals

The distribution of errors in the case of the same parameters as in Fig. 6 but for $noise_{\max} = 0{,}1$ Hz is presented in the Fig. 7.

Comparing Figs. 6 and 7 one can ascertain that for the linear changes of frequency the method of computing the frequency from two last pulses intervals enables to obtain good results in case of small changes of measured frequency. For the great values of noise this method can even worsen the accuracy of measurements.

Conclusions

In the paper the principles and realisation of pulse signal frequency measurement are presented in dynamic condition. The presented methods use linear interpolation. From the simulation of measurements for sinusoidal variations of measured signal the interpolation method gives good results for small amount of noises in the measured signal.

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Digital frequency measurements of small values of frequency are based on the measurement of the period of investigated signal. The result of measurement corresponds to the mean value of frequency in the defined time window. The instants which start the measurement depend on the position of the slope of the measured signal. For this reason it is impossible to compare the results of measurements in different measuring channels or to compare the results from frequency channels and voltage channels. The results are delivered in different intervals and therefore it is impossible to calculate Fourier transform. Good results can assure the linear interpolation. In the paper two methods (on-line and off-line mode) realized with buffered counters are presented. The simulation proves that good results of measurements can be obtained in case of a little amount of noises in the measured signal. Ill. 7, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

Л. Реферовский, Д. Свисульский. Цифровые измерения частоты с линейной интерполяцией в динамических истенах // Электроника и электротехника. – Каунас: Технология, 2007. – № 4(76). – С. 47–50.

Цифровые измерения сигнала для малых величин частоты, основаны на измерении его периода. Результат измерения – это средняя величина частоты в определённом пределе часа. Моменты, в которых начинается измерение, зависят от позиции склона измеренного сигнала. Поэтому невозможо сравнить результаты измерения в разных каналах или результаты измерений в каналах частоты и напряжения. Результаты получаются в разных периодах и поэтому невозможно вычислить трансформацию Фурье. Хорошие результаты возможно получить используя линейную интерполяцию. В статье представлены два метода (он-лан и офф-лан) связанные с буферными счётчиками. Результаты симуляции показывают, что хорошие результаты измерения можно получить при малом шуме в исследованном сигнале. Ил. 7, библ. 5 (на английском, русском и литовском яз.).

L. Referowski, D. Swisulski. Skaitmeninis dažnio matavimas naudojant tiesinę interpoliaciją dinaminėse sistemose // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 4(76). – P. 47–50.

Išnagrinėtas skaitmeninio signalo matavimas, esant mažoms dažnio reikšmėms, pagrįstas tiriamojo signalo periodo matavimu. Matavimo rezultatą atitinka vidutinė dažnio reikšmė tam tikru laiko momentu. Momentai, kuriais prasideda matavimai, priklauso nuo matuojamojo signalo formos. Dėl šios priežasties neįmanoma palyginti skirtinguose kanaluose arba dažnio ir įtampos kanaluose atliktų matavimų rezultatų. Rezultatai gaunami ir apdorojami skirtingais laiko momentais, tačiau Furjė transformacijos neįmanoma apskaičiuoti. Naudojant tiesinę interpoliaciją galima tikėtis gerų rezultatų. Išnagrinėti du metodai, susiję su buferiniais skaitikliais. Modeliavimo rezultatai rodo, kad geriausi matavimo rezultatai gaunami tik tada, kai matuojamame signale yra mažai triukšmų. Il. 7, bibl. 5, (lietuvių kalba; santraukos anglų, rusų ir lietuvių k.).