ŠARŪNAS KELPŠA

STEEL FIBRE INFLUENCE ON CRACKING AND STIFFNESS OF REINFORCED CONCRETE FLEXURAL MEMBERS

SUMMARY OF DOCTORAL DISSERTATION

TECHNOLOGICAL SCIENCES, CIVIL ENGINEERING (02T)

Kaunas
2017

KAUNAS UNIVERSITY OF TECHNOLOGY

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INTRODUCTION

Concrete is one of the most commonly used structural materials, with relatively high compressive strength, fairly low tensile strength and brittle collapse manner. In order to strengthen concrete structures and to avoid the brittle collapse of the structure, different types of reinforcement are used. Steel rebars are used in cases of ordinary reinforcement, where it takes over the tensile and shear stresses (Behbahani *et al.*, 2011; Vairagake and Kene, 2012). While reinforced concrete structures do not collapse after cracking, however, their deflection increases. Also, depending on the environmental conditions, the crack crossing reinforcement may start corroding. In order to ensure aesthetical, durability and, sometimes, waterproofing properties of concrete, the deflection and crack widths are limited (Mosley *et al.*, 2007; EN 1992-1-1:2004). Depending on the structure, the requirements and the environment, all of this could lead the significant increase of the cross-section height or reinforcement. In order to reduce the crack widths and deflections of the reinforced concrete structures, the additional steel fibre reinforcement can be used. The steel fibres take over the stress and thus cracks are restricted, and the stiffness of the cracked members is increased (ACI 544.3R-93; Ulbinas, 2012).

Steel fibre reinforced concrete (SFRC) structures have already been researched for more than four decades (Naaman, 2003a). Nevertheless, the common design method of steel fibre and combined (steel fibre and ordinary) reinforced concrete flexural members is not available yet. Although lots of different methods of SFRC properties estimation as well as combined reinforced concrete structure design can be found in scholarly literature, calculation results of these methods are fairly different (Kelpša *et al.*, 2014). Meanwhile, residual tensile strength of SFRC should be established by tests (Jansson, 2007). Considering the wide variety of steel fibres as well as the wealth of differences of the employed methods, the practical application of combined reinforcement becomes complicated. In order to solve this problem, new standards were developed in several countries (CNR-DT 204/2006; DafStb Guideline, 2012; SFRC Design Guideline, 2014; SS 812310:2014). However, in Lithuania as well as in many other countries, the design of the steel fibre and combined reinforced concrete members is not regulated.

The application of steel fibres in combined reinforced concrete structures is restricted due to the absence of a common and universally accepted calculation method. The selection of optimal, possibly combined, reinforcement is limited due to the necessary tests for the determination of the residual flexural tensile strength of SFRC.

Aim of the work:

To determine the influence of the steel fibre on the cracking and stiffness of reinforced concrete flexural members as well as to develop a new calculation method for the evaluation of this influence.

Tasks of the work:

1. To analyze steel fibre reinforced concrete (SFRC) parameters and its determination methods which are used in crack width and deflection calculations of steel fibre and combined (steel fibre and ordinary) reinforced concrete structures. To explore the statistical methods which are used in calculations of the characteristic values of SFRC properties;
2. To analyze the crack width and deflection calculation methods of steel fibre and combined reinforced concrete flexural members;
3. To carry out crack width and deflection experimental research of steel fibre and combined reinforced concrete flexural members and to determine experimentally the residual flexural tensile strength $f_{R,1}$ of different steel fibre reinforced concrete;
4. To analyze application possibilities of Naaman's and Sujivorakul's methods for estimation of residual tensile stress σ_{fb}. To create calculation methods of residual flexural tensile strength $f_{Rm,1}$ and its variation coefficient V_x of ordinary and self-compacting SFRC as a more accurate and more universally applicable alternative for σ_{fb} calculations;
5. To determine the suitability of the developed $f_{Rm,1}$ and V_x calculation methods for crack width and deflection estimations of combined reinforced concrete flexural members;
6. To develop a new plastic hinge calculation method which evaluates the stiffness along the plastic zone, and is suited for crack width and deflection calculations of SFRC flexural members. To determine the suitability of the developed $f_{Rm,1}$ calculation method in terms of the crack width and the deflection calculations of SFRC flexural members;
7. To create a simple modification method of residual flexural tensile strength $f_{R,1}$ which could be used in cases when $f_{R,1} > f_{ctm,fl,fb}$, and which would allow avoiding inaccurate curvature drop after cracking.

Scientific novelty of the work:

- The deduced adjustment coefficients of Naaman's and Sujivorakul's methods give the possibility to approximately calculate the residual tensile stress σ_{fb} of wavy and hooked end steel fibres reinforced concrete;
- The created calculation methods of average residual flexural tensile strength $f_{Rm,1}$ and its variation coefficient V_x can be applied for ordinary and self-compacting steel fibre reinforced concrete;

- The suitability of the created $f_{Rm,1}$ and V_x calculation methods to crack width and deflection estimations of combined reinforced concrete flexural members is determined;
- The new plastic hinge calculation method evaluating stiffness along the plastic zone of SFRC beams is created;
- The simple modification method of residual flexural tensile strength $f_{R,1}$ is developed. This method allows avoiding inaccurate curvature drop after cracking in cases when $f_{R,1} > f_{ctm,fl,fb}$.

Methods of the research:

The mechanical properties of concrete and SFRC were measured according to the guidance of the corresponding standards. Crack mouth opening displacement *CMOD* and deflection δ values were measured with two extensometers, while relative strains were measured with strain gages. Deflections of ordinary and combined reinforced concrete beams were measured with an electronic deformation gauge, whereas the values of crack widths were measured with crack width testing gauge.

The experimentally determined crack widths and deflections were compared with the calculation results. Compressive and residual tensile strengths of steel fibre reinforced concrete were calculated by using methods developed by separate scientists (such as Naaman, Sujivorakul, etc.). Characteristic values of residual flexural tensile strength of SFRC were calculated by using statistical (Bayesian and Classical) methods.

The adjustment coefficients k_P and k_{pc} were deduced after conducting comparative analysis of experimental and calculation results involving the use of statistical methods. $f_{Rm,1}$ as well as V_x estimation methods were created after performing regression and statistical analysis of experimental results. A plastic hinge calculation method was created with reference to energy method principles where numerical analysis was performed by using *Mathcad* software. Expressions of internal and external works as well as formulas of proposed coefficients A and B were obtained by using integration methods. Stresses in cracked cross-sections were calculated by using analytical and iterative (layer) methods. A reduction method of $f_{R,1}$ was proposed after completing numerical analysis which involved calculations performed by using analytical and iterative methods.

Statements presented for defence:

1. The residual tensile stress σ_{fb} can be approximately calculated by using Naaman's and Sujivorakul's calculation methods together with adjustment coefficients k_P and k_{pc} which are deduced in this thesis;

2. The average residual flexural tensile strength $f_{Rm,1}$, its variation coefficient V_x and characteristic values of $f_{R,1}$ can be calculated without any additional tests when the methods created and presented in this thesis are used;
3. Deflections and crack widths of SFRC flexural members can be easily calculated by using the created plastic hinge method evaluating the stiffness of the plastic zone;
4. The application of the developed $f_{Rm,1}$ and V_x calculation methods in crack width and deflection analysis of combined reinforced concrete beams is advisable due to the elimination of otherwise necessary tests and relatively small inaccuracies of the calculation results;
5. When the created residual flexural tensile strength $f_{R,1}$ modification method is used together with the simplified stress diagram of cracked cross-section in the deflection calculations of combined reinforced concrete beams, the curvature drop after cracking is avoided despite $f_{R,1} > f_{ctm,fl,fb}$.

<u>Practical relevance:</u>

- Crack width and deflection research results of steel fibre and combined reinforced concrete flexural members demonstrates the accuracy and reliability of the analyzed methods which are in use in practice;
- When average residual flexural tensile strength $f_{Rm,1}$ and its variation coefficient V_x calculation methods are used, the crack widths as well as the deflections of combined reinforced concrete flexural members can be estimated without any additional tests. The optimal structural members can be designed with simplicity within a brief timeframe in this case. Additional tests can be carried out only if the estimation results need to be verified;
- The application of the proposed plastic hinge method provides a possibility to relatively simply calculate the deflection and the crack width of SFRC flexural members, irrespective to the structural scheme;
- The calculated curvature increases after cracking of combined reinforced concrete flexural members when $f_{R,1} > f_{ctm,fl,fb}$ and calculations are performed by using the simplified stress distribution diagram together with the created $f_{R,1}$ modification method. The calculations still maintain their simplicity, and the results become more accurate.

1. LITERATURE REVIEW

1.1. Steel fibre and its application in concrete and reinforced concrete structures

Different fibre materials and their types can be used in order to improve the properties of concrete. Various classifications of fibres can be found in scholarly literature, for instance, according to the fibre material, according to their physical/ chemical or mechanical properties, according to their size, cross-section shape, type, applicability, manufacturing strategy, etc. (Behbahani *et al.*, 2011; Naaman, 2003a; Ulbinas, 2012). Due to its good properties (sufficient length, relatively high strength and the modulus of elasticity) as well as because of the relatively simple manufacturing process, steel fibre is the most commonly used fibre type in the world (Vairagake and Kene, 2012). When a steel fibres bridge cracks it transfer tensile stress through it and even changes the manner of concrete collapse from brittle to ductile (Jansson, 2007). Fig. 1 shows two popular steel fibre types which are analyzed in the thesis: wavy and hooked end steel fibre.

a) b)

Fig. 1. Analyzed steel fibre: a) wavy and hooked end steel fibre; b) hooked end steel fibre made by different manufacturers

Steel fibre is used in various monolithic and precast structures: industrial floors, overlays, sprayed concrete, precast concrete fence panels, precast concrete sewer pipes, bridge decks, precast concrete track slabs, thin shell structures, explosion resistant structures, precast bridge beams, tunnel linings, etc. (ACI 544.3R-93; Bathia *et al.*, 2012; Behbahani *et al.*, 2011; Brandt, 2008). In spite of such wide applicability, steel fibre is usually used for load bearing structures together with the ordinary reinforcement. The main purpose of steel fibre is to improve the properties and the durability of the structure: to restrict cracking, increase stiffness, improve resistance to impact or dynamic loading, resist material disintegration, etc. (ACI 544.3R-93; Bathia *et al.*, 2012; Behbahani *et al.*, 2011; Brandt, 2008; SFRC Design Guideline, 2014).

1.2. Steel fibre reinforced concrete properties and testing procedures

The addition of steel fibres into concrete changes its composition and the manner of collapse. However, such properties of concrete as compressive strength f_c, tensile strength f_{ct} and modulus of elasticity E_c change insignificantly. In some cases, these properties of steel fibre reinforced concrete (SFRC) are taken equal to the properties of ordinary concrete. In order to determine these properties, the test procedures are the same as for ordinary concrete although some specific empirical calculation methods are available (Bencardino *et al.*, 2007; CNR-DT 204/2006; Fib Model Code 2010; Jansson, 2007; Kelpša *at al.*, 2014; Naaman, 2003b; Sujivorakul, 2012).

When considering the ductile collapse manner of SFRC, a new property, the residual tensile strength, comes into consideration. The residual tensile strength is the tensile strength after the cracking of SFRC, and it depends on many factors, such as concrete properties, fibre properties, fibre content, casting and vibration procedures of SFRC, etc. The residual tensile strength can be even higher than the tensile strength of SFRC. For the determination of this SFRC property, some test procedures (uni-axial tension tests, three- and four-point bending tests, split tests, plate bending tests, etc.) as well as calculation proposals can be found in scholarly literature (Jansson, 2007; DafStb Guideline, 2012; CNR-DT 204/2006; Fib Model Code 2010; Naaman, 2003a; Naaman, 2003b; Sujivorakul, 2012; SS 812310:2014; SFRC Design Guideline, 2014; RILEM, 2001; EFNARC, 2011; EN 14651+A1:2007; Dupont, 2003; Thorenfeldt, 2003). While using the uni-axial tension test, the residual tensile strength can be determined directly; however, the three-point bending test is one of the most popular due to its simplicity (Jansson, 2007). The residual flexural tensile strength is used in many crack width and deflection calculation methods (Fib Model Code 2010; SS 812310:2014; SFRC Design Guideline, 2014; RILEM, 2003; Dupont, 2003). Here, the residual flexural tensile strength, which is obtained from the three-point bending test, is recalculated into the axial residual tensile strength or, in other words, into residual tensile stress σ_{fb}. The three-point bending test scheme is presented in Fig. 2. The residual flexural tensile strength is calculated according to Formula (1):

$$f_{R,i} = \frac{3F_{R,i}L}{2bh_{sp}^2}; \tag{1}$$

where $f_{R,i}$ is the residual flexural tensile strength when (i = 1...4); $F_{R,i}$ is load corresponding with $CMOD_i$ or deflection δ_i; L is the span length; b is the cross-section width; h_{sp} is the distance between the tip of the notch and the top of the specimen.

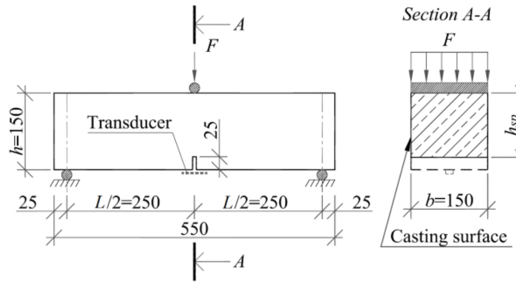


Fig. 2. Three-point bending test scheme with notched specimens (EN 14651+A1:2007)

Approximate calculations of the residual tensile strength can be performed according to Naaman's, Sujivorakul's, or other methods (Naaman, 2003b; Sujivorakul, 2012; Dupont, 2003; Thorenfeldt, 2003). However, none of the methods was intended for calculations of residual flexural tensile strength $f_{R,i}$. Maximum residual tensile strength σ_{pc} (Naaman's method) is calculated according to Formula (2):

$$\sigma_{pc} = \lambda \tau \frac{l}{d} V_f; \tag{2}$$

where λ is the coefficient depending on the fibre distribution and orientation in the current member as well as on the fibre bond strength; τ is the average bond strength at the fibre matrix interface; l is the fibre length; d is the fibre diameter; V_f is the fibre content (fibre dosage divided by the density of fibre material).

The mean and lower-bound values of residual tensile strength σ_P (Sujivorakul's method) are calculated according to Formulas (3) and (4):

$$\sigma_P = \sqrt{f_{ck}}\left(-0{,}0014V_f^2 + 0{,}0046V_f\right)\cdot(l/d)\cdot l^{0.2}; \tag{3}$$

$$\sigma_P = \sqrt{f_{ck}}\left(-0{,}001V_f^2 + 0{,}0038V_f\right)\cdot(l/d)\cdot l^{0.2}; \tag{4}$$

where f_{ck} is the characteristic compressive strength of concrete; l is the fibre length; d is the fibre diameter; V_f is the fibre content (%).

1.3. Statistical methods for the estimation of characteristic values of residual flexural tensile strength

In order to calculate crack widths and deflections of SFRC and combined (steel fibre and ordinary) reinforced concrete flexural members, the characteristic values of residual flexural tensile strength as well as other parameters are relevant (CNR-DT 204/2006; DafStb Guideline, 2012; Fib Model Code 2010; SS 812310:2014; SFRC Design Guideline, 2014). The statistically estimated characteristic values of material properties are characterized by their fractile and confidence level. For the serviceability limit state calculations, 5 % fractile and, depending on the statistical method, 95 % or 75 % confidence levels are used. Normal and log-normal distributions are commonly used in calculations of the characteristic residual flexural tensile strength as well as for other relevant material properties. Two cases are possible for the calculation of the characteristic values of material properties according to EN 1992:2002: when coefficient of variation V_x is known and when V_x is unknown. Usually, one of these cases is recommended by various design guidelines; meanwhile, Eurocode 0 prefers to use the case of V_x being known (EN 1990:2002; Gulvanessian *et. al.*, 2002; DafStb Guideline, 2012; RILEM, 2003; Fib Model Code 2010; SS 812310:2014; SFRC Design Guideline, 2014). In spite of that, no specific guidance for V_x selection of residual flexural tensile strength was found in the available literature.

The characteristic values of material properties can be calculated by using one of Formulas (5)...(8). The normal distribution is assumed together with V_x unknown and V_x known, when, respectively, Formulas (5) and (6) are used. Log-normal distribution is assumed analogically when Formulas (7) and (8) are used.

$$X_k = m_x \pm k_n s_x; \tag{5}$$

where m_x is the mean value of the sample; s_x is the standard deviation of the sample; k_n is the coefficient depending on the sample size, fractile, confidence level, the statistical method, and the coefficient of variation (whether V_x is known or unknown).

$$X_k = m_x \pm k_n \sigma_x; \tag{6}$$

where σ_x is the distribution of standard deviation (obtained by using the known V_x and the experimental m_x).

$$X_k = \exp\lfloor m_y \pm k_n s_y \rfloor; \tag{7}$$

$$X_k = \exp\lfloor m_y \pm k_n \sigma_y \rfloor; \tag{8}$$

where m_y is the mean of logarithmic values; s_y is the standard deviation of logarithmic values (calculated by using the experimental results); σ_y is the standard deviation of logarithmic values (calculated by using the known V_x).

There are additional provisions in SFRC Design Guideline (2014) when $f_{Rk,i}$ values are calculated.

1.4. Crack width calculation methods of steel fibre and ordinary reinforced concrete structures

It is assumed that the cross-section behaves elastically until cracking occurs (while $\sigma_{ct} \leq f_{ctm,fb}$ ($f_{ctm,fb,fl}$)) (ENV 1992-1-1:1991; EN 1992-1-1:2004).

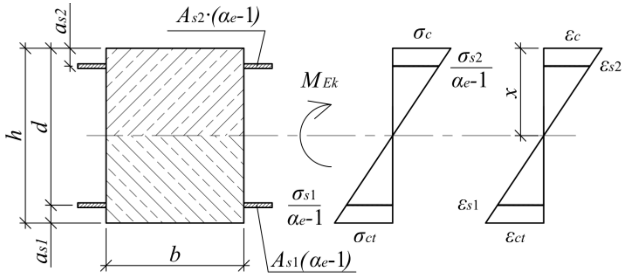


Fig. 3. Stress and strain distributions in uncracked steel fibre and ordinary reinforced concrete cross-section

After the cracking, the stress and strain distribution in the cross-section changes. The simplified stress distribution is given in Fig. 4.

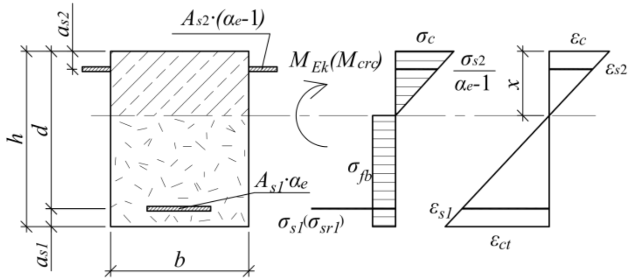


Fig. 4. The simplified stress and strain distributions in cracked steel fibre and in an ordinary reinforced concrete cross-section according to the guidance of RILEM (2003)

According to the RILEM (2003) method, the crack width is calculated by using Formula (9). Meanwhile, Formula (10) is used for crack width calculations according to the supplemented and corrected Eurocode 2 (1992-1-1:2004) methods as well as SFRC Design Guideline (2014), DafStb Guideline (2012) and SS 812310:2014 methods. Formula (11) is given in Fib Model Code 2010 (2012) for crack width calculations:

$$w_k = \beta\, s_{rm} \varepsilon_{sm}; \tag{9}$$

where w_k is the design crack width; s_{rm} is the average final crack spacing; ε_{sm} is the mean steel strain in the reinforcement allowed under the relevant combination of loads for the effects of tension stiffening, shrinkage, etc; β is the value of the coefficient relating the average crack width to the design value (for load-induced cracking, $\beta = 1.7$).

$$w_k = s_{r,\max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right); \tag{10}$$

where $s_{r,max}$ is the maximum crack spacing; ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, considering the tension stiffening; ε_{cm} is the mean strain in concrete between cracks.

$$w_d = 2l_{s,\max}\left(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}\right); \tag{11}$$

where w_d is the design crack width ($w_d = w_k$); $l_{s,max}$ is the length over which slip between concrete and steel occurs; ε_{sm} is the average steel strain over the length $l_{s,max}$; ε_{cm} is the average concrete strain over the length $l_{s,max}$; ε_{cs} is the strain of the concrete due to (free) shrinkage.

Although the crack width calculation Formula (10) is the same for supplemented and corrected Eurocode 2 methods, SFRC Design Guideline, DafStb Guideline, and SS 812310:2014 methods, however, the estimation of $s_{r,max}$, ε_{sm} and ε_{cm} differs depending on the method. The common parameter, which also differs depending on the method, is residual tensile stress σ_{fb}. For the RILEM (2003), Supplemented and Corrected Eurocode 2 methods, this parameter is calculated according to Formula (12). Meanwhile, for the method of FRC Design Guideline (2014), σ_{fb} is calculated according to Formula (13). For SS 812310:2014 and Fib Model Code 2010 (2012) methods, the residual tensile stress is calculated according to Formulas (14) and (15), respectively.

$$\sigma_{fb} = 0.45 f_{Rm,1}; \tag{12}$$

where $f_{Rm,1}$ is the average value of residual flexural tensile strength $f_{R,1}$ (when $CMOD = 0.5$ mm or $\delta = 0.46$ mm).

$$\sigma_{fb} = 0.40 \kappa_f \kappa_G f_{Rk,1}; \tag{13}$$

where κ_f is the fibre orientation factor which depends on the concrete and the type of the structure; κ_G is the factor which takes into account the influence of the member size on the coefficient of variation (for members subject to pure bending, $\kappa_G = 1.0 + 0.45A_c \leq 1.70$, where A_c is the cross-section area); $f_{Rk,1}$ is the characteristic value of residual flexural tensile strength $f_{R,1}$.

$$\sigma_{fb} = \eta_f \frac{0.45 f_{Rk,1}}{\gamma_f}; \tag{14}$$

where γ_f is the partial factor for material property (for SLS calculations, $\gamma_f = 1.0$); η_f represents the factor considering the fibre orientation (depending on the member dimensions, fibre length, and the casting procedure, $0.5 \leq \eta_f \leq 1.0$); $f_{Rk,1}$ features the characteristic value of residual flexural tensile strength.

$$\sigma_{fb} = \frac{0.45 f_{Rk,1}}{0.7}; \tag{15}$$

where $f_{Rk,1}$ is the characteristic value of residual flexural tensile strength $f_{R,1}$.

No comparative analysis of any of these methods and the relevant calculation results was found in scholarly literature.

1.5. Deflection calculation methods of steel fibre reinforced concrete structures

In order to calculate the deflection and the crack width of SFRC flexural members, the plastic hinge methods are used. It is assumed in the simplest methods that the crack surfaces remain plain and the angle of the crack is equal to the overall angular deformation of the plastic hinge (RILEM, 2002).

Schemes of two relatively simple methods are given in Fig. 5 and Fig. 6 (Meškėnas *et al.*, 2013; RILEM, 2002).

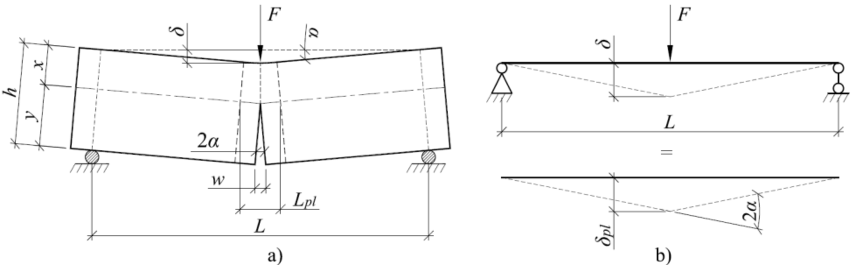


Fig. 5. Deflection calculation scheme of SFRC beams assuming that only the plastic hinge deforms: a) illustrative scheme of deformations in plastic hinge; b) deflection change along the beam

When scheme of Fig. 5 is used, the beam deflection as well as crack width can be calculated according to Formulas (16) and (17).

$$\delta = \delta_{pl} = \left(\frac{1}{r}\right)_c \frac{L \cdot L_{pl}}{4}; \tag{16}$$

where δ_{pl} is the deflection governed by plastic deformations; $(1/r)_c$ is the curvature of the cracked section; L is the beam span length; L_{pl} is the plastic hinge length.

$$w = \frac{4y\delta_{pl}}{L}; \tag{17}$$

where w is the crack width; y is the crack height.

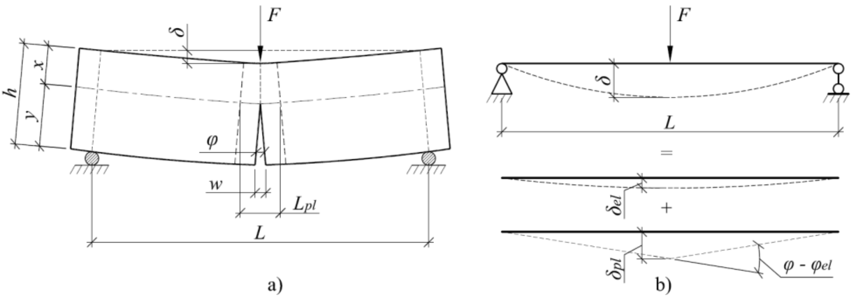


Fig. 6. Deflection calculation scheme of SFRC beams assuming elastic deformations along the whole beam and non-linear deformations in the plastic hinge: a) illustrative scheme of deformations in the plastic hinge; b) deflection change along the beam

In this case, beam deflection and crack width is calculated according to Formulas (18) and (19).

$$\delta = \delta_{el} + \delta_{pl} = \frac{FL^3}{48EI} + \left[\left(\frac{1}{r} \right)_c - \left(\frac{1}{r} \right)_{el} \right] \cdot \frac{L \cdot L_{pl}}{4}; \tag{18}$$

where δ is the total deflection; δ_{el} is the deflection governed by elastic deformation; δ_{pl} is the deflection governed by plastic deformation; F is the load; L is the beam span length; L_{pl} is the plastic hinge length; E is Young's modulus of beam material; I is the moment of inertia; $(1/r)_c$ is the total curvature of the cracked section; $(1/r)_{el}$ is the elastic curvature of the uncracked section taking the maximum moment into account.

$$w = \varphi \cdot y; \tag{19}$$

where φ is the rotation angle of crack planes.

In the above discussed methods, the curvature is considered constant along L_{pl}. However the change of the curvature in the plastic zone is taken into account in more advanced methods (Casanova and Rossi, 1996, SS 812310:2014).

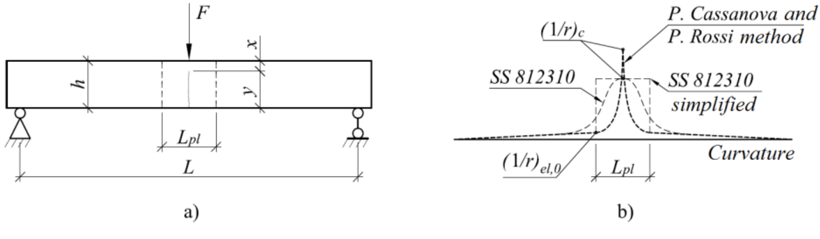


Fig. 7. Deflection calculation scheme of SFRC beams assuming the elastic deformations along the beam and non-linear deformations in the plastic hinge: a) an illustrative scheme of the plastic hinge zone; b) deflection change along the beam

Deflection of SFRC beam subjected to three-point bending according to Casanova's method can be calculated by Formula (20).

$$\delta = \delta_{el} + \delta_{pl} = \frac{FL^3}{48EI} + \left[\frac{\left(\frac{1}{r}\right)_c + 2\left(\frac{1}{r}\right)_{el,0}}{3} - \left(\frac{1}{r}\right)_{el} \right] \cdot \frac{L \cdot L_{pl}}{4}; \tag{20}$$

where $L_{pl} = 2y$ is the plastic hinge length; $(1/r)_{el,0}$ is the elastic curvature at the beginning of the plastic hinge.

There are no common recommendations for the selection of plastic hinge length L_{pl}; however, this parameter is important for all the calculation methods.

1.6. Deflection calculation methods of steel fibre and ordinary reinforced concrete structures

Deflections of combined (steel fibre and ordinary) reinforced concrete beams can be calculated according to Eurocode 2 (ENV 1992-1-1:1991; EN 1992-1-1:2004) methods (Formula (21) (Mosley et al., 2007)). However, the steel fibre influence should be additionally considered. In order to achieve that, the residual tensile stress in the crack is taken into account. The stress diagram in the cracked cross-section is given in Fig. 4; however, more accurate diagrams can also be used, which leads to a more difficult calculation process (DafStb Guideline, 2012; Dupont, 2003; SS 812310:2014; SFRC Design Guideline, 2014; RILEM, 2002).

$$\delta = k\left(\frac{1}{r}\right)_{avg} L^2; \tag{21}$$

where k is the deflection coefficient which depends on the structural scheme (supports and load distribution); $(1/r)_{avg}$ is the average curvature of cracked and uncracked sections; L is the span length.

The average curvature is calculated according to Formula (22):

$$\left(\frac{1}{r}\right)_{avg} = \varsigma\left(\frac{1}{r}\right)_{II} + (1-\varsigma)\left(\frac{1}{r}\right)_I; \tag{22}$$

where ς is the distribution coefficient allowing for tension stiffening at the section; $(1/r)_I$ is the curvature of the uncracked section; $(1/r)_{II}$ is the curvature of the cracked section.

The finite element or iterative methods can be used in order to calculate the deflection more accurately (Jansson, 2008; Ulbinas, 2012).

2. EXPERIMENTAL RESEARCH OF STEEL FIBRE AND COMBINED REINFORCED CONCRETE FLEXURAL MEMBERS

2.1. Experimental research of residual flexural tensile strength and compressive zone height

An experimental program was carried out where 168 three-point bending prisms with the notch and 148 cubes were tested. The prisms were tested according to the requirements of the EN 14651+A1:2007. In order to obtain the compressive strength values, 72 cubes were cast by using SFRC and 76 cubes from ordinary concrete. Two types of steel fibre were used in the program – hooked end and wavy (as shown in Fig. 1). The wavy steel fibre was used only in series from 101 to 103. In order to evaluate the manufacturing differences, the production of 13 different manufacturers was used. All the tests were performed by using a Toni Technik (600 kN) press. Information about the specimens and the tests series is presented in Table 1.

Table 1. Test series and specimens information

Series No.	600×150×150 No of prisms	No of cubes	Fibre content, kg/m^3	l/d, mm	f_y, MPa	$f_{cm,fb}$, MPa
1	6	4 / 4	25	50/1	1200	26.6
2	12	4 / 4	25	50/1	1150	30.5
3	12	4 / 4	30	50/1	1200	26.5
4	12	4 / 4	30	50/1	1150	32.8
5	12	4 / 4	35	50/1	1150	32.6
6	12	4 / 4	35	50/1	1150	33.0
7	12	4 / 4	15	52/0.75	1500	30.2
8	12	4 / 4	20	52/0.75	1500	35.6
9	12	9 / 4	30	50/0.75	1150	40.3
10	12	4 / 4	35	30/0.6	1150	32.1
101	12	4 / 4	35	50/1	1150	33.0
102	12	4 / 4	35	50/1	1150	30.9
103	12	4 / 4	35	50/1	1150	33.2
61	6	4 / 4	25	32/0.55	1450	26.9
62	6	4 / 16	25	52/0.75	1500	39.9
63	6	7 / 4	50	52/0.75	1500	43.2

Notations used in Table 1: l is the fibre length; d is the fibre diameter; f_y is the tensile strength of the fibre; $f_{cm,fb}$ is the mean value of SFRC cylinder compressive strength ($f_{cm,fb} = 0.81 \cdot 0.95 \cdot f_{cm,cube,100}$ and $f_{cm,fb} = 0.81 \cdot f_{cm,cube,150}$); $f_{cm,cube,100}$ and $f_{cm,cube,150}$ is the average cubic compressive strength of SFRC where the dimensions of the cubes are, respectively, 100×100×100 and 150×150×150 mm; $f_{Rm,1}$ is the average residual flexural tensile strength when $CMOD$ = 0.5 mm.

The average residual flexural tensile strength $f_{Rm,1}$ of all the series and the separate results of test series No. 9 are given in Fig. 8. The high scatter of $f_{R,1}$ results was determined in all the series. Coefficient of variation of $f_{R,1}$ was found to be in the range from 12.1 % to 32.2 %.

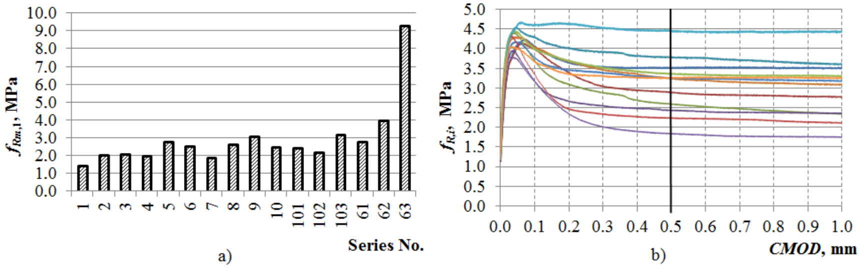


Fig. 8. Results of three-point bending tests according to EN 14651+A1:2007: a) $f_{Rm,1}$ results; b) $f_{R,i}$ – *CMOD* relations of test series No. 9 (Kelpša, 2014; Kelpša, 2015a; Kelpša, 2015b)

F–CMOD as well as *F–δ* relations were measured during the tests of series No. 62 and No. 63. Also, compressive zone height *x* and deflection *δ* relations of 2 specimens from these series were determined by using strain gages (HBM SG4wire).

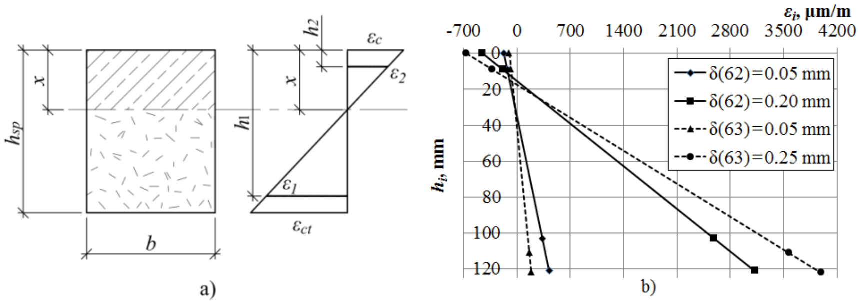


Fig. 9. Deformation distribution in cross-section: a) the calculating scheme of deformations ε and compressive zone height *x*; b) deformations ε_i of two specimens from test series No. 62 and No. 63

When the deflection reached 0.47 mm, the compressive zone height *x* of two specimens from tests series No. 62 and No. 63 was, respectively, 10.85 mm and 15.81 mm.

2.2. Experimental crack width research of small size combined reinforced concrete beams

In order to investigate the influence of the fibre on the cracking of combined reinforce flexural members, an experimental program with small-sized specimens was performed. 45 specimens were tested during the program. The information about the specimens is outlined in Table 2 (Kelpša, 2014). In all the cases where the geometry of specimens was 600×150×150 mm, the three-point bending scheme was used. In the second test series, specimens without the notch were used; however, in all other cases (Series No. 1, No. 3…No. 8), specimens with the notch were tested (Fig. 2). All the tests were conducted by using a Toni Technik (600 kN) press. The prisms were tested according to the guidance of EN 14651+A1:2007. The modulus of elasticity, the compressive strength and the flexural tensile strength were also determined according to the guidance of the relevant applicable standards.

Table 2. Tests and specimens information

Series No.	Geometry of specimens, mm	No. of speci-mens	Fibre content, kg/m^3	Rebars	Loading control	Measured parameter or relation
1	600×150×150	12	30	–	Under deformation	F–$CMOD$, $f_{Rm,1}$, LOP_m
2	600×150×150	3	–	–	Under deformation	$f_{ctm,fl}$
3	600×150×150	3	–	1ϕ6 S400	Under deformation	F-w
4	600×150×150	3	30	1ϕ6 S400	Under deformation	F-w
5	600×150×150	2	–	–	Under force	$f_{ctm,fl,notch}$
6	600×150×150	2	30	–	Under force	F-w
7	600×150×150	2	–	1ϕ6 S400	Under force	F-w
8	600×150×150	2	30	1ϕ6 S400	Under force	F-w
9	100×100×100	4	–	–	Under force	f_{cm}
10	100×100×100	9	30	–	Under force	$f_{cm,fb}$
11	300×100×100	3	–	–	Under force	E_{cm}

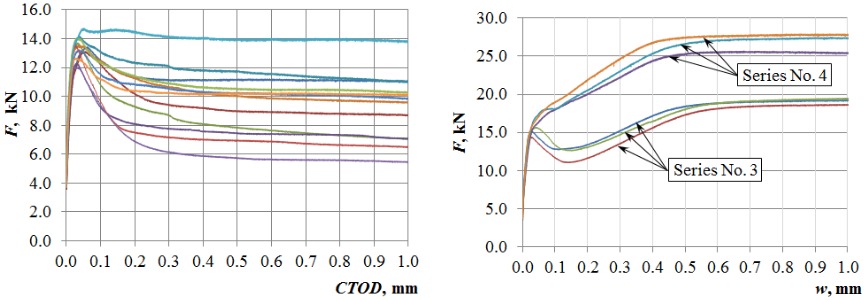
The following notations were used in Table 2: 1ϕ S400 is one S400 grade rebar with the diameter of 6 mm; F is the loading force; w is the crack width; $f_{Rm,1}$ is the average residual flexural tensile strength when $CMOD$ = 0.5 mm; LOP_m is the average first crack strength when specimens with the notch are used; $f_{ctm,fl}$ is the average flexural tensile strength; $f_{ctm,fl,notch}$ is the average flexural tensile strength when specimens with the notch are used; f_{cm} is the mean value of concrete cylinder compressive strength; $f_{cm,fb}$ is the mean value of SFRC cylinder compressive strength; E_{cm} is the secant modulus of the elasticity of concrete.

Test results are summarized in Table 3, Fig. 10 and Fig. 11.

Table 3. Summary of experimental results

Series No.	Fibre content, kg/m^3	Determined parameter	Value, MPa
1	30	$f_{Rm,1}$	3.07
1	30	LOP_m	4.24
2	–	$f_{ctm,fl}$	4.46
5	–	$f_{ctm,fl,notch}$	3.65
9	–	f_{cm}	47.04
10	30	$f_{cm,fb}$	49.66
11	–	E_{cm}	32988

The notation $CTOD$ in Fig. 11 part a means the crack tip opening displacement and can be approximately equated to crack width w of combined reinforced concrete specimens.



a) b)

Fig. 10. Three-point bending test results when the loading was under deformation control: a) results of tests series No. 2; b) results of tests series No. 3 and No. 4

Deflection softening behavior was determined during the tests with the specimens of 30 kg/m^3 fibre content. In the cases of notched specimens, lower cracking stress values in concrete were established. The same tendency is probable when SFRC is used. As it can be seen from Fig. 10, 30 kg/m^3 fibre content can effectively reduce the crack width and increase the bearing capacity of reinforced concrete members. The minor load increase after cracking of test series No.3 was governed by the small reinforcement ratio as well as the small cross-section dimensions.

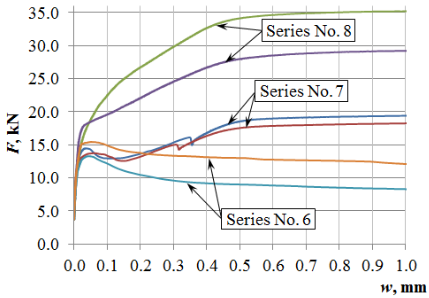


Fig. 11. Three-point bending test results when the loading was under force control

The loading control had no significant influence on the results of the experiment. The higher scatter of experiment result values was observed in cases where steel fibre was used.

2.3. Experimental crack width research of full-scale combined reinforced concrete beams

An experimental program was executed with full-scale combined reinforced concrete beams in order to determine the steel fibre influence on the crack width and deflection. 4 ordinary reinforced and 8 combined reinforced concrete full-scale beams (1300×200×160 mm) were tested. Together with these beams, 108 additional specimens (cubes and prisms) were tested in order to define their material properties. Two fibre contents were used in the experiments – 25 and 50 kg/m^3. The three-point bending tests are marked as test series No. 62 and No. 63 in Section 2.1. The prisms of test series No. 62 and No. 63 were tested according to the guidance of EN 14651+A1:2007. The compressive strength, the flexural tensile strength and the modulus of elasticity were determined according to the guidance of the relevant applicable standards. Reinforcement and loading schemes of the beams are presented in Fig. 12, whereas additional information about the beams is given in Table 4.

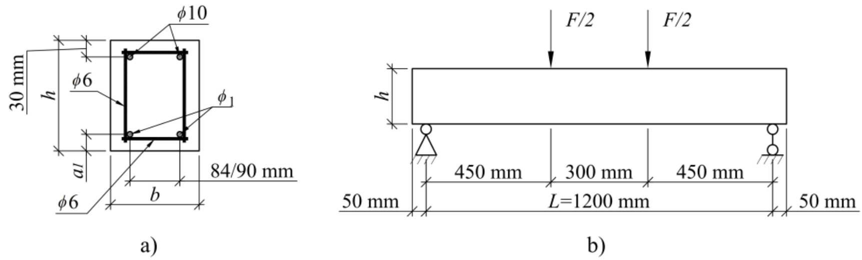


Fig. 12. Reinforcement and loading schemes of the tested beams: a) the reinforcement scheme; b) the loading scheme

Table 4. Full-scale beam information

Beam No.	Cross-section ($h \times b$), mm	a_1, mm	Bottom reinforce-ment $2\phi_1$ mm	Fibre content, kg/m^3	f_{cm} / $f_{cm,fb}$, MPa	$f_{ctm,fl}$ / $f_{ctm,fl,fb}$, MPa	f_{ctm} / $f_{ctm,fb}$, MPa	E_{cm} / $E_{cm,fb}$, MPa	$f_{Rm,1}$, MPa
1	198×161	28	2ϕ10 S500	-	43.9	4.89	3.49	30096	-
2	200×162	29	2ϕ10 S500	-	43.9	4.89	3.49	30096	-
3	200×160	28	2ϕ10 S500	25	41.6	5.54	3.96	30224	3.94
4	202×160	29	2ϕ10 S500	25	41.6	5.54	3.96	30224	3.94
5	200×161	30	2ϕ10 S500	50	43.3	5.96	4.26	31419	9.27
6	200×159	30	2ϕ10 S500	50	43.3	5.96	4.26	31419	9.27
7	199×161	30	2ϕ16 S500	-	43.8	4.89	3.49	30096	-
8	201×159	31	2ϕ16 S500	-	43.8	4.89	3.49	30096	-
9	199×161	29	2ϕ16 S500	25	41.6	5.54	3.96	30224	3.94
10	199×161	31	2ϕ16 S500	25	41.6	5.54	3.96	30224	3.94
11	200×161	30	2ϕ16 S500	50	42.0	5.96	4.26	31419	9.27
12	200×161	30	2ϕ16 S500	50	42.0	5.96	4.26	31419	9.27

The top concrete cover was 25 mm to the top longitudinal reinforcement whose diameter was 10 mm. An S500 rebar grade was used in the experiments with E_s = 200 GPa. The transverse reinforcement was designed to ensure satisfactory shear strength in all the cases.

All the beams were tested with hydraulic force equipment (200 kN), and the load steps were in the range from 2.0 to 5.0 kN. All the additional specimens were tested with a Toni Technik (600 kN) press. The test results are presented in Fig. 13, Fig. 14 and Fig. 15. The cracks were measured at the level of bottom reinforcement.

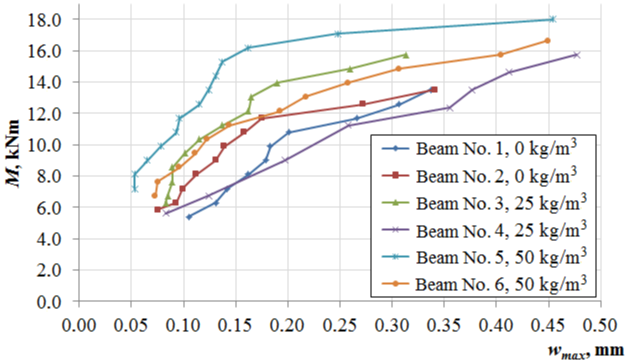


Fig. 13. Maximum crack widths of beams No. 1...No. 6 depending on bending moment M

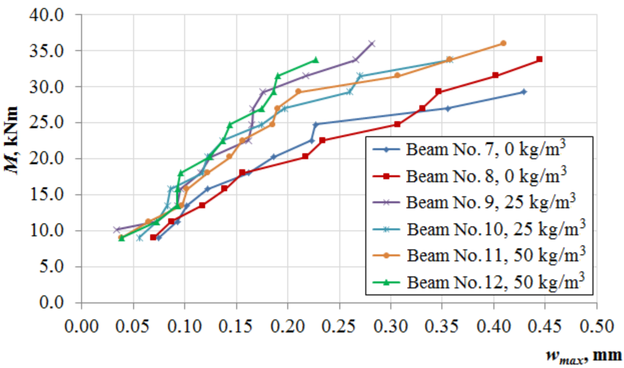
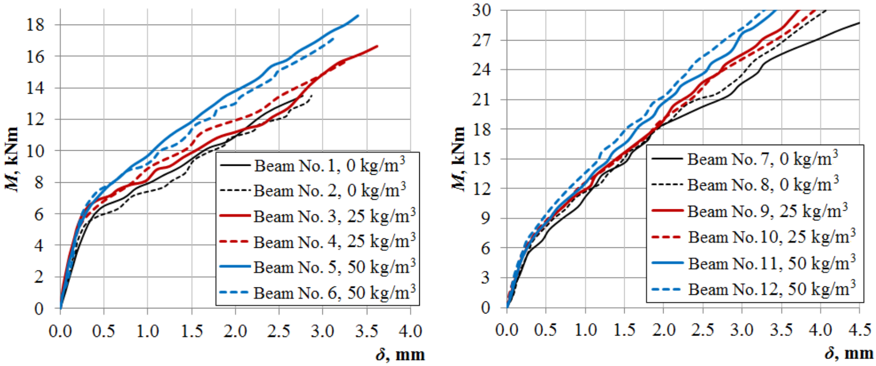


Fig. 14. Maximum crack widths of beams No. 7…No. 12 depending on bending moment M

It was observed from the test results that in most cases, the crack widths were reduced by steel fibre. However, the efficiency of the additional fibre reinforcement is restricted due to the uneven distribution of the fibre and its orientation in the structural member.



a) b)

Fig. 15. Experimental bending moment and deflection relationships ($M - \delta$) of beams No. 1…No. 12: a) bottom reinforcement: $2\phi10$; b) bottom reinforcement: $2\phi16$

The test results show that steel fibre reduces the deflection of the combined reinforced concrete beams. Steel fibre reinforcement efficiency is also influenced by the uneven distribution of the fibre and its orientation in the structural member.

3. NORMAL CRACK WIDTH ANALYSIS OF COMBINED REINFORCED CONCRETE FLEXURAL MEMBERS

3.1. Average residual tensile stress calculation

Residual tensile stress is a common and very important parameter in the crack width as well as deflection calculations of SFRC and combined reinforced concrete structures. As it was established above, this stress is calculated by using the experimental results. While there are some proposals for post-cracking strength calculations, none of these proposals are fitted directly for crack width and deflection analysis. In order to apply Naaman's and Sujivorakul's methods for the crack width and deflection calculations, adjustment coefficients k_{pc} and k_P were proposed. Consequently, the residual tensile strength can be calculated by Formulas (23) and (24).

$$\sigma_{fb} = k_{pc} \cdot \sigma_{pc}; \tag{23}$$

where σ_{pc} is the maximum post-cracking strength calculated by Naaman's method (Section 1.2); k_{pc} is the adjustment coefficient for Naaman's method.

$$\sigma_{fb} = k_P \cdot \sigma_P; \tag{24}$$

where σ_P is the post-cracking strength as calculated by employing Sujivorakul's method (Section 1.2); k_P is the adjustment coefficient for Sujivorakul's method.

For the estimation and verification of the adjustment coefficients, the experimental results of the three-point bending tests were used. A part of these tests were conducted at Kaunas University of Technology (Table 1) and the other part of was taken from the references (Table 5).

Table 5. Experiment results taken from references (Kelpša *et al.*, 2015a; Kelpša *et al.*, 2015b)

Series No.	No. of prisms	Fibre content, kg/m^3	Ratio l/d, mm	f_y, MPa	Series No.	No. of prisms	Fibre content, kg/m^3	Ratio l/d, mm	f_y, MPa
11	7	20	50/1.0	1100	37	6	40	50/0.62	1270
12	8	20	50/1.05	1000	38	6	40	50/0.62	1270
13	8	60	50/1.05	1000	39	6	40	50/0.62	1270
14	6	30	40/0.62	1050	40	8	40	35/0.45	1050
15	6	30	25/0.4	1700	41	8	60	35/0.45	1050
16	8	20	60/0.9	1000	42*	6	30	50/0.8	1550
17	8	20	60/0.9	1000	43*	6	30	40/0.62	1050
18	6	20	60/0.9	1000	44*	6	30	40/0.62	1050
19	6	30	60/0.9	1000	45*	6	30	40/0.62	1050
20	8	40	60/0.9	1000	46*	6	30	25/0.4	1700
21	8	40	60/0.9	1000	47*	6	30	25/0.4	1700
22	8	60	60/0.9	1000	48*	6	30	25/0.4	1700
23	8	60	60/0.9	1000	49*	6	30	60/0.9	1000
24	6	60	60/0.9	1000	50*	6	30	60/0.9	1000
25	16	75	60/0.9	1000	51*	6	30	60/0.9	1000
26	6	20	35/0.55	1100	52*	6	39	60/0.9	1160
27	6	30	35/0.55	1100	53*	5	80	60/0.9	1160
28	4	39	35/0.55	1100	54*	6	78	60/0.9	1160
29	6	40	35/0.55	1100	55*	6	78	60/0.9	1160
30	6	60	35/0.55	1100	56*	9	50	35/0.55	1100
31	5	79	35/0.55	1100	57*	4	50	35/0.55	1100
32	6	20	60/0.75	1050	58*	9	40	60/0.75	1050
33	5	39	60/0.75	1050	59*	9	40	60/0.75	1050
34	6	40	60/0.75	1050	60*	6	80	60/0.75	1050
35	5	79	60/0.75	1050	104	6	40	50/1.0	–
36	6	40	50/0.62	1270	***notation*** * – self compacting SFRC				

The wavy steel fibre was only used in test series number 104. The average compressive strength of the entire SFRC test series is presented in Fig. 16.

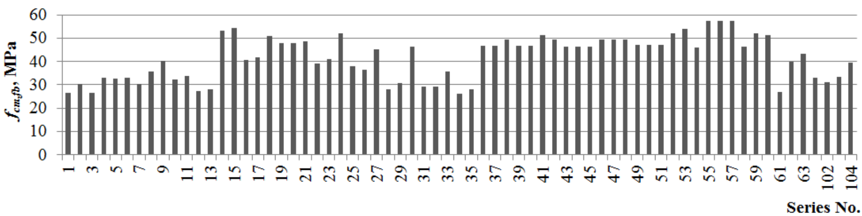


Fig. 16. Average compressive strength $f_{cm,fb}$ of SFRC used in the analysis

Comparative analysis was performed, and its results are given in Fig. 17 and Fig. 18.

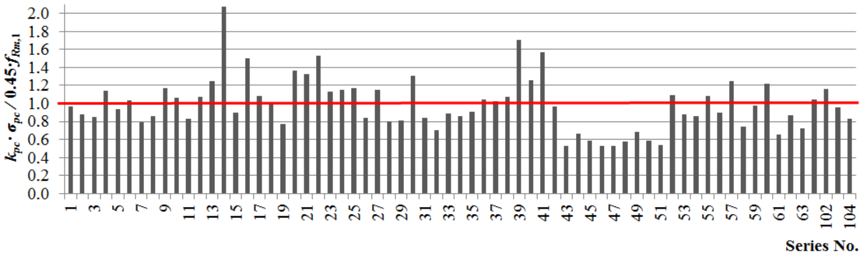


Fig. 17. Residual tensile stress ratios obtained by using Naaman's method and k_{pc}

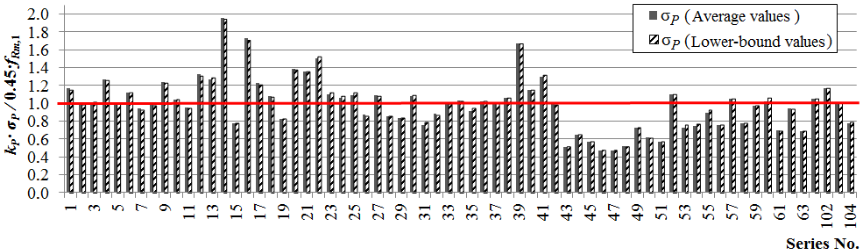


Fig. 18. Residual tensile stress ratios obtained by using Sujivorakul's method and k_P

As it can be seen from Fig. 17 and Fig. 18, the residual tensile stress can be approximately calculated by employing Naaman's and Sujivorakul's methods when the adjustment coefficients are used. However, relative errors in some cases were significant; they could consequently lead to inaccuracies of the calculated crack widths and deflections.

In order to develop a more accurate calculation method, the main factors having influence on the post-cracking properties of SFRC were analyzed. The main analyzed factors were fibre length l, fibre diameter d, aspect ratio l/d, fibre material properties $f_{y,fb}$, fibre shape, fibre content V_{fb}, fibre orientation, the bond strength between fibre and matrix of concrete, as well as a few others. After conducting analysis of these factors and obtaining the experiment results (Table 1 and Table 5), a new calculation method of average residual flexural tensile strength $f_{Rm,1}$ was created (Formula 3.1-3). Comparative results of the calculated and experimental $f_{Rm,1}$ values are given in Fig. 19.

$$f_{Rm,1} = k_{adj}\left(16.5 f_{cm,fb} - 0.185 f_{cm,fb}^{\,2} - 155\right) \cdot \eta_0 \left(\frac{l}{d}\right)^{-\frac{1}{3}} \left(\frac{f_{y,fb}}{1000}\right)^{\frac{1}{2}} \cdot \left[27.658\left(k_{fb}^{\,1.5} V_{fb}\right) - 590.63\left(k_{fb}^{\,1.5} V_{fb}\right)^2 + 0.0024\right]; \tag{25}$$

where k_{adj} is the adjustment coefficient (k_{adj} = 0.96); η_0 is the capacity factor depending on the fibre orientation; k_{fb} is the fibre reinforcement efficiency factor ($k_{fb} = l / 50d$).

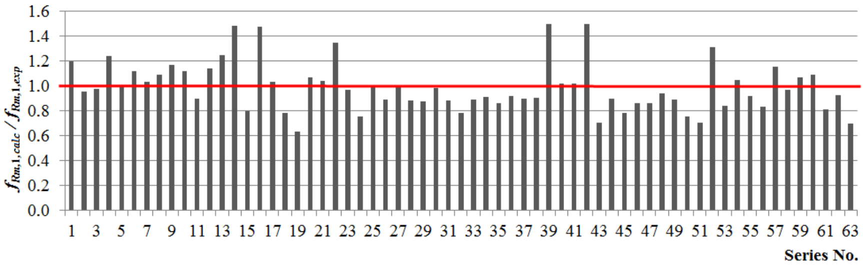


Fig. 19. Ratio of the calculated and experimental values of $f_{Rm,1}$

When $f_{Rm,1}$ is calculated, the residual tensile stress σ_{fb} can be easily determined. The precision of the calculated $f_{Rm,1}$ values is noticeably better than in the cases of calculating σ_{fb} according to Naaman's or Sujivorakul's methods. Considering the relative errors between $f_{R,1}$ results of the same series, the precision of calculated $f_{Rm,1}$ values is deemed as acceptable.

3.2. Characteristic values of residual tensile stress calculations

Depending on the crack width or the deflection calculation method, the average or characteristic values of $f_{R,1}$ are required. However, according to some statistical calculation methods, coefficient of variation V_x should be known from experience. This coefficient should be known in all the cases when $f_{Rm,1}$ is calculated. Due to this reason, after analysis of all experimental results, an empirical method of the coefficient of variation V_x of $f_{R,1}$ was created.

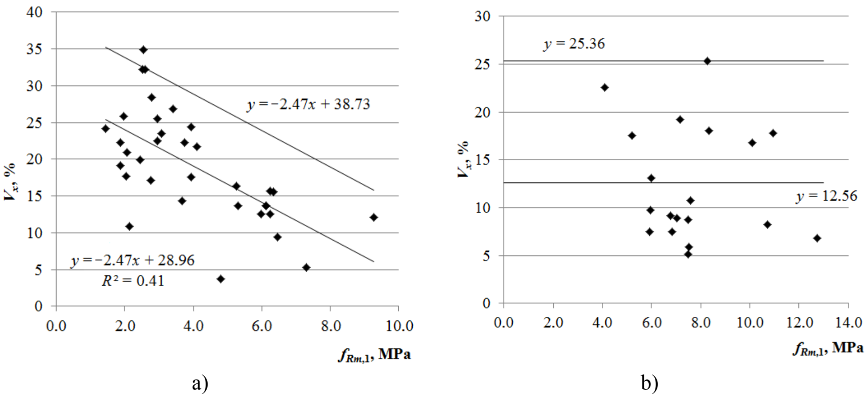


Fig. 20. Empirical relationship between V_x and $f_{Rm,1}$: a) for ordinary (vibrated) SFRC b) for self-compacting SFRC

In order to establish discrepancies of the calculation results received by using different methods as well as the applicability of developed V_x calculation method, extensive comparative analysis was performed. Ordinary and self-compacting concrete types were analyzed separately. The calculation information is in Table 6.

Table 6. Notations and explication of comparative analysis calculations

Notation	Distribution	Method	fr / γ	V_x	Additional info
NB-1	Normal	Bayesian	5% / 95%	Unknown	$f_{R,1,exp}$
NB-2	Normal	Bayesian	5% / 95%	Known	$f_{R,1,exp}$
NK-1	Normal	Classical	5% / 75%	Unknown	$f_{R,1,exp}$
NK-2	Normal	Classical	5% / 75%	Known	$f_{R,1,exp}$
LK-1	Log-normal	Classical	5% / 75%	Unknown	$f_{R,1,exp}$
LK-2	Log-normal	Classical	5% / 75%	Known	$f_{R,1,exp}$
LK-3	Log-normal	Classical	5% / 75%	Unknown	$f_{R,1,exp}$, *
LK-4	Log-normal	Classical	5% / 75%	Known	$f_{R,1,exp}$, *
NB-3	Normal	Bayesian	5% / 95%	Known	$f_{Rm,1,calc}$
NK-3	Normal	Classical	5% / 75%	Known	$f_{Rm,1,calc}$
LK-5	Log-normal	Classical	5% / 75%	Known	$f_{Rm,1,calc}$,
LK-6	Log-normal	Classical	5% / 75%	Known	$f_{Rm,1,calc}$, *

Notations used in Table 6: fr is the fractile; γ is the confidence level; V_x is the coefficient of variation; $f_{R,1,exp}$ are the $f_{R,1}$ experiment values; $f_{R,1,calc}$ are the $f_{Rm,1}$ calculated values according to Formula (25); * denotes cases when additional requirements of SFRC Design Guideline (2014) were applied.

Comparative analysis showed that in most of the cases, the lesser values of $f_{Rk,1}$ were obtained by using V_x known calculations. Also, lower values of $f_{Rk,1}$ were obtained when the classical method was used. Additional requirements of SFRC Design Guideline (2014) lead to lesser values of $f_{Rk,1}$ in most of the cases. The results of the comparative analysis when $f_{Rk,1}$ values are calculated by using $f_{Rm,1}$ and V_x estimation proposals together are presented in Fig. 21 and Fig. 22.

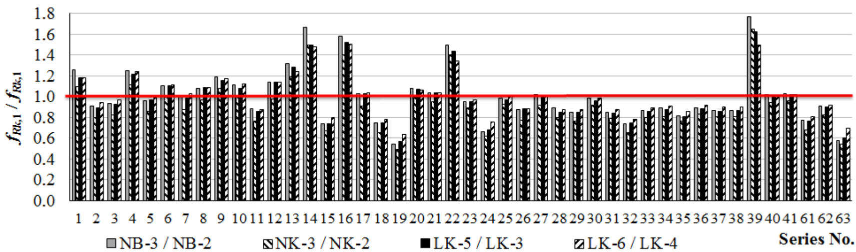


Fig. 21. Ratios of $f_{Rk,1}$ values when $f_{Rm,1}$ are calculated according to Formula (25) and the $f_{R,1}$ values are taken from experiments (case of ordinary concrete)

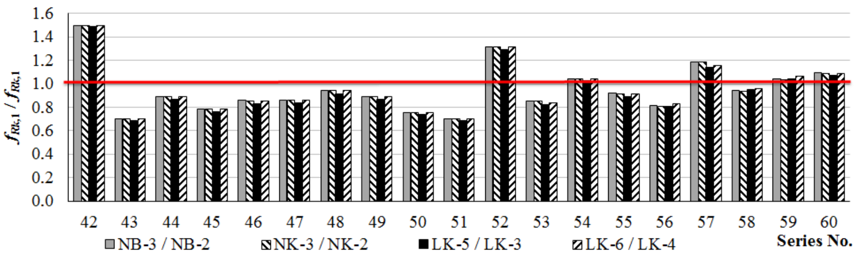


Fig. 22. Ratios of $f_{Rk,1}$ values when $f_{Rm,1}$ are calculated according to Formula (25) and the $f_{R,1}$ are taken from experiments (case of self-compacting concrete)

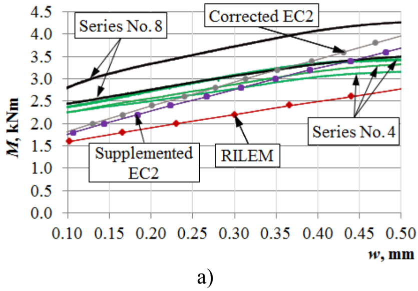
It was deduced that the relative errors of the calculated $f_{Rk,1}$ depend on the errors made by calculating $f_{Rm,1}$. In all the analyzed cases, the calculated values of $f_{Rk,1}$ were lower than the experimental $f_{Rm,1}$ values.

3.3. Crack width calculations of steel fibre and ordinary reinforced concrete flexural members

In order to evaluate the precision of the crack width calculation methods of combined reinforced concrete members, extensive analysis was performed. This analysis was divided into three stages:

- **Stage 1**. The calculations were done according to RILEM, Supplemented Eurocode 2 (EC2) and Corrected Eurocode 2 methods. The calculations were verified with the experimental results given in Section 2.2 as well as the experimental results of 2 Ulbinas' beams (Ulbinas, 2012).
- **Stage 2**. Calculations were done according to RILEM, Supplemented Eurocode 2, Corrected Eurocode 2, SFRC Design Guideline, SS 812310:2014, and Fib Model Code 2010 methods. The calculation results were verified with the test results given in Section 2.
- **Stage 3**. Calculations were done by using the same methods as in Stage 2; however, the experiment results of the seven beams were taken from Dupont's doctoral thesis (Dupont, 2003) for the verification of the calculation.

Results of the comparative analysis of **Stage 1** are presented in Fig. 23. The most accurate results were obtained by using the corrected Eurocode 2 method.



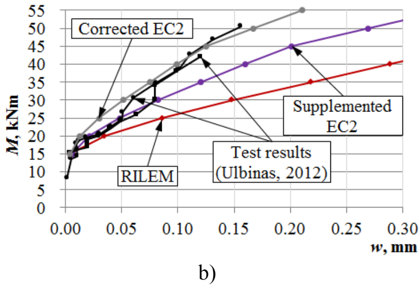


Fig. 23. Experiment and calculated crack width results of combined reinforced concrete beams: a) experiment results presented in Section 2.2; b) experiment results taken from the doctoral thesis by Ulbinas (Ulbinas, 2012)

A part of the comparative analysis results of **Stage 2** is presented in Fig. 24, Fig. 25 and Fig. 26. Tendency of the remaining results is fairly similar.

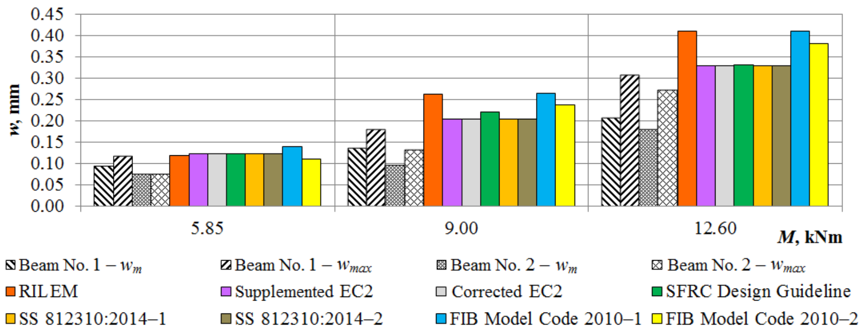


Fig. 24. Calculated and experimental crack width results of beams No. 1 and No. 2 (reinforcement: 2ϕ10 and 0 kg/m^3 of fibre)

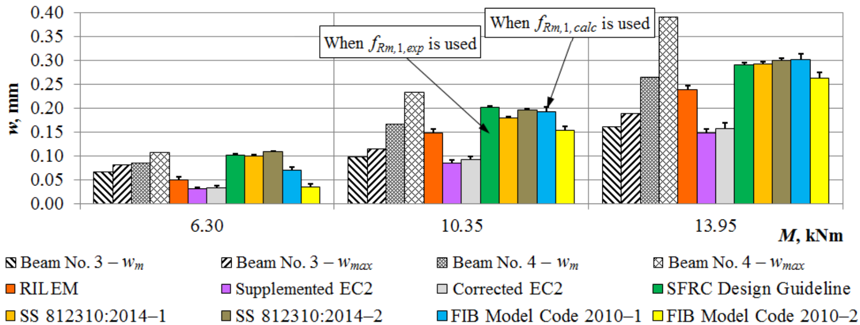


Fig. 25. Calculated and experimentally obtained crack width results of beams No. 3 and No.4 (reinforcement: 2ϕ10 and 25 kg/m^3 of fibre)

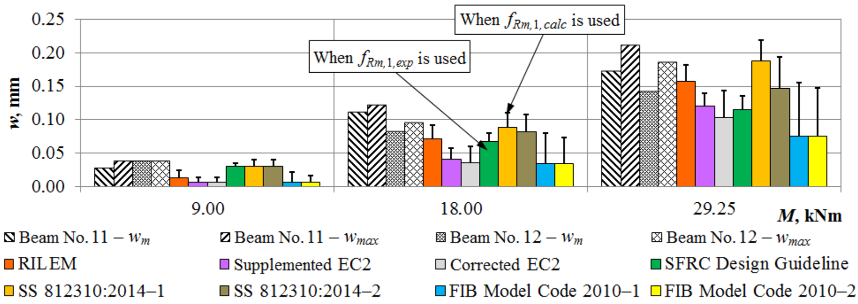


Fig. 26. Calculated and experimentally obtained crack width results of beams No. 11 and No. 12 (reinforcement: 2ϕ16 and 50 kg/m^3 of fibre)

Since results of **Stages 2** and **3** are similar, the corresponding findings are discussed jointly. The fibre orientation has a significant influence on the crack widths. Therefore, the calculated crack widths are more secure when the characteristic values of $f_{R,1}$ are used. The widest and the most reliable at the same time crack widths were given when the SS 812310:2014 method was used.

The influence of $f_{R,1}$ errors on the crack widths strongly depends on the overall combined reinforcement ratio – the higher is the percentage of fibre reinforcement, the stronger is the influence of the calculated $f_{R,1}$ inaccuracies. There were no analyzed cases for which the inaccuracies of the calculated $f_{R,1}$ values would be critical. The calculated crack width differences were higher when using separate calculation methods in comparison with the results obtained under the influence of $f_{R,1}$ inaccuracies. As a result, it was determined that the developed $f_{Rm,1}$ and V_x calculation methods can be used in the crack width calculations of the combined reinforced concrete members.

4. DEFLECTION ANALYSIS OF STEEL FIBRE AND COMBINED REINFORCED CONCRETE FLEXURAL MEMBERS

4.1. Deflection calculations of SFRC flexural members

A new plastic hinge method is developed for the deflection and crack width calculations of SFRC flexural members. It is assumed in this method that in the zone near the crack behavior of the SFRC flexural members is not elastic. The curvature ($1/r$) in that zone depends on bending moment M and stiffness of the current section $E_c \cdot I_{pl}$. It is assumed that modulus of elasticity E_c remains constant; however, moment of inertia I_{pl} varies along the plastic zone under the assumed manner. Deflection is calculated according to Formula (26) where shear deformations can also be considered:

$$\delta = \delta_{el} + \delta_{pl} + \delta_{shear} = 2\int_0^{L_{el}} \frac{M_0 M}{E_c I_{el}} dz + 2\int_{L_{el}}^{L/2} \frac{M_0 M}{E_c I_{pl}} dz + 2\int_0^{L/2} \mu \frac{V_0 V}{GA} dz; \qquad (26)$$

where δ_{el} is the deflection due to elastic deformations; δ_{pl} is the deflection due to plastic deformations; δ_{shear} is the deflection due to shear deformations; E_c is the SFRC modulus of elasticity; I_{el} is the moment of inertia of the elastic zone; M_0 is the internal virtual moment in the beam expressed as a function of z, which is caused by the external virtual unit load; M is the internal moment in the beam expressed as a function of z, which is caused by the real external load; I_{pl} stands for the assumed moment of inertia in the plastic zone expressed as a function of z; μ is the shear coefficient depending on the cross-section only; V_0 is the internal virtual shear force in the beam expressed as a function of z, which is caused by the external virtual unit load; G is the shear modulus of SFRC; A is the cross-section area; L is the beam span; L_{el} is the elastic zone length; L_{pl} is the plastic zone length.

Three variation functions of the assumed moment of inertia I_{pl} as well as its expressions are proposed for the calculations. These variation functions of I_{pl} are presented in Fig. 27 – parabolic, curvilinear and constant (Formulas (27), (28) and (29), respectively).

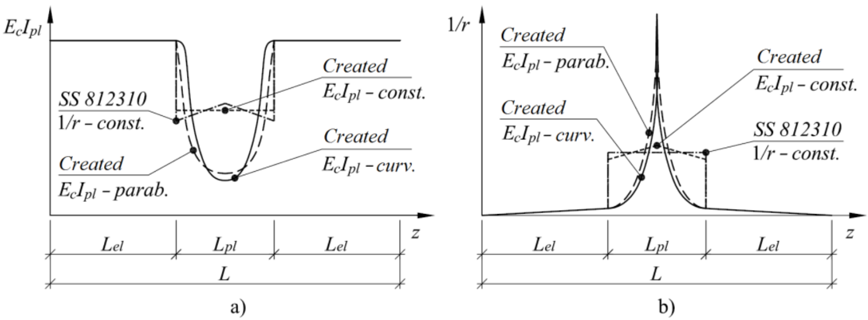


Fig. 27. Scheme of stiffness and curvature variations along the SFRC beam (loading scheme is given Fig. 7): a) stiffness variation; b) curvature variation

$$I_{pl} = Az^B; \tag{27}$$

where A and B are coefficients; z is the analyzed cross-section distance from the beginning of the beam.

$$I_{pl} = \frac{I_{el}}{1+\left(\frac{z-L_{el}}{B}\right)^A}; \tag{28}$$

$$I_{pl} = \frac{b\left(x^3 + 3hx^2\right)}{12}; \tag{29}$$

where b is the cross-section width; x is the height of the compressive zone of the cracked cross-section.

In order to verify the developed method and to define plastic zone length L_{pl}, extensive comparative analysis was performed. The experiment results of the 2 specimens of test series No. 62 and No. 63 were used in the analysis. The crack width, the compressive zone height, the residual tensile stress coefficient and the deflection errors caused due to residual stress inaccuracies were analyzed. Two stress diagrams (simplified and relatively accurate) in the cracked cross-section were applied in the analysis. The previously discussed (Section 1.5) plastic hinge methods were also analyzed. The information about all the employed methods is given in Table 7. The compressive zone height results are presented in Fig. 28.

Table 7. Numbering and description of the analyzed calculation methods

Method No.	Description of the method	Stress diagram	Deflection composition	Notations
1	Created; E_cI_{pl} – *parab.*	1	$\delta_{el}+\delta_{pl}+\delta_{sh}$	–
2	Created; E_cI_{pl} – *curv.*	1	$\delta_{el}+\delta_{pl}+\delta_{sh}$	$B = 1/6 \cdot L_{pl}$
3	Created; E_cI_{pl} – *curv.*	1	$\delta_{el}+\delta_{pl}+\delta_{sh}$	$B = 1/8 \cdot L_{pl}$
4	Created; E_cI_{pl} – *const.*	1	$\delta_{el}+\delta_{pl}+\delta_{sh}$	–
5	SS 812310:2014, $(1/r)_c$ – *const.*	1	$\delta_{el}+\delta_{pl}+\delta_{sh}$	–
6	Meškėnas *et al.* (2013), $(1/r)_c$ – *const.*	1	δ_{pl}	–
7	RILEM (2002), $(1/r)_c$ – *const.*	1	$\delta_{el}+\delta_{pl}$	–
8	Casanova and Rossi (1996) method	1	$\delta_{el}+\delta_{pl}$	–
9	Meškėnas *et al.* (2013), $(1/r)_c$ – *const.*	2	δ_{pl}	–
10	Casanova and Rossi (1996) method	2	$\delta_{el}+\delta_{pl}$	–
11	Created; E_cI_{pl} – *curv.*	2	$\delta_{el}+\delta_{pl}+\delta_{sh}$	$B = 1/6 \cdot L_{pl}$
12	Created; E_cI_{pl} – *curv.*	2	$\delta_{el}+\delta_{pl}+\delta_{sh}$	$B = 1/8 \cdot L_{pl}$
13	Created; E_cI_{pl} – *curv.*	2	$\delta_{el}+\delta_{pl}+\delta_{sh}$	$B = 1/100 \cdot L_{pl}$

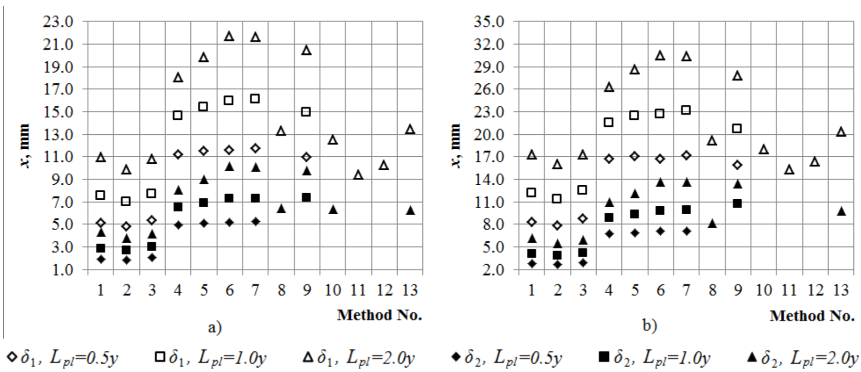


Fig. 28. Compressive zone height x relationship to plastic zone length L_{pl}, deflection δ and the analytical method; a) fibre content: 25 kg/m^3; b) fibre content: 50 kg/m^3; where δ_1 = 0.47mm; δ_2 = 3.02mm

It was deduced during the analysis that optimal plastic zone length L_{pl} depends on the method. The application of a simplified stress diagram is adequate in the range of small deflections. Even slight inaccuracies of residual tensile stresses cause significant errors of SFRC flexural member deflections. Therefore, the application of the developed $f_{Rm,1}$ and V_x calculation methods for plastic hinge methods is limited.

4.2. Deflection calculations of steel fibre and ordinary reinforced concrete flexural members

In order to establish the extent of developed $f_{Rm,1}$ and V_x estimation methods pertaining to deflection calculations of combined reinforced concrete flexural members, extensive comparative analysis was performed. The experiment results presented in Section 2.3 are used in the analysis together with the results of the eight additional beams tested by Dupont (2003). Four beams were reinforced only ordinarily, and their results are presented in Fig. 29.

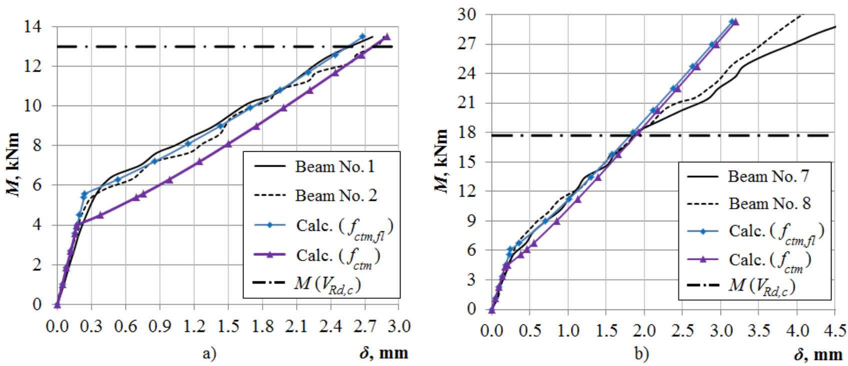


Fig. 29. Experimental and calculated $M-\delta$ curves of ordinary reinforced beams: a) bottom reinforcement: 2ϕ10; b) bottom reinforcement: 2ϕ16

As it can be seen from Fig. 29, more precise results were obtained when $f_{ctm,fl}$ was used. Due to this reason, $f_{ctm,fl,fb}$ was used in all the further calculations of the cracking moment. An unexpected increment of deflection δ is visible when bending moment M reaches the point of calculated shear strength $V_{Rd,c}$ of the concrete beam (without shear reinforcement). Also, span L of the beams was lower than 10h (where h is the beam height). Therefore, it was assumed that this deflection increment is caused by shear deformations and shear cracks which were not considered in the calculations. The similar tendency is visible across all the results of combined reinforced concrete beams where L/h is low.

A part of the analysis results of combined reinforced concrete beams is presented in Fig. 30 and Fig. 31. In the cases where $f_{Rk,1}$ is used, the residual tensile stress was calculated according to Formula (14) while Formula (12) was used in the remaining cases. Notations "$f_{Rm,1}$ – calc." and "$f_{Rk,1}$ – calc." mean that $f_{Rm,1}$ and V_x were calculated by using the developed methods which were discussed in Sections 3.1 and 3.2.

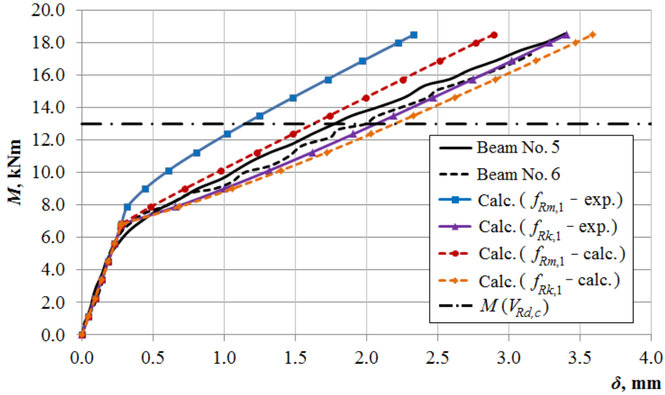


Fig. 30. Experimental and calculated $M-\delta$ curves of beams No. 5 and No. 6 (bottom reinforcement: 2ϕ10; fibre content: 50 kg/m^3)

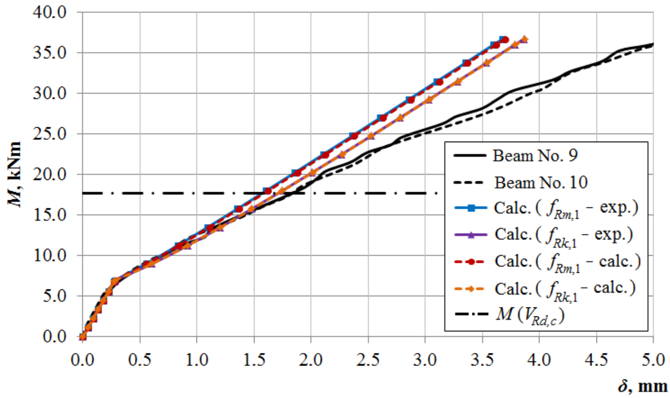


Fig. 31. Experimentally obtained and calculated $M-\delta$ curves of beams No. 9 and No. 10 (bottom reinforcement: 2ϕ16; fibre content: 25 kg/m^3)

It was deduced during the analysis of all the combined reinforced concrete beam results that more reliable and, in most cases, more precise results were obtained when characteristic values $f_{R,1}$ were used. The analysis showed that in the cases where calculated $f_{Rm,1}$ and V_x values were used, inaccuracies of the calculated deflections were not critical. Therefore, it can be considered that the created $f_{Rm,1}$ and V_x calculation methods can be used in deflection calculations of combined reinforced concrete members. Meanwhile, when the simplified stress diagram is used in calculations and when $f_{Rm,1}$ exceeds $f_{ctm,fl,fb}$, the curvature decrease after cracking is obtained. As it can be seen in Fig. 30, the curvature decrease leads to the increase of the bending moment (Blue line – "Calc. ($f_{Rm,1}$ – exp.)"). It is advisable to use a more precise stress diagram or to adjust the calculation method in cases where $f_{R,1} \geq f_{ctm,fl,fb}$.

4.3. Deflection calculations of steel fibre and ordinary reinforced concrete flexural members in cases when $f_{R,1} > f_{ctm,fl,fb}$

In order to eliminate the curvature increment after cracking of combined reinforced concrete flexural members (when $f_{R,1} \geq f_{ctm,fl,fb}$), the curvilinear or bilinear stress diagrams could be used. However, it leads to a complicated calculation process. Therefore, as a simpler alternative, a modification method is proposed where curvature ($1/r$) can be calculated by using the simplified stress diagram in the cracked cross-section (as given in Fig. 4). In this case, the residual tensile stress can be calculated by using modified residual flexural tensile strength $f_{Rm,1,mod}$ instead of $f_{Rm,1}$. An explanatory scheme of the modified residual flexural tensile strength $f_{R,1,mod}$ establishment is given in Fig. 32.

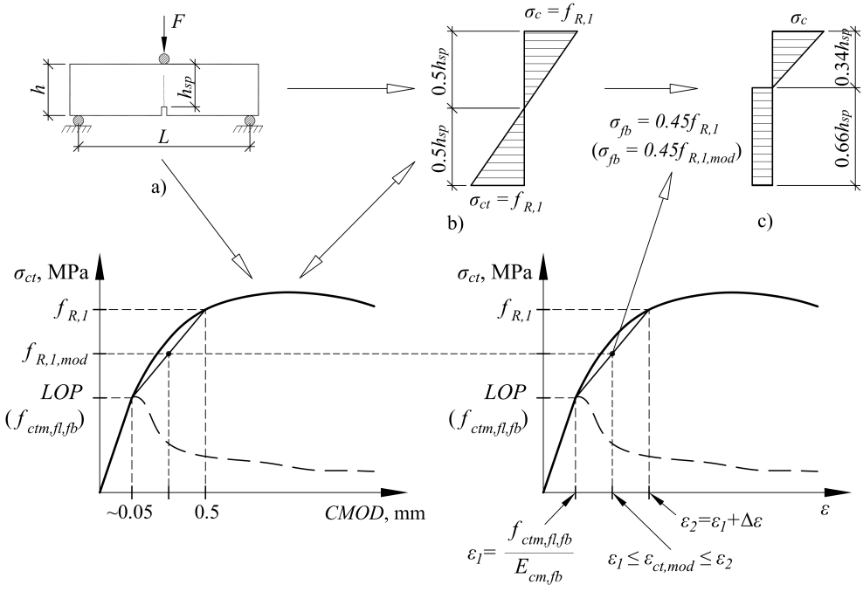


Fig. 32. Modification and application scheme of modified residual flexural tensile strength $f_{R,1,mod}$: a) a three-point bending scheme according to EN 14651+A1:2007; b) stress diagram of $f_{R,1}$ calculation; c) a simplified stress diagram which is used in SLS calculations (RILEM, 2003); d) σ–CMOD relation given from three-point bending tests; e) σ–ε relation which explains the calculation of $f_{R,1,mod}$

The tensile stress which is taken by steel fibres is lower than $f_{R,1}$ while deformations are not sufficient. Therefore, modified residual flexural tensile strength $f_{R,1,mod}$ allows evaluating the residual tensile stress properly, i.e. according to the relevant deformations. The proposed $f_{R,1}$ modification procedure is a simple approach which could easily be used in practical calculations. In order to verify the created method, comparative analysis was performed. A part of its results is submitted in Fig. 33 and Fig. 34.

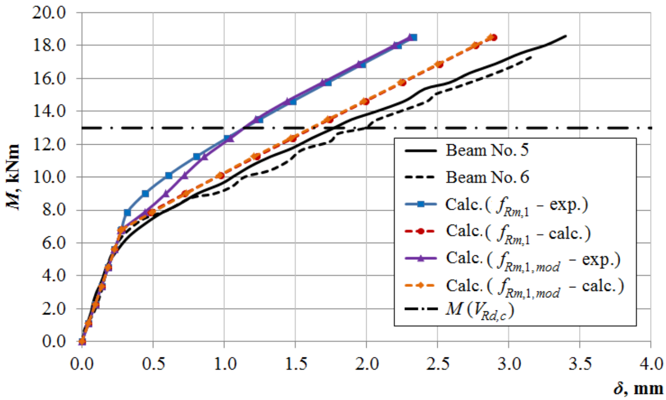


Fig. 33. Experiment-based and calculated $M - \delta$ curves of beams No. 5 and No. 6 when modified $f_{R,1}$ values ($f_{R,1,mod}$) are used (bottom reinforcement: $2\phi10$, fibre content: 50 kg/m^3)

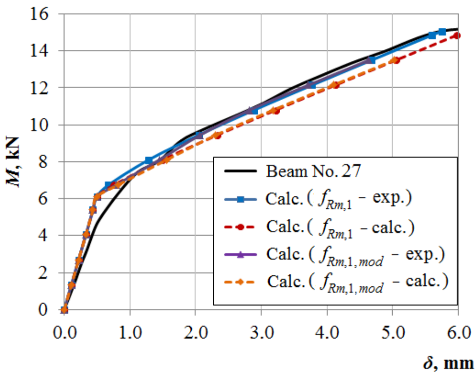


Fig. 34. Experiment-based and calculated $M - \delta$ curves of Dupont (2003) beam No. 27 when modified $f_{R,1}$ values ($f_{R,1,mod}$) are used (bottom reinforcement: $2\phi8$; fibre content: 50 kg/m^3; span: 2.0 m)

Influence of modified residual flexural tensile strength is clearly visible in Fig. 33 (Blue line – "Calc. ($f_{Rm,1}$ – exp.)"). However, in this case, the characteristic value of $f_{R,1}$ was a better fit; therefore, no real improvement was achieved. Whereas, more precise results were obtained for Dupont (2003) beam No. 27 when $f_{R,1,mod}$ was used. Such results were obtained because the $f_{Rm,1}$ value was more accurate in this case than $f_{Rk,1}$.

CONCLUSIONS

1. Compressive and tensile strengths as well as the elasticity modulus of SFRC change insignificantly in the cases of low fibre content ($V_{fb} \leq 1.0$ %). However, the collapse manner of tensioned SFRC still changes from brittle to ductile. The outlined properties of SFRC can be measured by performing tests or can be estimated by using approximate methods proposed by various scientists. The experimentally measured residual flexural tensile strength $f_{R,1}$ is used in the majority of the crack width and deflection calculation methods of combined reinforced concrete flexural members. Normal and log-normal distributions are common in calculations of the characteristic values of residual flexural tensile strength. Calculations differ depending on the method as well as on the assumption whether coefficient of variation V_x is known or unknown.
2. Many analyzed crack width and deflection calculation methods of combined reinforced concrete flexural members are created on the basis of the Eurocode 2 method. Residual tensile stress σ_{fb} is obtained differently depending on the calculation method. The plastic hinge-based methods are used for the crack width and deflection calculations of SFRC flexural members. The plastic hinge curvature change is considered in the analyzed methods; however, the distribution of inner forces is not involved in calculations.
3. $F–CMOD$, $F–\delta$, $F–\varepsilon$, $F–w$ and $M–x$ relations, $f_{R,1}$ values and other parameters of concrete as well as SFRC were measured during the experimental researches. A large number of the tested specimens (469 specimens) was necessary considering the random and uneven steel fibre distribution in concrete. All the results of the extensive experimental program were used in comparative and theoretical researches of the present thesis.
4. Adjustment coefficients k_{pc} and k_P for Naaman's and Sujivorakul's methods were deduced after the analysis of experiment (488 specimens tested by the author of the thesis and other scientists) results. Residual tensile stress σ_{fb} of the hooked end as well as wavy steel fibre-reinforced concrete can be estimated without any additional tests when these coefficients are used. Also, calculation methods for $f_{Rm,1}$ as well as for its variation coefficient V_x have been developed; they are suitable to ordinary and self-compacting hooked end steel fibre-reinforced concrete. Relative errors of σ_{fb} values reduced from 22–23 % to 15 % when the created $f_{Rm,1}$ calculation method was used instead of Sujivorakul's and Naaman's methods together with adjustment coefficients k_{pc} and k_P. Considering the significant scatter of the results, the

reliability of the deduced adjustment coefficients k_{pc} and k_P as well as the developed $f_{Rm,1}$ and V_x calculation methods was verified by using experiment results of 3 independent test series involving 18 specimens.

5. It was deduced during the comparative crack width and deflection analysis that the created $f_{Rm,1}$ and V_x calculation methods had no critical influence on the accuracies of the calculation results. The most reliable results were mostly delivered by SS 812310:2014 method, where average relative error Δw was equal to +1.9 %. The crack width differences of the specific methods were higher than the inaccuracies influenced by relative errors of $f_{R,1}$. The influence of $f_{R,1}$ inaccuracies on the crack width results strongly depended on the fibre reinforcement percentage in the overall reinforcement. More reliable and more precise deflection results in most of the cases were obtained when $f_{Rk,1}$ values were used instead of $f_{Rm,1}$, where the average relative errors were respectively equal to +12.5 % and –16.6 %. More precise cracking moments as well as deflection curves were given by using $f_{ctm,fl}$ ($f_{ctm,fl,fb}$) instead of f_{ctm} ($f_{ctm,fb}$). The discrepancies of the experiment deflection curves could be possibly governed by shear deformations and shear cracks which were not considered in calculations. The present research showed that the application of the developed $f_{Rm,1}$ and V_x calculation methods is possible in crack width and deflection calculations of combined reinforced concrete flexural members, and the methods have a great practical benefit.
6. A new plastic hinge method was devised, in which the stiffness change in the hinge is taken into consideration together with the change of the bending moment along the plastic zone. The method can be easily applied for crack width and deflection calculations of SFRC flexural members in different cases of the structural scheme. A comparative analysis where the 2 % relative error of σ_{fb} led to inaccuracies of deflection in the range of 29...55 % revealed that the precision of residual tensile stress is significant when all the plastic hinge methods are used. Therefore, the application of the developed $f_{Rm,1}$ calculation method is limited to the cases of crack width and deflection calculations of SFRC beams.
7. A simple modification method of $f_{R,1}$ was created; it may be applied in deflection calculations of combine reinforced beams where $f_{R,1} > f_{ctm,fl,fb}$. The curvature decrease after cracking is eliminated when the created method is applied in calculations of σ_{fb}, and the analysis process retains its simplicity.

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RESIUMĖ

Nors plieno plaušu armuoto betono konstrukcijos yra tyrinėjamos jau kelis dešimtmečius, tačiau praktinis plaušo pritaikymas vis dar yra ribotas dėl vieningos skaičiavimo metodikos nebuvimo, didelės plaušo rūšių įvairovės ir atsitiktinio jo pasiskirstymo betone. Siekiant nustatyti plieno plaušo įtaką plaušu ir kombinuotai armuotų konstrukcijų pleišėjimui bei įlinkiui buvo atliktas tyrimas ir jo pagrindu parengta disertacija. Šio tyrimo metu atlikta analizė ir pasiūlytos metodikos turi didelę praktinę vertę.

Disertaciją sudaro 4 skyriai, bendrosios išvados, naudotos literatūros sąrašas ir publikacijų disertacijos tema sąrašas.

Pirmajame skyriuje apžvelgiama mokslinė literatūra, kur nagrinėjamas plieno plaušo pritaikymas konstrukcijose, plaušu armuoto betono savybės, jų nustatymas ir statistinis įvertinimas. Taip pat išanalizuotos metodikos plaušu ir kombinuotai armuotų konstrukcijų plyšių pločiams ir įlinkiams apskaičiuoti.

Antrajame skyriuje pateikiami atliktų eksperimentinių plaušu ir kombinuotai armuotų konstrukcijų tyrimų rezultatai.

Trečiajame skyriuje, remiantis eksperimentų rezultatais, yra nustatyti suderinimo koeficientai Naaman ir Sujivorakul metodams, kurių taikymas leidžia apytiksliai apskaičiuoti liekamuosius tempimo įtempius σ_{fb}. Taip pat sukurti $f_{Rm,1}$ bei V_x apskaičiavimo metodai ir išnagrinėtos jų pritaikymo galimybės kombinuotai armuotų sijų plyšių pločių skaičiavimams.

Ketvirtajame skyriuje sukurtas praktiškas plastinio lanksto metodas, skirtas plaušu armuotų sijų plyšių pločių ir įlinkių apskaičiavimui. Atlikta šio ir kitų plastinio lanksto metodų skaičiavimo rezultatų analizė. Taip pat atlikta kombinuotai armuotų sijų įlinko skaičiavimų analizė bei sukurtas liekamojo tempimo stiprio lenkiant modifikavimo metodas, kurį taikant gaunami tikslesni įlinkio rezultatai, kai $f_{Rm,1} > f_{ctm,fl,fb}$.

Darbo uždaviniai

1. Išanalizuoti plieno plaušu armuoto betono parametrus ir jų nustatymo metodus, kurie taikomi plieno plaušu ir kombinuotai (plieno plaušu ir armatūra) armuotų konstrukcijų plyšio pločio ir įlinkio skaičiavimams. Apžvelgti statistinius metodus, taikomus charakteristinėms medžiagų savybių reikšmėms apskaičiuoti.
2. Išanalizuoti plieno plaušu bei kombinuotai armuotų konstrukcijų plyšio pločio ir įlinkio apskaičiavimo metodikas.
3. Atlikti plieno plaušu bei kombinuotai armuotų lenkiamų gelžbetoninių elementų plyšio pločio ir įlinkio eksperimentinius tyrimus bei eksperimentiškai nustatyti skirtingai plieno plaušu armuotų betonų liekamąjį tempimo stiprį lenkiant $f_{R,1}$.
4. Išanalizuoti Naaman ir Sujivorakul metodų pritaikymo galimybes liekamųjų tempimo įtempių σ_{fb} apskaičiavimui. Kaip tikslesnę ir universalesnę σ_{fb}

apskaičiavimo alternatyvą sukurti vidutinio liekamojo tempimo stiprio lenkiant $f_{Rm,1}$ ir jo variacijos koeficiento V_x apskaičiavimo metodus įprastam ir savaime sutankėjančiam plieno plaušu armuotam betonui.
5. Nustatyti sukurtų $f_{Rm,1}$ ir V_x apskaičiavimo metodų tinkamumą kombinuotai armuotų lenkiamų gelžbetoninių elementų plyšių pločiams ir įlinkiams apskaičiuoti;
6. Sukurti standumo kitimu pagrįstą plastinio lanksto metodą, plieno plaušu armuotų lenkiamų betoninių elementų įlinkiams ir plyšių pločiams apskaičiuoti. Ištirti sukurto $f_{Rm,1}$ apskaičiavimo metodo tinkamumą lenkiamų plaušu armuotų betoninių elementų įlinkio ir plyšio pločio skaičiavimams;
7. Sukurti liekamojo tempimo stiprio lenkiant modifikavimo metodą, taikytiną $f_{R,1} > f_{ctm,fl,fb}$ atvejais, kuris leistų išvengti skaičiuojamojo kombinuotai armuotų sijų kreivio sumažėjimo atsivėrus plyšiui.

Mokslinis naujumas

- Nustatyti Naaman ir Sujivorakul metodikų suderinimo koeficientai, suteikiantys galimybę apytiksliai apskaičiuoti banguotu plieno plaušu ir plaušu lenktais galais armuoto betono liekamuosius tempimo įtempius σ_{fb}.
- Sukurti vidutinio liekamojo tempimo stiprio lenkiant $f_{Rm,1}$ ir jo variacijos koeficiento V_x apskaičiavimo metodai, taikytini plieno plaušu lenktais galais armuotam įprastam ir savaime sutankėjančiam betonui.
- Nustatytas sukurtų $f_{Rm,1}$ ir V_x apskaičiavimo metodų tinkamumas plieno plaušu ir kombinuotai armuotų sijų plyšių pločių ir įlinkių skaičiavimams.
- Sukurtas plastinio lanksto metodas, kuriuo įvertinamas standumo kitimas plaušu armuoto betono sijų plastinėje zonoje.
- Sukurtas nesudėtingas liekamojo tempimo stiprio lenkiant $f_{R,1}$ modifikavimo metodas, leidžiantis išvengti skaičiuojamojo kombinuotai armuotų sijų kreivio sumažėjimo po plyšio atsivėrimo, kai $f_{R,1} > f_{ctm,fl,fb}$.

Tyrimo metodika

Mechaninės betono ir plieno plaušu armuoto betono savybės nustatytos bandymų metodais, apibrėžtais atitinkamuose standartuose. Plaušu armuotų betoninių elementų *CMOD* ir įlinkio δ reikšmės išmatuotos ekstensiometrais, o santykinės deformacijos – tenzometrine įranga. Kombinuotai armuotų sijų įlinkiai matuoti skaitmeniniais poslinkių matuokliais, o plyšių pločiai – skaitmeniniu plyšių pločių matuokliu.

Bandymais nustatyti plyšių pločiai ir įlinkiai palyginti su teorinių skaičiavimų rezultatais. Gniuždymo ir liekamasis plieno plaušu armuoto betono tempimo stipriai apskaičiuoti taikant kitų mokslininkų (Naaman, Sujivorakul ir kt.) sukurtus metodus. Charakteristinės liekamojo tempimo stiprio lenkiant reikšmės apskaičiuotos statistiniais – klasikiniu ir Bayesian metodais.

Suderinimo koeficientai k_P ir k_{pc} nustatyti atlikus palyginamąją eksperimentų ir skaičiavimų rezultatų analizę bei pritaikius statistinius metodus. $f_{Rm,1}$ ir V_x apskaičiavimo metodai sukurti atlikus regresinę ir statistinę eksperimentų rezultatų analizes. Plastinio lanksto metodas sukurtas remiantis energetiniais medžiagų mechanikos principais, atlikus skaitinę analizę programa „Mathcad". Vidinių ir išorinių darbų išraiškos bei siūlomos A ir B koeficientų formulės gautos taikant integravimo metodus. Įtempiai skerspjūviuose apskaičiuoti analitiniu ir iteraciniu (sluoksnių) metodais. $f_{R,1}$ redukavimo metodas sukurtas atlikus skaitinę analizę analitiniu ir iteraciniu metodais.

IŠVADOS

1. Nedideliu plieno plaušo kiekiu ($V_{fb} \leq 1{,}0$ %) armuoto betono gniuždomasis bei tempiamasis stipriai ir tamprumo modulis nežymiai skiriasi nuo įprasto betono. Tačiau net ir toks plaušo kiekis tempiamo betono suirimo pobūdį pakeičia iš trapaus į plastišką. Minėtos plieno plaušu armuoto betono savybės gali būti nustatytos bandymais arba apytiksliai apskaičiuotos pagal skirtingų mokslininkų pasiūlytus metodus. Daugelyje kombinuotai armuotų gelžbetoninių konstrukcijų plyšio pločio ir įlinkio apskaičiavimo metodų yra naudojamas tritaškio lenkimo bandymais nustatytas liekamasis tempimo stipris lenkiant $f_{R,1}$. Charakteristinės liekamojo tempimo stiprio lenkiant reikšmės apskaičiuojamos darant prielaidą, kad $f_{Ri,1}$ pasiskirsto pagal normalųjį arba lognormalųjį skirstinius. Skaičiavimai skiriasi pagal tai, ar variacijos koeficientas V_x yra iš anksto žinomas, ar gautas bandymo metu.
2. Daugelis kombinuotai armuotų lenkiamų gelžbetoninių elementų plyšio pločio ir įlinkio apskaičiavimo metodikų yra pagrįstos *Eurocode 2* pateikta metodika. Skirtingose metodikose nevienodai įvertinami plaušo perimami liekamieji tempimo įtempiai σ_{fb}. Plaušu armuotų lenkiamų betoninių elementų įlinkio ir plyšio pločio apskaičiavimams yra taikomi plastinio lanksto metodai. Visuose nagrinėtuose metoduose yra aprašomas kreivio kitimas plastiniame lankste, tačiau nėra įvertinamas įrąžų pokytis išilgai lanksto.
3. Eksperimentinių tyrimų metu nustatytos bandinių *F–CMOD*, *F–δ*, *F–ε* ir *F–w*, *M–x* priklausomybės, plieno plaušu armuoto betono $f_{R,1}$ reikšmės bei kiti betono ir plaušu armuoto betono parametrai. Reikalingą didelį bandinių kiekį (469 bandiniai) lėmė atsitiktinis ir netolygus plaušo pasiskirstymas bei orientacija. Visi šie išsamių eksperimentinių tyrimų rezultatai naudoti palyginamiesiems ir teoriniams darbe aprašytiems tyrimams.
4. Remiantis eksperimentinių tyrimų rezultatais (488 disertacijos autoriaus ir kitų mokslininkų išbandyti bandiniai), buvo nustatyti Naaman ir Sujivorakul metodų suderinimo koeficientai – k_{pc} ir k_P. Pritaikius siūlomus suderinimo koeficientus, liekamuosius plaušu lenktais galais ir banguotu plieno plaušu armuoto betono tempimo įtempius σ_{fb} apytiksliai galima apskaičiuoti be

bandymų. Taip pat sukurti $f_{Rm,1}$ ir jo variacijos koeficiento V_x apskaičiavimo metodai, skirti plaušu lenktais galais armuotam įprastam ir savaime sutankėjančiam betonui. Taikant sukurtą $f_{Rm,1}$ metodą, vidutinė σ_{fb} paklaida sumažėjo nuo 22–23 % iki 15 %, lyginant su skaičiavimų pagal Naaman ir Sujivorakul metodus rezultatais, gautais naudojant k_{pc} ir k_P koeficientus. Kartu taikant $f_{Rm,1}$ ir V_x apskaičiavimo metodus apytiksliai, be papildomų bandymų, galima apskaičiuoti charakteristines $f_{R,1}$ reikšmes. Atsižvelgiant į didelę skaičiavimo rezultatų sklaidą, k_{pc} ir k_P koeficientų bei $f_{Rm,1}$ ir V_x apskaičiavimo metodų patikimumas patikrintas naudojant 3 papildomų bandymų serijų (18 bandinių) rezultatus.

5. Remiantis palyginamosiomis kombinuotai armuotų sijų plyšių pločių ir įlinkių analizėmis nustatyta, kad pasiūlytų $f_{Rm,1}$ ir V_x apskaičiavimo metodų paklaidos kritinės įtakos plyšio pločių ir įlinkių rezultatams neturėjo. Daugeliu atvejų patikimiausios plyšio pločio reikšmės buvo gautos taikant SS 812310:2014 metodiką, pagal kurią vidutinė plyšių pločių paklaida Δw buvo lygi +1,9 %. Atskiromis metodikomis apskaičiuotų plyšių pločių skirtumai buvo didesni nei skirtumai, kuriems įtakos turėjo $f_{R,1}$ paklaidos. Šių paklaidų įtaką plyšio pločiui nulėmė armavimo plaušu procentinė dalis kombinuotame armavime. Patikimesni ir daugeliu atvejų tikslesni įlinkiai skaičiavimais gauti naudojant $f_{Rk,1}$, o ne $f_{Rm,1}$ reikšmes, kur vidutinės δ paklaidos atitinkamai buvo +12,5 % ir –16,6 %. Tikslesni pleišėjimo momentai ir kartu įlinkių kreivės gautos naudojant $f_{ctm,fl}$ ($f_{ctm,fl,fb}$), o ne f_{ctm} ($f_{ctm,fb}$) reikšmes. Eksperimentinių įlinkio kreivių nukrypimus galimai paveikė šlyties deformacijos ir įstrižieji plyšiai, kurių įtaka skaičiuojant nevertinama. Tyrimas atskleidė, kad sukurtus $f_{Rm,1}$ ir V_x apskaičiavimo metodus galima taikyti kombinuotai armuotų sijų plyšių pločių ir įlinkių skaičiavimuose, o tai turi didelę praktinę naudą.
6. Sukurtas plastinio lanksto metodas, aprašantis standumo kitimą plastiniame lankste, taip įvertinant įrąžų pokytį išilgai lanksto. Šis metodas nesudėtingai gali būti taikomas plieno plaušu armuotų betoninių sijų įlinkių ir plyšių pločių apskaičiavimui įvairių skaičiuojamųjų schemų atvejais. Palyginamoji analizė, kurioje 2 % σ_{fb} paklaida lėmė net 29–55 % įlinkio paklaidas, parodė, kad liekamųjų tempimo įtempių tikslumas yra ypač svarbus taikant visus plastinio lanksto metodus. Todėl sukurto $f_{Rm,1}$ apskaičiavimo metodo pritaikymas plaušu armuotų sijų plyšių pločių ir įlinkių skaičiavimuose yra ribotas.
7. Sukurtas nesudėtingas liekamojo tempimo stiprio lenkiant modifikavimo metodas taikytinas $f_{R,1}$ reikšmėms viršijus $f_{ctm,fl,fb}$. Liekamuosius tempimo įtempius apskaičiuojant sukurtu metodu, yra išvengiama kreivio sumažėjimo po elemento supleišėjimo, o įlinkio skaičiavimai išlieka nesudėtingi.

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Padėka

Autorius dėkoja moksliniam vadovui, KTU Statybos technologijų katedros vedėjui, prof. dr. Mindaugui Daukšiui už pagalbą rengiant disertaciją. Taip pat nuoširdžiai dėkoja konsultantui, KTU Statybinių konstrukcijų katedros vedėjui, doc. dr. Mindaugui Augoniui už rekomendacijas ir visokeriopą pagalbą disertacijos rengimo metu.

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