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Determination critical stresses of buckling on basis of stresses and geometrical parameters analyses

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1. Introduction

Solving the problem of unstability for constructional elements in case of stress conditions, especially in case of triaxial loading appears to be insufficient. Loads, not stresses, are concerned most of all in unstability solutions. Therefore, physico-mechanical characteristics of certain materials are partly omissed. Critical equivalent stresses appearing in cases of biaxial or triaxial stress conditions are not valued correctly.

Classical solutions obtained by Euler and other scientists in cases of simple loading are applied for loads not exceeding the limit of elasticity. Hutchiston [1] made an important hypothesis to the nonlinear branching theory of structures loaded in the plastic range. The important theory of shear-flexural buckling of columns presented Timoschenko [2]. Galambos [3] introduced the stability criteria for steel structures. Nowadays, the studies [4] pay most attention to avoiding the stability loss in thin-wall constructions using new achievements in engineering. Standards are also intended for this [5, 6]. However, theoretical solutions for stability criteria are insufficient. Thus, this study aims to obtain the criterion of stability loss on the analogy of strength criteria first of all using the probability prognosis.

2. Beams loaded by flexural-torsional buckling

In case of flexural-torsional buckling or lateral buckling (Fig. 1) the unstability equation is recorded as [7]:

$$\left.\begin{aligned} EI_y\frac{d^2u}{dz^2} &= -M_x\phi; \\ GI_s\frac{d\phi}{dz} &= M_x\frac{du}{dx}+M_z, \end{aligned}\right\} \tag{1}$$

where ϕ is angle of torsion; M_x, M_z are moments of bending and torsion about axes x and z; I_y are minimum moment of inertia with respect to axis y; I_s are inertial moment of torsion; G is modulus of shear.

According to Eq. (1) the critical force can be expressed [7]:

$$F_c = \frac{4.0126}{L^2}\sqrt{EI_yGI_s}\,, \tag{2}$$

where F_c is critical unstability force in case of flexural-torsional buckling.

According to Eq. (2) instability appears on influence of two kinds of stresses, i.e. normal (bending) and tangential (torsion).

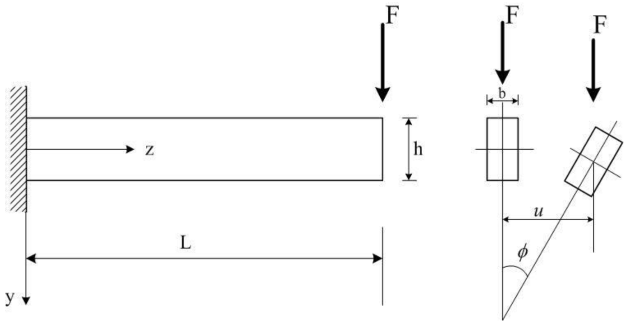


Fig. 1 Contilever beam of rectangular cross-section in bending

Therefore, the case of lateral buckling is the case of complicated loading. Then, stress intensity σ_i is calculated as follows [7]:

$$\sigma_i = \frac{1}{\sqrt{2}}\left[\begin{aligned}&\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\\ &+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\end{aligned}\right]^{\frac{1}{2}}, \tag{3}$$

where σ_x, σ_y, σ_z, are normal stresses in directions x,y,z; τ_{xy}, τ_{yz}, τ_{zx} are tangential stresses in plane surfaces.

In case of torsional bending that makes

$$\sigma_i = \sqrt{\sigma_1^2-\sigma_1\sigma_2+\sigma_2^2}\,, \tag{4}$$

where σ_1, σ_2, are principal stresses.

Thus, the stability and strength as well is evaluated using criteria that depend on normal and tangential influences. These criteria are determined by the fact if the material is brittle or plastic. In order to apply universal criteria, characteristics of determination instability referring to probability analysis must be described.

3. Evaluation of buckling probability in case of complicated stresses

3.1. Accidental events of stability loss

Let us mark the event of unstability A, and the event of stability $\overline{A}$. These two events will be opposite and impossible to happen at the same time. This can be shown in a diagram (Fig. 2). If certain element is under influence of three critical stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$, multitude of events $\{A_i\}$ and $\{\overline{A}_i\}$ in all three directions of stresses (i = 1, 2, 3) using the principle of independence between the effects of certain forces is to be analyzed (Fig. 2).

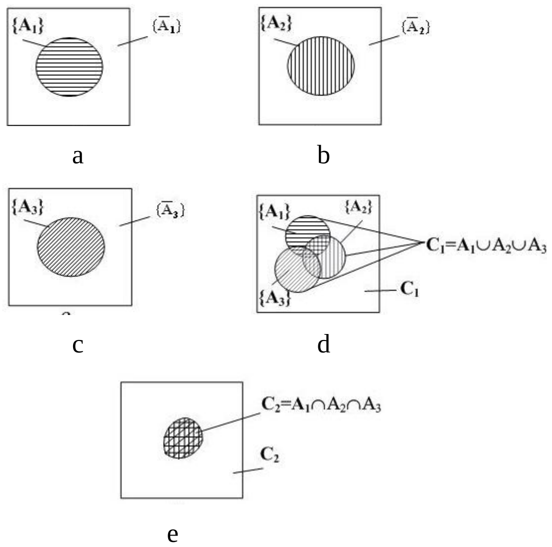


Fig. 2 Diagrams of stability events in case of various events multitude

If only the first critical stress σ_1 influences the events multitudes of $\{A_1\}$ and $\{\overline{A}_1\}$ can be shown in Fig. 2, a. If the events are influenced by σ_2 or σ_3 multitudes $\{A_2\}$ and $\{\overline{A}_2\}$ or $\{A_3\}$ and $\{\overline{A}_3\}$ as well can be shown in Fig. 2, b, c. Areas covered with horizontal lines correspond to multitude $\{A_1\}$, vertical – $\{A_2\}$, diagonal – $\{A_3\}$ shown in Fig. 2, d, represent common area $A_1 \cup A_2 \cup A_3 = C_1$, where instability appears because of one kind of stress σ_1, σ_2, σ_3 or because of all three stresses.

Area covered in Fig. 2, e show the identification of multitudes $\{A_1\}$, $\{A_2\}$, $\{A_3\}$ as $A_1 \cap A_2 \cap A_3 = C_2$ and the fact that all these events are happening at the same time influenced by all three stresses σ_1, σ_2, σ_3.

3.2. Relation between probable instability conditions and critical stress

Let us mark critical stresses for instability σ_{ci}, then main influencing stresses $\sigma_i (i = 1, 2, 3)$ are distributed according to functions $p(\sigma_{ci})$ and $p(\sigma_i)$ (Fig. 3).

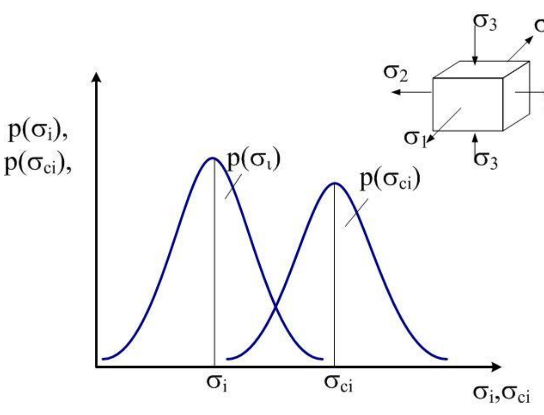


Fig. 3 Connection between the chance of instability and certain stresses

Let us suggest the fact when $P(A_i)$ reaches $P_{ekv}^{(i)}$ the condition of statistical equivalence is met and the probability for instability in case of linear stress condition P equals to equivalent probability P_{ekv} in case of complicated stress condition.

$$P = P_{ekv} = P_{123} = P_e \,, \tag{5}$$

where P_e is experimentally obtained probability for instability in case of complicated stress condition.

Referring to the theory of probability, the probability for instability can be expressed as:

$$P_{123} = 1 - \prod_{i=1}^{3} \left[1 - P(A_i) \right] = P_e \,. \tag{6}$$

Each main stress σ_1, σ_2, σ_3 after the analogy of the theory of probability [7] equals to critical instability stress multiplied by probability $P(A_i)$ and can be expressed as:

$$\left. \begin{aligned} \sigma_1 &= \sigma_{cr} P(A_1); \\ \sigma_2 &= \sigma_{cr} P(A_2); \\ \sigma_3 &= \sigma_{cr} P(A_3). \end{aligned} \right\} \tag{7}$$

Then the Eq. (7) considering the events as independent is can be expressed as:

$$\begin{aligned} P_{123} &= P(A_1) + P(A_2) + P(A_3) - P(A_1 A_2) - \\ &- P(A_2 A_3) - P(A_3 A_1) + P(A_1 A_2 A_3) = P_e. \end{aligned} \tag{8}$$

Considering Eq. (7) and putting the members of Eq. (8) as functions of first stress invariant I_1, second stress invariant I_2 and third stress invariant I_3 we have the following equation:

$$\left. \begin{aligned} & P(A_1) + P(A_2) + P(A_3) = F_1(\sigma_1 + \sigma_2 + \sigma_3) = F_1(I_1); \\ & P(A_1)P(A_2) + P(A_2)P(A_3) + P(A_3)P(A_1) = \\ & = F_2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = F_2(I_2); \\ & P(A_1)P(A_2)P(A_3) = F_2(\sigma_1\sigma_2\sigma_3) = F_3(I_3). \end{aligned} \right\} \tag{9}$$

Thus:

$$P_{123} = F_1(I_1) - F_2(I_2) + F_3(I_3) = P_e. \tag{10}$$

Instability condition:

$$P_{ekv} = P_{123} = P_e \le P_{all} \,, \tag{11}$$

where P_{all} is allowed probability.

Instability condition can be expressed in stresses:

$$\sigma_{ekv} = \sigma_{123} \le \sigma_{c,all} \,, \tag{12}$$

where σ_{ekv} is equivalent stress corresponding to the instability chance P_{ekv} in case of triaxial stress condition; σ_{123} is stress obtained in case of linear stress condition corresponding to the instability chance P_{123}; $\sigma_{c,all}$ is possible instability stress corresponding to normative instability probability P_{all}.

Equivalent σ_{ekv} stresses in case of complicated condition depend on stress invariants I_1, I_2, I_3. As shown in

(9, 10) equational relationship between stresses and instability probability is unlinear.

4. Determination instability stresses according to invariants of stress condition

Certainly [8], in case of complicated stress condition equivalent stresses σ_{ekv} are calculated under criterion:

$$m_a\sigma_i + m_b\sigma_o = \sigma_{ekv}, \tag{13}$$

where σ_i is stress intensity; σ_0 is average stress; m_a, m_b are constant of materials.

However, measuring peculiarities of instability it is clear that critical stresses are distributed nonlinearly. Criterion (13) can be expressed as:

$$m_1(\sigma_i)^2 + m_2(\sigma_o)^2 = \sigma^2{}_c, \tag{14}$$

where σ_c are critical instability stresses.

In case of instability (bending and torsion) that makes:

$$m_1\left(\sigma_1^2 + 3\tau^2\right) + m_2\left(\frac{\sigma_1}{3}\right)^2 = \sigma_c{}^2. \tag{15}$$

Critical cases of deformation are as follows:

1. The force functions in vertical plane.

Critical stress makes yield limit σ_Y, thus $\sigma_1 = \dfrac{M_x}{W_x}$ and $\tau = 0$. Then, Eq. (15) can be changed as follows:

$$\left.\begin{aligned} &m_1\sigma_1^2 + m_2\frac{\sigma_1^2}{9} = \sigma_Y^2,\ \text{if}\ \ \sigma_1 = \sigma_Y;\\ &m_1 + \frac{m_2}{9} = 1. \end{aligned}\right\} \tag{16}$$

2. In case the cross-section is buckled irreversibly and moves horizontally and vertically critical instability stresses depend on the element length and geometrical characteristics of the cross-section. Critical stress F_c in case:

$$\sigma_c = \frac{F_c}{F_Y}\sigma_Y, \tag{17}$$

where F_Y is yield calling force calculated as:

$$F_Y = \frac{\sigma_Y W_x}{L}. \tag{18}$$

Referring to Eq. (2) we can change this as follows:

$$F_c = \frac{D}{L^2}, \tag{19}$$

where $D = 4.0126\sqrt{EI_yGI_s}$ and:

$$\sigma_c = \frac{D}{F_Y L^2}\sigma_Y. \tag{20}$$

Constant D can also be obtained from experimental data i.e. applying Eq. (19). Constant D can be calculated as:

$$D = F_c L^2. \tag{21}$$

Therefore, Eq. (15) can be changed as follows:

$$m_1\left(\sigma_1^2 + 3\tau^2\right) + m_2\left(\frac{\sigma_1}{3}\right)^2 = \left(\frac{D\sigma_Y}{F_Y L^2}\right)^2. \tag{22}$$

Evaluating dependence (16) that makes:

$$\frac{m_1F_Y^2L^4}{D^2}\left(\sigma_1^2 + 3\tau^2\right) + \frac{9(1-m_1)F_Y^2L^4}{D^2}\left(\frac{\sigma_1}{3}\right)^2 = \sigma_Y^2. \tag{23}$$

Thus, after obtaining results of stability loss in single research and calculating constant m_1 is possible to analyze other cases of complicated loading by modelling under Eq. (15).

5. Experimental data

For experiments, two types of steel profiles are chosen, i.e. girded and double-T. Geometric and mechanical characteristics for tested beam profiles is: modulus of elasticity - $2{\cdot}10^5$ MPa ; shear modulus - $8{\cdot}10^4$ MPa; yield limit in bending – 325 MPa; strength limit in bending – 520 MPa .

Chosen profiles were fixed rigidly as shown in Fig. 1 and loaded with force F concentrated on free end. Complicated loading is obtained without bending force adding torsion moment in the cross-section. Beam stability loss is determined measuring beam displacement and twisting angle.

During the experiment, critical bending forces in various torsion moments were measured

While testing the gird 50 × 5 (mm) with length 1.5 m loaded by torsion moment M_{tor} = 10 Nm, stability loss is obtained with bending force F = 205 N, the beam lost its stability with σ_1 = 147.17 MPa and τ = 25.61 MPa. The beam of 2 m length, 100 × 6 (mm) loaded with same torsion moment lost its stability with bending force F = 396 N, the beam lost its stability with σ_1 = 79.2 MPa and τ = 8.66 MPa. Bending force corresponding to yield limit was F_Y = 451 N.

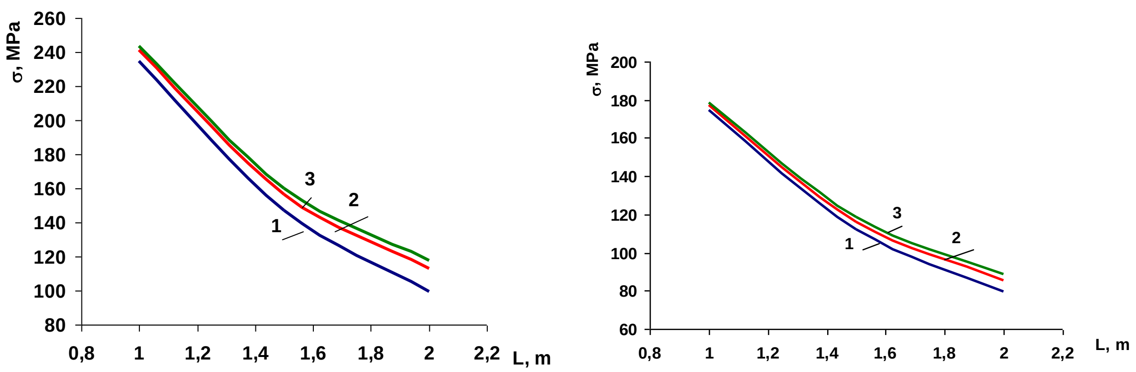
Dependencies of critical stability stresses calculated under formula (23) on length of the beam given in Fig. 4. Results obtained during the experiment presented in Table 1.

Dependencies presented in Fig. 4 show that adding extra torsion moment without bending force decreases critical stability force according to the value of the moment.

Table 1

Experimentally obtained data of steel beams

Profile cross-section	Length, m	Torsion moment M_{tor}, Nm	Critical stress of experimental data:		
			Average, MPa	Standard deviation	Coefficient of variation, %
Steel beam 50 mm × 5 mm	1	10	234.82	3.24	1.38
	1.5		147.17	5.06	3.44
	2		99.84	3.90	3.91
	1	5	240.96	1.76	0.73
	1.5		156.53	4.71	3.01
	2		113.09	2.91	2.57
	1	2.5	244.22	2.70	1.11
	1.5		161.57	3.40	2.10
	2		119.81	3.98	3.32
Steel beam 100 mm × 6 mm	1	10	177.40	3.85	2.17
	1.5		116.70	5.85	5.01
	2		86.00	6.32	7.35
	1	5	178.50	3.04	1.70
	1.5		118.35	3.84	3.24
	2		88.00	5.48	6.22
	1	2.5	179.00	2.92	1.63
	1.5		119.10	3.45	2.90
	2		89.00	5.92	6.65



a b

Fig. 4 Dependencies between critical stability stresses σ_1 (using Eq. (23)) and length for steel beam, with M_{tor} as constant: *1* – 10 Nm; *2* – 5 Nm; *3* – 2.5 Nm; a - steel beam 50 × 5 (mm) ; b - steel beam100 × 6 (mm)

Then, Eq. (23) is presented as:

$$m_1 F_Y^2 L^4 \left(\sigma_1^2 + 3\tau^2\right) + 9\left(1 - m_1\right) F_Y^2 L^4 \left(\frac{\sigma_1}{3}\right)^2 = \\ = \left(k_0 \sqrt{B_1 C}\right)^2 \sigma_Y^2, \tag{24}$$

where k_0 is numerical coefficient depending on ratio $\frac{L^2}{a^2}$ ($a^2 = \frac{Dh^2}{2C}$, D is shelf stiffness in bending) $C = GI_s$ is stiffness of the whole profile in torsion. $B_1 = EI_y$ is stiffness of the whole profile in bending. When $L^2/a^2 > 40$ k_0 obtained from formula: $k_0 = \frac{4.01}{(1 - a/L)^2}$.

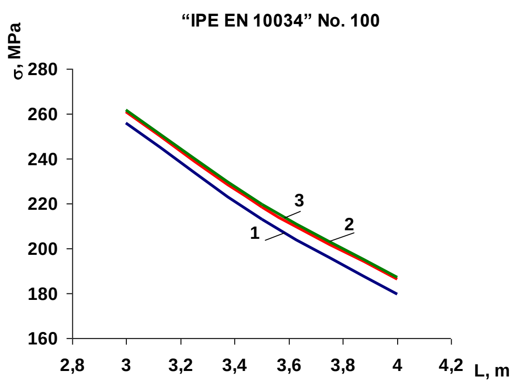


Fig. 5 Dependencies between critical stability stresses σ_1 (using Formula (24)) and length L for double-T profile beam, with M_{tor} as constant: *1* – 50 Nm; *2* – 30 Nm; *3* – 10 Nm

Table 2

Experimentally obtained data of steel beams

Profile cross-section	Length, m	Torsion moment M_{tor}, Nm	Critical stress of experimental data :		
			Average, MPa	Standard deviation	Coefficient of variation, %
Double-T profile „IPE EN10034“ Nr.100	3	10	261.46	4.41	1.69
	3.5		212.47	3.36	1.58
	4		176.52	4.80	2.72
	3	30	260.71	3.16	1.21
	3.5		211.41	3.81	1.80
	4		176.12	4.36	2.47
	3	50	254.66	3.54	1.39
	3.5		205.24	4.20	2.05
	4		168.26	4.73	2.81

While testing double -T profile IPE EN 10034 holder beam of 4 m length and loading torsion moment M_{tor} = 50 Nm, stability loss was found with bending force F = 1670 N, the beam lost its stability with σ_1 = 168.26 MPa and τ = 22.11 MPa.

Dependencies of critical stability stresses calculated under Eq. (24) and length for double-T profiled IPE EN 10034 presented in Fig. 5. Results obtained during the experiment given in Table 2.

After the experiments, calculating constant m_1, that is invariable with constant beam cross-section and $\tau = const$, using Eqs. (23) or (24) various cases of stability loss are to be foreseen and critical instability stresses found in case of rotated bending.

6. Conclusions

1. Complicated stress condition obtained in cases of lateral buckling is evaluated on strength criterion.
2. Probability forecast shows the relation between the stress invariants and the probability of instability.
3. Stability loss can be described by nonlinear strength criterion and geometrical characteristics of the constructional element.
4. Constant values for the strength criterion are included into general equation of stability loss are obtained experimentally.
5. After loading the gird subjected to bending with the moment of torsion, stability loss occurs in case of less bending load.

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DETERMINATION CRITICAL STRESSES OF BUCKLING ON BASIS OF STRESSES AND GEOMETRICAL PARAMETERS ANALYSES

S u m m a r y

This paper suggests the method for evaluating critical stresses of instability in case of complicated deformation based on probability forecast. It is shown that in case of lateral buckling the stress condition is evaluated on criterion of rigidity, and the latter is applied for stability calculations. This is possible because of obtained relation between the probable instability condition and critical stresses, in form of stress invariants. Analytical research is based on experiments, testing simply supported steel girds of small thickness and big height subjected to bending force on free end.

Keywords: buckling, probability, stress, thin-walles construction, torsion, unstability.

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