KAUNAS UNIVERSITY OF TECHNOLOGY

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STRENGTH OF LAYER STRUCTURAL ELEMENTS AND MODELLING OF FRACTURE

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KAUNO TECHNOLOGIJOS UNIVERSITETAS

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INTRODUCTION

Layer structural elements usually are produced by connecting two or more different materials (e.g.: fibber – matrix structures, particles – matrix structures). Strength and elasticity properties of layer structural elements are different from properties of initial materials. In some cases, due to mechanical interaction among different components, those properties of layer structures can be even better than properties of strongest element's material. Therefore layer structures should be considered as a part of larger construction but not as a material of it.

Technological defects are one of the most important reasons when layer structures show worse properties and decompose earlier than expected. During exploitation of those structures, small technological defects can grow into critical. Defects of material's structure (fracture, voids, and weak bonds) are cause of crack's growth while stresses are quite insignificant. Growth of cracks gradually lowers level of integrity and some of components are affected by increased strains. That determinates constant fracture of layer element and in some cases can cause early breakage of whole structure functionality.

High interlayer stresses are typical for spatial stresses state of layer structural elements. They are caused by peculiar anisotropy of such structures. High interlayer stresses cause delaminating in layer structures and fracture zone grows rapidly. That is initial indication of whole structure's fracture. Therefore it is important to determinate interlayer stresses in order to examine possibilities of structural layer element.

Object of the Research:

Strength of layer structural elements with defects and methods of strength evaluation; characteristics of structural element's layers and interlayer zones, influencing strength of whole structure, are investigated in this research.

Cracks propagation in layer structural elements; influence of crack's position direction of interlayer and way of layering on process of fracture; dependence of plastic deformations zone, located next to crack's head, size upon thickness of layer are investigated too.

Influence of remote plastic deformations zones on fracture is analyzed in details.

Goals of the Research

Fracture of layer structural elements depends on fracture of different layers and interlayer zones. Cases of layer structural elements fracture are analysed in this research. Due to materials heterogeneity and technologies of manufacturing, medium layer (interlayer) appears. This layer influences deformation, stresses distribution and fracture of the structure. Often medium layer is weakest link of structural layer element and interlayer fracture called delaminating can appear. Therefore it is aimed:

- Determinate suitability of fraction criterions for evaluation of layer structural element's strength and fracture;
- Propose mathematical model for predicting of layer structural element's fracture;
- Analyse and compare analytical and numerical models of layer structures;
- Propose methodology for analysis of fracture process applying finite elements.

Scientific Novelty

Mathematically model of layer structural element fracture is formulated for a case, when size of plastic deformations zone is similar to thickness of a layer. This model allows calculation of layer structural element's plastic deformations and size of crack's head.

Presented description of development of plastic deformation zones is based on classical laws of fraction mechanics. This description is verified using means of Finite Elements Method (FEM) and experimentally.

Proposed methodology for modelling of fracture process is based on mathematical equations for calculation of plastic deformations zone and numerical finite elements method (FEM). This methodology will have essential meaning for development of new layer structures characterized by better mechanical properties.

Presented for the Defence

There are no universal criterions for evaluation of fracture initiation in layer structural elements. Interlayer of such structures influences field of stress distribution next to crack's head due to rise of stress intensity coefficient. When crack is located in interlayer zone, distribution of stresses in some angles depends on elasticity modules of structure's materials.

Interlayer of layer structural elements influences field of stress distribution next to head of crack, because coefficient of stress intensity increases.

Fracture in layer structural element is affected by mechanical characteristics of inserted layer, interlayer bonding force, thickness of inserted layer, angle of crack's head rising.

Fracture process in layer structural elements are not affected by remote zone of plastic deformations in case of transverse bending, but fracture initiates in inserted layer next to remote zones of plastic deformations in case of pure bending.

Method of finite elements can be used for analysis of layer structural elements with some limitations only.

Original methodology for usage of FEM in evaluation of layer structural element's fracture is composed. It includes mathematical calculation of plastic deformation zones and does not depend on mesh of finite elements.

Structure and Volume of the Dissertation:

Dissertation consists of an introduction, four chapters, conclusions, list of author's publications and list of references. Total volume of dissertation is 111 pages, 80 pictures and 11 tables.

CONTENT OF THE DISSERTATION WORK

1 REVIEW OF LITERATURE

Quantitative criterions for evaluation of tensile as well as elastic fracture of layer structural elements are not perfectly universal in case of combinative load, but they are acceptable for evaluation of designed structure's state and for obtaining of main mechanical characteristics of layer structural element. Differently from metal elements, layer structural elements are heterogeneous and anisotropic. Macrostructure of those elements is designed by bonding one layer on another. Layers can have different mechanical properties.

Layer is assumed as the main structural element in most of engineering calculations of layer structural elements. Layers are characterised by elastic constants (obtained experimentally or using methods of micromechanics), ultimate strength and geometry.

Analysis of recent scientific works, where mechanical behaviour of layer structural elements is discussed, shows that there are no universal criterions for describing beginning of fracture. Criterions of strength and fracture are based classical theories of strength and fracture mechanics.

Fracture can be described using energetic criterion J-integral and conditions of fracture can be described by critical value of the J-integral in evaluation of elastic – plastic fracture behaviour of layer structural elements.

2 DETERMINATION OF LAYER STRUCTURAL ELEMENT'S FRACTURE LAWS

Comparison analysis of non-local fracture criteria

Comparison analysis of non-local fracture criteria is performed by research of two problems (fig. 1 and fig. 2). Analysis is preformed for three types of non-local fracture criteria: average stress fracture criterion, minimum stress fracture criterion and fictitious crack fracture criterion.

Every problem of strength and fracture can be assumed as equality of specific shape function of general non-local force. Criteria of comparison consist of two parameters: typical length and ultimate stress or critical stress intensity factor.

In practice, there are several problems of strength and fracture mechanics that can not be solved using classical conditions of strength and fracture. Some examples of those problems are shown in figures 1 and 2.

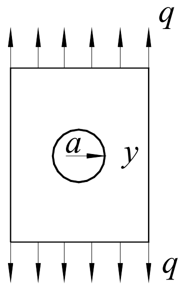


Fig. 1 Plate with hole of radius *a*

Example of small defect effect on strength is shown in fig. 1. Here infinite elastic plate with round hole is loaded by infinite load *q*. It is known form theory of elasticity, that maximum stresses are equal to *3q*, are located in analyzed point *y* and do not depend on radius of hole *a*. Strength of the plate is evaluated by fracture criterion $\sigma_{\theta\theta} = \sigma_c$ and will be equal to one third from σ_c for plate without of crack (continuous line in fig. 1).

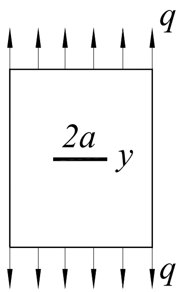


Fig. 2 Plate with crack's length *a*

Another example of small defect effect on strength is plate with *2a* crack (fig. 2). Linear elasticity is characterized by value of the stress intensity factor $K_I(y) = q\sqrt{\pi a}$. From the linear fracture mechanics it is known that every fracture criterion can be given in form $K_I(y) = K_{IC}$.

Those two expressions can be used for theoretical dependence of plate's strength on length of the crack. Strength of the plate is increase till the infinity if length of the crack is decreasing to the zero. However experiments with short cracks show that strength has finite values.

Three non-local criteria for solution of problems in plane

Fracture criteria, based on average stress for characteristic length, can be obtained integrating in direction of θ_0. Then criterion has simplest form:

$$\frac{1}{d_1}\int_0^{d_1}\sigma_{\theta\theta}\left(y+\rho\eta(\theta)\right)d\rho=\sigma_C\,. \tag{1}$$

Here σ_C is strength of specimen without concentrators, when unitary load is applied; d_I is constant of material, related with criterion of length.

The second fracture criterion is based on minimum stress for characteristic length. Minimum stress criteria can be presented in simple form, when direction θ_0 corresponds with maximum of function (1):

$$\min_{0\le\rho\le d_2}\sigma_{\theta\theta}\left(y+\rho\eta(\theta_0)\right)=\sigma_C \tag{2}$$

The third fracture criterion is based on fictitious crack model in characteristic length. It is assumed that fictitious crack exist, has characteristic length d_3 and starts in point of specimen y. After some manipulations and assuming that direction of the crack θ_0 is known, criterion of fictitious crack obtains form:

$$\min\left(K_I(y),K_I\left(y+d_3\eta(\theta_0)\right)\right)=K_{IC} \tag{3}$$

Here K_{IC} and d_3 are constants of material, $K_{II}=K_I(y)$ and $K_{12}=K_I\left(y+d_3\eta(\theta)\right)$ are factors of stress intensity on sides of the fictitious crack, oriented in direction $\eta(\theta)$.

Every of those three criteria involve two parameters: characteristic length d_i and parameter of strength σ_C or K_{IC}.

Analyzing strength of plate with central crack, straight crack of length $2a$ is located in infinite plate (fig. 2). Beginning of a coordinate system (x_1,x_2) is superposed with centre of the crack. If uniform stress σ acts parallel to axis x_2, near head of the crack $\sigma_{22}(x_1,0)$ can be approximated asymptotically and obtains form:

$$\sigma_{22}(x_1,0)=\frac{K_I x_I}{\sqrt{\pi a\left(x_I^2-a^2\right)}}\,. \tag{4}$$

Direction of the maximum tensile stresses corresponds with direction of crack's propagation x and all non-local criteria of fracture can be used in simple expressions (1), (2) and (3). Strength of plate with central straight crack can be written using average stress fracture criterion (1) and expression (4):

$$K_{IC} = \sigma_C \sqrt{\frac{\pi d_1 \eta_1}{1+\eta_1}} \,. \tag{5}$$

Here $\eta_1 = a/(a+d_1)$ is normalized length of crack. Besides, if $q = \sigma_C$, ultimate strength of the plate with crack is reached.

Then relation between normalized strength of plate and length of crack can be obtained from equations (4) and (5).

$$\frac{q}{\sigma_C} = \sqrt{\frac{1-\eta_1}{1+\eta_1}} \tag{6}$$

Equation (6) says that $q/\sigma_C \to 1$ when $\eta_1 \to 0 (a \to 0)$ for a short crack and $q/\sigma_C \to 0$ when $\eta_1 \to 1 (a \to \infty)$ for a long crack.

Strength of plate with central straight crack can be written using minimum stress fracture criterion (2) and equation (4). Equalities of critical intensity of stresses for length of the crack are:

$$K_{IC} = \sigma_C \sqrt{\pi d_2 \eta_2 (1+\eta_2)} \,. \tag{7}$$

Here $\eta_2 = \dfrac{a}{a+d_2}$. Analyzing equations (4), (7) and from condition of ultimate strength (then $\sigma_C = q$) it can be obtained:

$$\frac{q}{\sigma_C} = \sqrt{1-\eta_2^2} \tag{8}$$

Equation (8) also shows that $q/\sigma_C \to 1$ when $\eta_2 \to 0 (a \to 0)$ for a short crack and $q/\sigma_C \to 0$ when $\eta_1 \to 1 (a \to \infty)$ for a long crack.

Analyzing strength of plate with central straight crack by fictitious crack fracture criterion, fictitious crack with length d_3 is formatted next to head of the main crack $2a$. Factor of stress intensity for such formatted crack is $K_I = \sigma \sqrt{\pi \left(a + \dfrac{d_3}{2} \right)}$. Criterion (3) gives $K_{IC}^{\infty} = q \sqrt{\pi \left(a + \dfrac{d_3}{2} \right)}$.

$a = 0$ and $K_{IC}^{\infty} = \sigma_C \sqrt{\pi d_3 / 2}$ for a plate without crack. Rewriting linear prognosis of fracture strength mechanics in normalized form gives equation, which can be compared with other generalized criteria of fracture in the same system of coordinates.

$$\frac{q}{\sigma_C} = \sqrt{\frac{1-\eta_3}{2\eta_3}}\sqrt{1-\frac{q^2}{\sigma_C^2}} \tag{9}$$

Common non-dimensional parameters of crack's length $\eta = a/(a+d_0)$ and stress $\lambda = q/\sigma_C$ are introduced before results in the same coordinates are shown.

Dotted line in fig. 3 corresponds to prognosis of linear fracture mechanics. Non-local criteria give $q/\sigma_C \to 1$ when $\eta \to 0$ for narrow cracks, while criterion of linear fracture mechanics and criterion of fictitious crack give unrealistic prognosis $q/\sigma_C \to \infty$.

In case when $\eta \approx 0.5$, $q/\sigma_C \approx 0.5...0.6$ for fracture criteria of average stress and fictitious crack. Description of this case is incorrect using criterion of minimum stresses and criterion of linear fracture mechanics.

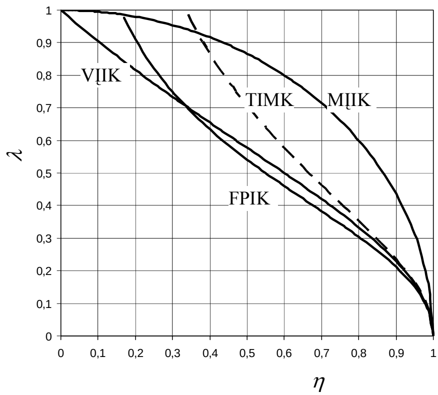


Fig. 3Comparison of fracture criteria for a plate with a central hole

As can be seen, all non-local fracture criteria evaluate cases with long crack's quite good ($q/\sigma_C \to 0$ when $\eta \to 1$).

Summarising it can be said that average stress fracture criterion is more precise for fracture evaluation of the plate with central hole.

Analysing a circular hole of a radius a in an isotropic infinite plate (fig. 1), beginning of coordinate system (x_1, x_2) coincides with centre of the hole. If uniform tensile stress σ acts parallel to axis x_2, then the distribution of the normal stress $\sigma_{22}(x_1, 0)$ along the x_1 axis is given by the expression:

$$\frac{\sigma_{22}(x_1,0)}{\sigma} = 1 + \frac{1}{2}\left(\frac{a}{x_1}\right)^2 + \frac{3}{2}\left(\frac{a}{x_1}\right)^4 . \tag{10}$$

The stress distribution in the non-dimensional coordinates does not dependent on the hole size, and the stress concentration factor at the edge of hole does not dependent on the hole radius too, $\sigma_{22}(a,0) = 3\sigma$. However, the size of stress concentration region depends on the hole's radius.

Due to average stress fracture criterion, strength of a plate with circular hole can be calculated by substituting equation (1) into (10) and performing the integration. Fracture criterion for the most stressed point at the plate with a crack (*a*, 0) is:

$$\frac{q}{\sigma_C} = \frac{2}{(1+\eta_1)(2+\eta_1^2)} \tag{11}$$

For large values of the radius a (when $\eta_1 \to 1$) reduction of plate's strength is caused by the hole and evaluated by factor $\frac{q}{\sigma_C} \to \frac{1}{3}$, while for small values of a (when $\eta_1 \to 0$) no strength reduction is predicted $q/\sigma_C \to 1$.

According to minimum stress fracture criterion, strength of a plate with circular hole can be calculated from equation (2) and (10):

$$\frac{q}{\sigma_C} = \frac{2}{2+\eta_2^2+3\eta_2^4} \tag{12}$$

Equation (12) obtains values $\frac{q}{\sigma_C} \to \frac{1}{3}$ and $\frac{q}{\sigma_C} \to 1$ for big holes $\eta_2 \to 1$ and small holes $\eta_2 \to 0$.

Analyzing strength of plate with circular hole by fictitious crack fracture criterion, fictitious crack with length d_3 is formatted in point (*a*, 0) in direction of x_1 axis. Concentrator of stresses has asymmetric form. According a linear elastic solution of such shape concentrator:

$$\frac{q}{\sigma_C} = \frac{1}{\sqrt{2} f(\eta_3)} . \tag{13}$$

For hole of radius $a \to \infty$, $\eta_3 \to 1$ and if geometric limits exist, analogy for half-plate with crack on it's side and load 3σ can be written

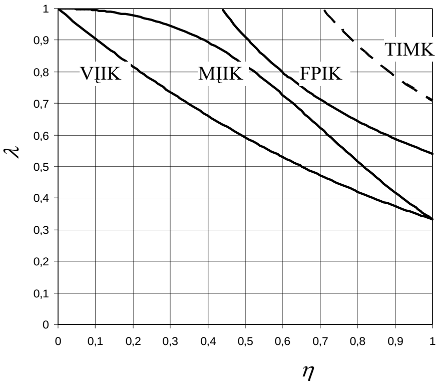


Fig. 4 Comparison of fracture criteria for a plate with circular hole

Comparison of fracture criteria for a plate with circular hole is made using normalization $\lambda = \dfrac{q}{\sigma_C}$ and $\eta = \dfrac{a}{a + d_0}$. Fictitious crack fracture criterion is not exact for bid radius holes.

Results of comparison are presented in figure 4. Average stress fracture criterion and fictitious crack fracture criterion give similar results for plates with symmetrically loaded central crack.

Average stress fracture criterion gives best results for analysed examples. Therefore this criterion will be used in further analysis of layer structural element's strength and fracture.

Propagation of Crack

Propagation of cracks is illustrated by special shape samples composed of two elements glued with polymeric adhesives. Intensity of released energy G_I for certain length of a crack for those specimens can be given as

$$G_I = \frac{F^2}{2B}\left(\frac{24}{EB}\right)\left[\frac{(a + a_0)^2}{h^3} + \frac{1}{3h}\right]. \tag{14}$$

Since values a_0 and $1/3h$ are not big, G_I does not depend on the length of crack a, in condition if proportion a^2/h^3 is constant. Therefore during the experiments, aiming to keep proportion a^2/h^3 constant, shape of specimens should have oblique surface. 7° angle was used (fig. 5) for obtaining constant value of proportion a^2/h^3.

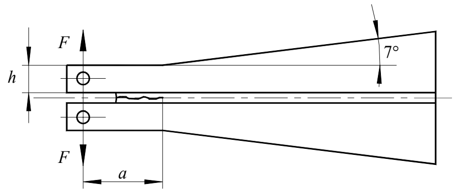


Fig. 5 Layer specimen for analysis of crack's opening

Proportion a^2 / h^3 should remain constant, if length of sample is much larger then height (equation 14). Therefore, aiming to eliminate influence of geometric parameter *h* to the opening of crack, height of the specimen should start increasing from the point where analysis of crack opening is started.

Proportion a^2 / h^3 changes non-linearly, as it is shown in fig. 6. Since producing of such shape surface is technically difficult, it can be simplified, because insignificant deviation of proportion values will not affect result of calculations badly.

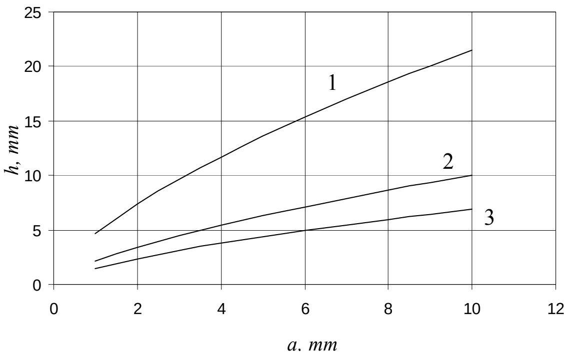


Fig. 6 Dependence geometric parameter *h* on length of crack *a*

Curves 1, 2 and 3, shown in fig. 6, are obtained with different values of proportion, respectively: 1 – $a^2 / h^3 = 0.01$, 2 – $a^2 / h^3 = 0.1$ and 3 – $a^2 / h^3 = 0.3$. After linear approximation and simplifying value of specimen's surface angle is obtained approximately 7°. In this case dependence of parameters G_I and *F*, when crack is located in middle part of the specimen will be:

$$G_I = 12{,}7 \cdot 10^{-6} \cdot F^2 . \qquad (15)$$

Loading layer structural elements with increasing load G_I (work of cracks propagation), marginal value G_{Ic} can be obtained than crack is growing. Speed of crack's growing can be obtained analyzing irregularities of fracture surface in head of crack.

Stopping of a crack usually is unpredictable. If crack stopped, force of crack's propagation G_a can be calculated using value of used load.

Influence of Interlayer Orientation to the Process of Fracture

Three main cases of crack's surface movements, when different loads are applied, are shown in fig. 7. First type of cracks surface movement and fracture of the first type start in isotropic solid with a crack located in plane *Oxz* when surfaces of fracture move away from each other and displacement of surface planes are oriented along axis *y* in reverse directions (fig. 7 a). In case of the second type fracture (fig. 7 b), displacements of fracture planes are oriented along axis *x* perpendicularly to the head of crack and case of shear is obtained. In the third case of fracture (longitudinal shear), planes of fracture move along axis *z* and parallel to the head of crack (fig. 7 c).

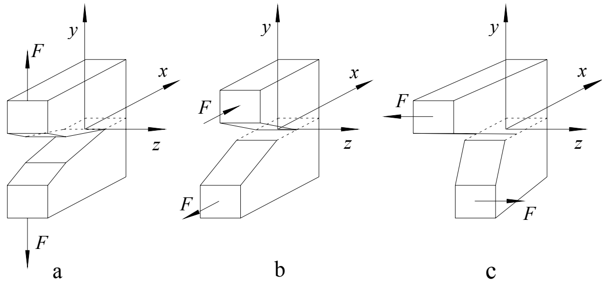


Fig. 7 Cases of fracture and displacements of crack's planes

Lamination of practical structures is possible in all directions. Compact layer specimen can be presented as an example. It is laminated parallel to horizontal, profile and frontal plates. Fracture of specimens will depend not only on direction of lamination, but also on direction of loading forces *F*.

Dependence of fracture process on inserted layer mechanical properties, interlayer bonding force and thickness of inserted layer is analysed in this chapter too.

For the first case of fracture (fig.7 a), analyzed specimens are loaded by concentrated forces parallel to axis *y*.

Fracture type of horizontally laminated specimen is influenced by mechanical properties of inserted layer, thickness of this layer and force of bonding with the main material. If thickness of inserted material is smaller than radius of plastic deformation areas and main material is mechanically stronger, zone of plastic deformation grows even by 20%.

Fracture under load starts next to head of crack in specimen layered parallel to profile plane. Crack changes its direction when reaches inserted material (it grows into two cracks in ideal case). Change of crack's direction is caused by shape of plastic deformations area and by mechanical properties of inserted material.

Case, shown in figure 7 c, is the most complicated. Results of fracture evaluation depend on type of loads. There are two possible ways to apply force. In the first case force is applied to both layers and they are deformed equally. Another case is when force is divided into two components and applied to each layer separately; loading with equal forces is used.

In analysis of **the second type of fracture** (fig. 7 b) specimens are loaded with concentrated forces parallel to axis *x*. In this case fracture of isotropic material is caused by shear and starts in the weakest point of specimen (next to a stress concentrator). Major shear stresses are dominant and direction of specimen lamination does not matter.

Fig. 7 shows typical case of pure shear. Surfaces of crack will slide on each other in plane *Oxz* along direction of load. Fracture does not depend on thickness of layer and can be calculated by classic formulas of the second type of fracture and criterions of second type fracture (K_{II}) can be used.

In case, shown in figure 7, shear appears next to stress concentrator and surface of shear is anisotropic. If thickness of inserted layer is similar to thickness of surrounding layers, influence of inserted layer to fracture process is insufficient and can be neglected in most of engineering calculations.

Results of fracture calculations depend on type of loading for the last case. First loading type is than specimen is deformed equally and second is, than equal forces are loaded to different layers. In the first case fracture starts due to shear in more brittle material and process of fracture starts when values of forces *F* are lover. Alternatively, in the second case, fracture starts in mechanically stronger material and will followed by two shear surfaces perpendicular to each other.

Analysing **the third type of fracture** (fig. 7 c), specimens are loaded by concentrated forces oriented parallel to axis *z*.

For a type of laminating typical case of the third type of fracture can be obtained as subject of inserted layer's thickness. Surfaces of crack will slide on each other in plane *Oxz* in direction of load. Fracture does not depend on thickness of inner layer and can be calculated by classic formulas of the third type of fracture and criterions of third type fracture can be used. Point of stress concentration moves into point *K* as it is shown in fig. 8. Distance till the head of crack, used in K_I equation, can be calculated using this formula: $a' = a - \frac{h}{2} \cdot ctg\frac{\alpha}{2}$.

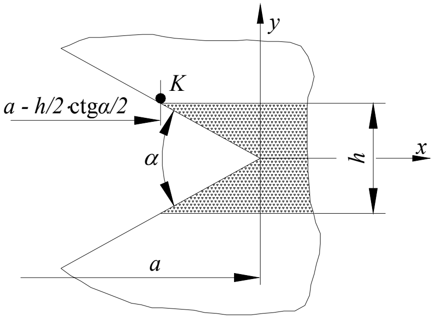


Fig. 8 Development of crack than stronger layer is inserted

Force of interlayer bonding has largest influence to process of fracture in case of laminating. If value bonding force is high and mechanical properties of inserted layer is significantly lower than characteristics of other layers, crack will branch in inner layer (plane *Oyz*) after fracture of layer with a stress concentrator.

In case shown in fig. 7 c, process of fracture will start next to head of crack in layer of more brittle material. Weaker layer will be deformed plastically. In process of crack's growing it will be cut by edge of crack in brittle material.

Still there is no universal strength or fracture criterion capable to describe exactly all types of fracture. Need of such criterion is undoubted, because real structures can be loaded by combinative forces. Prognostication of strength or fracture is even more complicated in this case.

Main Results

1. Non-local criteria of fracture consist of two parameters: typical length and ultimate stress or critical stress intensity factor. Due to those parameters, criteria of fracture are divided into three types: criterion of average stress fracture, minimum stress fracture criterion and fictitious crack fracture criterion. Criteria of average stress fracture describe layer structural elements with defects more exactly.

2. Analysis of layer structural elements crack's propagation showed, that specimen (model) has be prepared in a way that ratio of specimen height and depth of crack should remain constant.

3. All three types of fracture are influenced not only by direction of layering, but also by mechanical properties of inserted layer, force of interlayer bonding, thickness of inserted layer, angle of crack's head.

3 COMPARISON OF ANALYTIC AND NUMERICAL METHODS AND CREATION OF MATHEMATICAL AND FEM MODELS

Experiments and Mathematical Modelling

Most of structural elements are loaded by bending during exploitation. Often constructions are loaded in a way that pure point bending is created. Such type of loading reduces maximum moment of bending and distributes stresses more equally in every cross-section. Layer structural elements, loaded by transversal bending, fracture in same manner as homogeneous elements: crack opens at the concentrator of stresses in the first layer, later it increases through the second layer and so on. Loading layer structure by pure point bending, fracture starts in inner material close to interlayer. In case of strong adhesion, crack increases in inserted material along layers, and in case of weak adhesion, delaminating begins.

Goal of experiments: determinate emerging of secondary areas of plastic deformations in inserted layer in case of pure point bending and analyse influence of those zones on direction of crack's growth.

Methodology of Experiments:

In classical fracture mechanics calculations of homogeneous isotropic materials are limited to calculations of plastic deformation's zones where stresses are higher than ultimate yield limit. It is also known that decrease of stresses is exponential receding from head of a crack. Even in distance from head of crack equal to 5 – 10 values of radius of plastic deformation areas, decrease of stresses is 2 – 3 times and stresses are lower then limit of ultimate yield. Deformations at head of a crack in case of plane state of stresses are calculated:

$$\varepsilon = \frac{K_I}{E\sqrt{2\pi r}} \cos\frac{\Theta}{2} \sqrt{1 + 3\sin^2\frac{\Theta}{2}} \, \frac{1 + \frac{r}{l}}{\sqrt{1 + \frac{r}{2l}}} \, . \qquad (16)$$

In case of plane deformations:

$$\varepsilon = \frac{K_I}{E\sqrt{2\pi r}} \cos\frac{\Theta}{2} \sqrt{(1 - 2\mu)^2 + 3\sin^2\frac{\Theta}{2}} \, \frac{1 + \frac{r}{l}}{\sqrt{1 + \frac{r}{2l}}} \, . \qquad (17)$$

Factor of pressure intensity K_I for beam on two supports in case of pure point bending:

$$K_I = \frac{F(L - L_1)}{t\sqrt{b^3}} Y_4 . \tag{18}$$

Here L is distance between supports, b is height of specimen, t is thickness of specimen, L_1 is distance between supports of medium element.

$$Y_4 = 3.494\left(1 - 3.396\left(\frac{l}{b}\right) + 5.839\left(\frac{l}{b}\right)^2\right). \tag{19}$$

If layer of material characterized by few times lower limit of yield is inserted in distance from head of crack similar to value of radius of plastic deformation areas, secondary zones of plastic deformations emerge. Change of normal stresses in any layer of structural element, travelling from one layer into another, is proportional to proportion of elasticity modules of those layers:

$$\sigma_{i+1} = \sigma_i \frac{E_{i+1}}{E_i} . \tag{20}$$

When load is increasing, stresses reach critical values and process of fracture starts. Crack grows in direction of largest deformations. In same time, secondary zones of plastic deformations emerge in stressed layer. Crack grows perpendicularly to a plane of stressed layer. Head of crack changes its direction when reaches inner layer with already emerged zones of plastic deformations. In ideal case, crack propagates into two cracks, because two zones of ultimate plastic deformations are formatted.

Proceeding of Experiments:

Layer specimens in case of bending collapse similarly to isotropic. Main difference is that layer structures delaminate before they collapse. Fracture of layer structures under bending is not main aim of those experiments, but illustrates differences of fracture due to type of load.

Load in case of transversal bending (force F) is applied on the centre of specimen in reverse side from a cut. Load in case of pure point bending is applied in the same way as in transversal bending, but using symmetric medium element. Here load is divided into two components ($F/2$).

Experiments with First Type of Specimens

In experiments of pure point bending, top support is changed by symmetrically located supports. Distance between centres of supports is 5 cm. Specimen is deformed and delaminating starts between first and second layers. Signs of fracture emerge in the first layer and areas of secondary plastic deformations develop in the second layer. When force reaches critical limit, first layer collapse and appearance of cracks in second layer is caused by areas of secondary plastic deformations. Cracks appear not in a point of initial crack, but in zones of secondary plastic deformations. Comparing stresses on head of a crack and stresses in areas of secondary deformations, located in some distance, it can be noticed that stresses analysed areas are signally lower, however stresses exceed yield stresses in interlayer.

Experiments with Second Type of Specimens

In experiments of pure point bending, top support is changed by symmetrically located supports. Distance between centres of supports is 37 mm. Specimen is deformed and delaminating starts between first and second layers. Areas of secondary plastic deformations develop in the inner layer. Inner layer collapses in areas of secondary deformations perpendicularly to layers. First layer collapses if load is increased and maximum limit is reached. If specimen is even more deformed, second and third layers delaminate.

Analysis of Results:

Forces of bending strength for structure (F), forces of bending strength for inserted element (F_I) and radius of secondary plastic deformation's areas (r) were measured during experiments.

Strength limits in bending for the first type of specimens were obtained: for structure $F = 5222.9 \pm 281$ N; for inserted element $F_I = 4603.5 \pm 216$ N.

Strength limits in bending for the second type of specimens were obtained: for structure $F = 5025.4 \pm 158$ N; for inserted element $F_I = 4559.8 \pm 103$ N.

Strength limits in bending for the third type of specimens $F = 9398.2 \pm 265$ N.

Distance from initial crack till secondary crack in a horizontal direction for the first type of specimens $r_1 = 10.40 \pm 0.45$ mm.

Distance from initial crack till secondary crack in a horizontal direction for the second type of specimens $r_2 = 17.31 \pm 0.46$ mm.

Secondary cracks do not develop for the third type of specimens.

Mathematical Modelling of Experiment

Mathematically fracture process of layer materials can be described using equations 16 – 20. Correction ratio for the first type of specimens is $Y_4 = 1.83$ and for the second type of specimens is $Y_4 = 1.94$.

Ratio of stress intensity K_I for the first type of specimens is $K_I = 7.2 \cdot 10^6$ N/m$^{-3/2}$ and for the second type is $K_I = 16.0 \cdot 10^6$ N/m$^{-3/2}$.

Relative deformation next to head of crack in case of plane state of stresses for the first type of specimens, when $r = 10.4$ mm and angle is 80°, is $\varepsilon = 3.5 \cdot 10^{-4}$. In case of plane deformations: $\varepsilon = 2.6 \cdot 10^{-4}$.

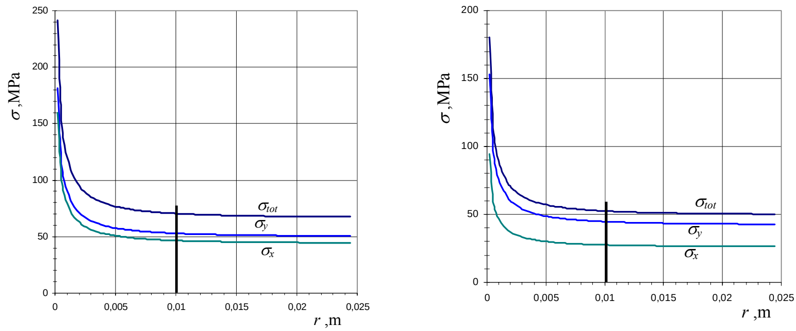
Relative deformation next to head of crack in case of plane state of stresses for the second type of specimens, when $r = 20.8$ mm and angle is 80°, is $\varepsilon = 0.27 \cdot 10^{-4}$. In case of plane deformations: $\varepsilon = 0.20 \cdot 10^{-4}$.

Stresses in inserted element for the first type of specimens are $\sigma = 4.93 \cdot 10^{6}$ Pa $= 4.93$ MPa ≈ 5.00 MPa (5 MPa is yield stress for lead) and for the second type of specimens are $\sigma = 66.75 \cdot 10^{6}$ Pa $= 66.75$ MPa ≈ 65.00 MPa (65 MPa is yield stress for epoxy).

Differences of experimental results do not exceed 5%.

Stresses in distance r from head of crack in inserted layer are similar to yield stress for both types of specimens.

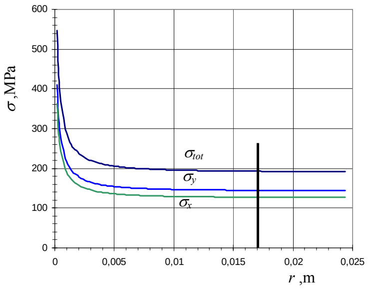
Stresses calculated on head of crack in case of plane state of stresses and in case of plane deformations for the first type of specimens are as shown in figures 9 a and 9 b respectively. Distribution of stresses is shown in case of 80° angle from axis of crack's head y. Results of experiments are presented by thick vertical line.

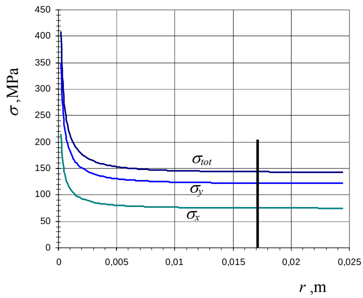


a b

Fig. 9 Distribution of stresses on head of crack in case of plane state of stresses (a) and in case of plane deformations (b)

Stresses calculated on head of crack in case of plane state of stresses and in case of plane deformations for the first type of specimens are as shown in figures 10 a and 10 b respectively. Distribution of stresses is shown in case of 80° angle from axis of crack's head y. Results of experiments are presented by thick vertical line.





a b

Fig. 10 Distribution of stresses on head of crack in case of plane state of stresses (a) and in case of plane deformations (b)

Stresses in distance r from the head of crack in inserted layer, calculated using given expressions, are similar to yield stresses of materials for both types of specimens. Experimental results show concentration of stresses and change of crack's propagation direction in the same areas of specimens. Therefore conclusion that used mathematical model describes fracture and experiments precisely can be maid.

Main Results

Processes of fracture in layer structural elements under transversal and pure point bending starts from phenomenon of delaminating, specific for layer structures only.

In case of pure point bending, layer structural elements with inserted layer of weaker mechanic properties, fracture after delaminating continues in zones of secondary plastic deformations.

Experiments showed that mathematical model of beam with two supports loaded by pure point bending, describes fracture of layer structural elements in case of pure point bending precisely when accepted boundary conditions are used.

4 METHODOLOGY OF FRACTURE PROCESS MODELLING BY METHOD OF FINITE ELEMENTS

Method of Finite Elements and Its Use for Analysis of Structures

Method of finite elements is a universal approximate mathematical method, intended for calculations of differential equations with partial derivatives. This method is realized by standard algorithm which is almost unaffected by contents of analysed problem and is almost the same for problems of different physical systems.

Although method of finite elements has advantages in solving problems of mechanics of structural elements and especially problems of the plastic analysis, problems arise frequently:

• Programs are huge and demand huge resources of computer technical equipment;

• Locations of plastic zones should be indicated previously to calculations.

Results of all numerical methods including method of finite elements are received only for a specific problem. They are not universal and are not focused on daily use, for analytical modelling of objects or processes.

Therefore solving of all problems only numerically is not useful or meaningful. In some cases, analytical methods are more effective and more practical to use, comparing with method of finite elements.

Methodology of Fracture Modelling by FEM

Solving fracture problems by a method of finite elements, input of the primary data does not differ from input of this data, solving problems of strength. A first step is a choice and the description of an element.

The following step is input of mechanical characteristics of material. Points of external loadings and supports (they can be typical points of structure) are appointed for analysed structure.

Model of structure is divided into finite elements. Aiming to receive more exact results of zones of stress concentration, it is necessary to divide them into smaller finite elements.

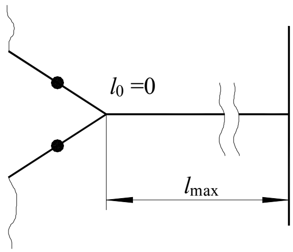


Fig. 11 Model of structure in the first cycle

Usually model of structure is divided into finite elements only once. Then model is loaded by forces and distribution of stresses according accepted criterion can be obtained. Due to suggested technique, model of structure is divided into finite elements many times. It depends on values of loading, mechanical properties of material, other factors. Loop of dividing model into finite elements is used; if condition $l_i = 0.8 l_{\max}$ is not satisfied at the end of every step of cycle, cycle is repeated. Model of structure during the first cycle is shown in figure 11. In the initial moment the structural element is non-loaded. Geometric parameters are unchanged. There are to nodes on the head of crack in the initial moment.

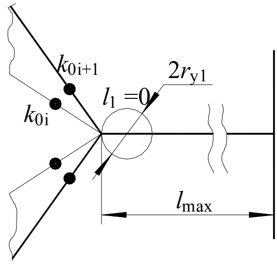


Fig. 12 Model of structure in the second cycle

New model (fig. 12) is designed before second dividing of structure model into finite elements. Coordinates of nodes in this model are already changed (model of structure is deformed). Radius of plastic deformations zone r_{yi} is calculated and added to head of a crack of $i-1$ cycle, before dividing of structure model into finite elements. Thus, head of a crack moves ahead by distance $2r_{yi}$, from position of the last cycle.

When $l_i = 0.8l_{\max}$ (depth of a crack reaches 80 % of the maximum depth of this crack), cycle stops, because fast and unrealistic growth of a crack begins. In next step of cycle, area of plastic deformations $2r_y$ would grow up to value exceeding a remaining part of structure and $2r_y > l_{\max} - l_i$ as it is shown in figure 13.

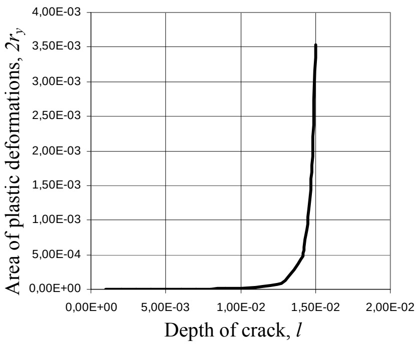


Fig. 13 Growth of radius of plastic deformations zone due to growth of crack's depth

Crack's direction is influenced by angle α, which is determined visually or calculated using specialized techniques between axis x and an axis going trough head of a crack and trough the geometrical centre of zone of ultimate stresses.

Before the beginning of the second cycle, second crack in the size of $2r_y$ is formed. Two new nodes are created: the first node at head of the crack (k_i) received in cycle i–1, the second node at head of the second crack (l_i). Two new lines are created: the first line between node l_{i-1} and l_i, the second line between node l_i and k_i. Model is divided into finite elements after modelling and evaluation of displacement in other nodes of structure.

Distribution of stresses at head of a crack is shown in figure 13. Depth of a crack is increased by $2r_y$, coordinates of nodes k_i and l_{i-1} do not coincide, the maximum stress is at node l_i. Opening of a crack is defined from differences of coordinates of nodes k_i and l_{i-1}. Condition $l_i = 0.8l_{\max}$ is checked before beginning of the third cycle. If $l_i < 0.8l_{\max}$, information for the following cycle is collected.

Opening of a crack between nodes k_i and l_{i-1} has increased. Opening of a crack is recalculated from coordinates of nodes after the third cycle. Thus, opening of a crack or depth of a crack, position of crack's head can be calculated in any cycle.

Main Results

Although the method of finite elements is widely used for modelling of different physical nonlinear problems and processes, but it can be used for the analysis of layer structures only with restrictions.

The main disadvantage of method of finite elements in solving problems of fracture is that position of plastic zones should be predicted before calculations are started.

Methodology for usage of FEM in problems of fracture of layer structural elements is proposed.

CONCLUSIONS

1. Non-local criteria of fracture consist of two parameters: typical length and ultimate stress or factor of critical stress intensity. It is advisable to divide those criteria of fracture into three types: criterion of average stress fracture, criterion of minimum stress fracture and criterion of fictitious crack fracture. It is shown, that criteria of average stress fracture describe layer structural elements with defects more exactly.
2. It is obtained, that strength of layer structural element is influenced by mechanical properties of inserted layer, bonding force of interlayer, thickness of inserted layer and rising angle of crack's head. Angle of crack's head rising should have numerical value of 7 degrees in case of constant ratio between length of a crack and height of a specimen.
3. It is proved, that for a plate with longitudinal crack, crack approaches to interface surface, but does not reach it during the process of fracture. Distribution of stresses next to the head of crack by some angle is calculated considering radius of plastic deformations zone, factor of stress intensity and angle next to the head of crack. Field of deformations should be calculated considering thickness of a plate for a narrow plate in an infinite body with longitudinal crack. Criterion of stress intensity depends on ratio between a height of a plate and length of a crack in case tearing of thin plate with a crack. Herewith factors of tearing and shear stresses intensity should be calculated in case of inserted plate with a crack, located close to a surface of interlayer.
4. In case than crack is located in an interlayer, distribution of stresses depends on material's Young module, bonding force of interlayer, thickness of inserted layer and angle of crack's head rising. Fracture of interlayer will start in more brittle material with higher value of Young module if load is applied perpendicularly; fracture will start in weaker material and will be followed by shear in case than load is applied parallel to the interlayer.
5. Model of layer structural element with stiffly embedded ideal plastic plate is proposed. It shows fracture behaviour of structural elements and gives characteristics of fracture (size of plastic zone and opening of crack) quite precisely. This mathematical model of inserted elastic –plastic layer is validated by model finite elements.
6. It is determined, that in case of transversal bending, remote areas of plastic deformations do not influence fracture processes of layer structural elements. In case of pure point bending, fracture in layer structural element with inner layer of lower mechanical characteristics propagates next to remote areas of plastic deformations (after delaminating occurs). Correctness

of mathematical and FEM model is validated by experimental data of layer beam pure bending.

7. Original methodology for evaluation of layer structural element's fracture is proposed. This methodology combines FEM and analytical calculations of plastic deformation areas and does not depend on the mesh of finite elements.

LIST OF AUTHOR'S PUBLICATIONS

Publications Corresponding to the List of Lithuanian Department of Science and Education

1. Navasaitis, J., Jutas, A., Žiliukas, A., Leišis, V., Mockaitis, J., Žaldarys, G., Meslinas, N. Analysis of Microstructure and Mechanical Properties of Wrought Iron // Materials Science (Medžiagotyra). ISSN 1392–1320. Vol. 9. No. 1. 2003. Kaunas: Technologija.
2. Antanas Žiliukas, Nerijus Meslinas. Model of Fracture with Elastic – Plastic Tile // Mechanics (Mechanika). ISSN 1392–1207. Kaunas: Technologija, 2003, No. 39, p. 11-15.
3. Antanas Žiliukas, Artūras Keršys, Nerijus Meslinas. Estimation Criteria of Elasto-Plastic Behavior of Composites // Mechanics (Mechanika). ISSN 1392–1207: Technologija, 2003, No. 40, p. 21-24.

Proceedings of Lithuanian Conferences

1. Meslinas, N., Žiliukas, A. Sluoksniuotų konstrukcinių elementų irimas // Tarptautinės konferencijos Mechanika – 2000 pranešimų medžiaga. Kaunas: Technologija, 2000. p.420-425.
2. Meslinas, N., Žiliukas, A. Daugiasluoksnių konstrukcijų stiprumo ir irimo kriterijai // Mechaninė technologija : mokslo darbai. 2001. T. 29, p. 78-85
3. Meslinas, N., Žiliukas, A. Plyšio plitimas sluoksniuotame bandinyje su plastišku tarpiniu elementu // Mechaninė technologija: mokslo darbai. 2003.

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REZIUMĖ

Disertacinį darbą sudaro įvadas, keturi skyriai, išvados, autoriaus publikacijų bei naudotos literatūros sąrašai. Darbo apimtis yra 111 puslapių, 80 paveikslų, 11 lentelių.

Pirmame skyriuje yra pateikiama mokslinės literatūros, straipsnių ir kitų šaltinių apžvalga. Aprašomas sluoksniuotų konstrukcinių elementų panaudojimas šiuolaikiniuose inžineriniuose sprendimuose. Pateikiamas tokių konstrukcijų gamybos bei eksploatacijos problemos, apžvelgiami privalumai ir trūkumai. Šiame skyriuje taip pat aptariami sluoksniuotų konstrukcinių elementų tyrimo sunkumai. Aprašomos kelios sluoksniuotų konstrukcinių elementų stiprumo ir irimo vertinimo metodikos bei atliekama sluoksniuotų konstrukcinių elementų analitinių irimo tyrimų metodikų apžvalga. Apžvelgiami šios srities specialistų atlikti darbai. Skyriuje taip pat nustatomi sluoksniuotų konstrukcinių elementų tampriai plastinės elgsenos įvertinimo dėsningumai, analizuojama su įtempių sluoksnio riboje skaičiavimo metodika. Apžvelgiamos pasaulyje naudojamos tiesinės irimo mechanikos ir energinės sluoksniuotų konstrukcijų stiprumo ir irimo vertinimo metodikos.

Antrajame skyriuje sugrupuojami sluoksniuotų konstrukcinių elementų irimo kriterijai ir pateikiama šių apibendrintų irimo kriterijų lyginamoji analizė; analizuojamas plyšio plitimas sluoksniuotuose konstrukciniuose elementuose. Nagrinėjant sluoksniuotų konstrukcinių elementų tarpsluoksninį irimą, sprendžiami: begalinės siauros plokštelės su išilginiu plyšiu, begalinės siauros plokštelės su baigtinio ilgio išilginiu plyšiu, pusiau begalinės plokštelės su plyšiu ties tarpsluoksniu, begalinės plokštelės su baigtinio ilgio plyšio tarpsluoksnyje bei dviejų medžiagų plokštelės su plyšio viršūne tarpsluoksnyje uždaviniai. Būtent tokie uždaviniai su plyšio padėtimi viename iš sluoksnių, tarpsluoksnyje arba statmenai sluoksnių galimi nagrinėjant sluoksniuotus konstrukcinius elementus plokštumoje. Taip pat šiame skyriuje nagrinėjama tarpsluoksnio krypties įtaka irimo procesams, atliekama sluoksniavimo krypties įtakos analizė trims tipiniams irimo atvejams ir irimo priklausomybė nuo įterpiamo sluoksnio mechaninių charakteristikų, tarpsluoksninio ryšio jėgos dydžio, įterpiamo sluoksnio storio, plyšio viršūnės kampo.

Trečiajame skyriuje tęsiama antrame skyriuje nagrinėjamų uždavinių analizė ribinio būvio atveju, kai įterpto sluoksnio storis artėja į nulį, o tai atitinka įtempių pasiskirstymo tarpsluoksnyje bei atsisluoksniavimo atvejus. Šiam atvejui matematiškai ir baigtinių elementų metodais analizuojamas plastinių deformacijų zonų susidarymas plokštelėje su tampriai plastiniu tarpsluoksniu. Iškeliama nutolusių plastinių deformacijų zonų formavimosi hipotezė, kurios idėjai patvirtinti palyginami eksperimentiškai, matematiškai ir baigtinių elementų metodu gauti stiprumo ir irimo duomenys dviem grynuoju lenkimu apkrautoms sluoksniuotoms dviejų atramų sijoms.

Ketvirtajame skyriuje giliau analizuojamos baigtinių elementų metodo taikymo galimybės ir apribojimai sprendžiant irimo uždavinius bei siūloma šių uždavinių sprendimo metodika, apjungianti matematinį ir baigtinių elementų metodus. Aptariami siūlomos metodikos taikymo stiprumo bei irimo skaičiavimuose privalumai bei trūkumai. Pagal rekomenduojamą metodiką atliekami keli irimo modeliavimo žingsniai.

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