Kaunas University of Technology

Faculty of Mechanical Engineering and Design

Mathematical Modelling of Launching a Rocket via a Quadcopter

Master's Final Degree Project

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Supervisor

Kaunas, 2024

Kaunas University of Technology
Faculty of Mechanical Engineering and Design

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Masters's Final Degree Project
Aeronautical Engineering (6211EX024)

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Task of the Master's Final Degree Project

Given to the student – Arda Zumrutkaya

1. **Topic of the project**

Mathematical Modelling of Launching a Rocket via a Quadcopter

(In English)

Raketos iš kvadrakopterio paleidimo matematinis modeliavimas

(In Lithuanian)

2. **Aim and tasks of the project**

Aim: to create the mathematical model of a quadcopter during the launching of a rocket.
Tasks:
1. to analyse similar works of literature for quadcopter and rocket kinematics and dynamics, rocket physics, recoil effects, and mathematical model validation methods;
2. to determine the methodology of the work, covering mathematical model development for both pre- and post-launch;
3. to create a mathematical model of quadcopter behaviour before and after missile launch that includes both the quadcopter and the missile;
4. to validate the mathematical model by the dichotomy method;
5. to write the conclusions.

3. **Main requirements and conditions**

The quadcopter and rocket masses are set to be 4 kg and 1 kg respectively. The system is modelled when it is at 50 meters AGL. Normal air conditions without including aerodynamic effects are set for the model. The methodology converges when the $\upsilon_r \approx 59.99$ m/s within the specified bounds, validating the mathematical model. The bounds are set between 50 and 60 m/s.

4. **Additional requirements and conditions for the project, report and appendices**

Not applicable.

Project author	Arda Zumrutkaya *(Name, Surname)*	2023-09-15 *(Date)*
Supervisor	Assoc. Prof. Saulius Japertas *(Name, Surname)*	2023-09-15 *(Date)*
Head of study field programs	Assoc. Prof. Saulius Japertas *(Name, Surname)*	2023-09-15 *(Date)*

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Summary

In this study, a quadcopter's mathematical model during a rocket launch is presented and analysed using MATLAB to explore the quadcopter's behaviour. The validation of the model is conducted through the dichotomy method. This comprehensive mathematical model covers pre- and post-launch scenarios, covering the kinematics, dynamics, and system linearisation. By employing MATLAB and the dichotomy method, the study simulates the quadcopter's behaviour, solving the nonlinear representations of the system.

The research incorporates literature analyses on quadcopter and rocket dynamics, recoil effects, model validation, and methodological aspects, including the problem statement, mathematical model formulation, model validation employing the dichotomy method, and MATLAB implementation. While no direct studies cover the exact focus of this study, literature findings on quadcopter and rocket behaviour offer sufficient information to understand the kinematics and dynamics of the system and comprehend rocket behaviour post-launch.

The mathematical model, formulated based on Newton-Euler equations and Newton's laws of motion, captures pre- and post-launch kinematics and dynamics. It includes equations defining the system's position and orientation in space, linear and angular velocity, total thrust, total moment, torque, rotational and translational acceleration, all linearised and presented in state space form.

The model considers the quadcopter hovering at an altitude of 50 meters AGL during a rocket launch, with the rocket launched in the direction of the positive X-axis, producing a negative displacement of the quadcopter along the same axis and a downward pitch angle of 30°. Dependent variables include the quadcopter's reaction force, velocity, and acceleration, while independent variables involve the quadcopter and rocket mass, system mass, gravitational acceleration, rocket launch altitude, and rocket launch velocities.

Section 5 outlines the principles of the dichotomy method, detailing the setup of initial bounds and a convergence tolerance level. The bounds range between 50 and 60 m/s rocket velocities with increments of 0.5 m/s, with a tolerance level set at one millionth (0.000001). However, the minimum rocket launch velocity is approximately $\upsilon_{0_{rocket\_min}} \approx 55$ m/s, the model tests between 50 and 60 m/s, observing successful convergence of both quadcopter velocity and acceleration equations around a 59.99 m/s rocket launch velocity.

Furthermore, the study explores the quadcopter's behaviour across various rocket launch velocities within a specific timeframe, providing data on velocity, acceleration, position trajectory, total energy, and more. It concludes by noting the absence of assessment regarding stability, control systems, or

trajectory movement for both quadcopter and rocket pre- and post-launch. The methodology employed ensures a systematic validation of the system's mathematical model, with success determined by root convergence for each bound. The generated MATLAB graphs illustrate the relationship between quadcopter velocity and acceleration for specific timeframes across different rocket launch velocities in the post-launch scenario.

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Studijų kryptis ir sritis (studijų krypčių grupė): Aeronautikos inžinerija (E14), Inžinerijos mokslai.

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Kaunas, 2024. 53 p.

Santrauka

Šiame tyrime pateikiamas matematinis kvadrakopterio modelis raketos paleidimo metu ir analizuojama jo elgsena naudojant MATLAB. Modelio patvirtinimas atliekamas taikant dichotomijos metodą. Šis išsamus matematinis modelis aprašo scenarijus prieš raketos paleidimą ir po paleidimo, ir apima kinematiką, dinamiką ir sistemos linearizaciją. Taikant MATLAB ir dichotomijos metodą, tyrimas imituoja kvadrakopterio elgseną, sprendžiant netiesines sistemos reprezentacijas.

Tyrimas apima literatūros analizę apie kvadrakopterių ir raketų dinamiką, atatrankos efektus, matematinių modelių korektiškumo patvirtinimą ir metodologinius aspektus, įskaitant problemos apibrėžimą, matematinio modelio formulavimą, modelio patvirtinimą naudojant dichotomijos metodą ir įgyvendinimą MATLAB programoje. Nors trūksta mokslinių tyrimų apie raketos iš kvadrakopterio paleidimo matematinių modeliavimų, literatūros išvados apie kvadrakopterių ir raketų elgesį suteikia pakankamai informacijos, kad galima būtų suprasti sistemos kinematiką ir dinamiką bei raketų elgesį po paleidimo.

Siūlomas matematinis modelis remisi Niutono-Eulerio lygtimis ir Niutono judėjimo dėsniais, fiksuoja kinematiką ir dinamiką prieš ir po raketos paleidimo. Šį modelį sudaro lygtys, apibrėžiančios sistemos padėtį ir orientaciją erdvėje, tiesinį ir kampinį greitį, bendrą trauką, bendrą momentą, sukimo momentą, sukimosi ir transliacijos pagreitį; visos lygtys yra tiesinės ir pateikiamas sistemos būsenos erdvės forma.

Sukurtas matematinis modelis buvo patikrintas darant prielaidas, kad raketos paleidimo metu kvadrakopteris sklando 50 metrų AGL aukštyje, o raketa paleidžiama teigiamos X ašies kryptimi, todėl keturkopteris pasislenka neigiama kryptimi išilgai tos pačios ašies ir 30° nuolydžio kampu žemyn. Priklausomi kintamieji apima kvadrakopterio reakcijos jėgą, greitį ir pagreitį, o nepriklausomi kintamieji apima kvadrakopterio ir raketos mases, sistemos masę, gravitacinį pagreitį, raketos paleidimo aukštį ir raketos paleidimo greitį.

Dichotomijos metodo principai yra aptariami 5 skyriuje. Šis metodas reikalauja nustatyti pradinį intervalą ir konvergencijos toleranciją. Intervalai nustatyti nuo 50 iki 60 m/s raketos greičių su 0,5 m/s žingsniu, o tolerancijos lygis siekia vieną milijoninę dalį (0,000001). Pavyko apskaičiuoti mažiausią raketos paleidimo greitį, kuris apytiksliai yra $v_{0_{rocket\_min}} \approx 55$ m/s. Modelio testavimas atliktas nuo 50 iki 60 m/s su 0,5 m/s padidėjimu, siekiant nustatyti dichotomijos metodo sėkmę. Kvadrakopterio greičio ir pagreičio lygtys konvergavo sėkmingai į maždaug 59,99 m/s raketos paleidimo greitį.

Be to, tyrime tiriamas kvadrakopterio elgsena įvairiais raketų paleidimo greičiais per tam tikrą laikotarpį, pateikiant duomenis apie greitį, pagreitį, padėties trajektoriją, bendrą energiją ir kt.

Tyrime pažymima, kad nebuvo išnagrinėtas kvadrakopterio ir raketos stabilumas, valdymo sistemų ar trajektorijos judėjimas tiek prieš, tiek po raketos paleidimo. Metodika siūlo sistemingą požiūrį į matematinio modelio teisingumo validavimą, o tyrimo sėkmė vertinama pagal kiekvienos ribos šaknies konvergenciją, kuri skaičiuojama kiekvienam intervalui. Sukurti MATLAB grafikai iliustruoja ryšį tarp kvdarakopterio greičio ir pagreičio tam tikru laikotarpiu, esant skirtingiems raketų paleidimo greičiams po jų paleidimo.

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List of abbreviations and terms

Abbreviations:

AA – air-to-air;

AGL – above ground level;

Assoc. Prof. – associate professor;

CAD – computer-aided drawing;

COM – centre of mass;

DLSRR – drone-launched short-range rocket;

DOF – degrees of freedom;

Eq. – equation;

Lect. – lecturer;

LQ – linear quadratic;

LQG – linear quadratic Gaussian;

MSL – mean sea level;

No. – number;

PD – proportional-derivative;

PID – proportional-integral-derivative;

RTSA – reconnaissance, surveillance, and target acquisition;

UAV – unmanned aerial vehicle.

Introduction

Armed UAVs are becoming more dominant in modern combat [23,27,41]. These types of drones have attracted attention for their ability to conduct surveillance and launch attacks, granting them an advantage in modern combat.

Recent developments in armed UAVs have sparked discussions around the idea that it may no longer be necessary to put soldiers in harm's way, as UAVs can accomplish the same mission. UAVs have transformed modern combat and are a significant reason why countries continue to invest in this type of technology. Although this technology is not a new concept [21,34,39], UAVs have experienced significant technological improvements in recent years, making them even more valuable tools for military and law enforcement operations.

Because of their potential and rapidly advancing technology, the rising industry of UAVs has garnered a lot of attention in recent years. What sets UAVs apart from other systems is their capability to carry out operations without the need for close or onboard human involvement.

UAVs can be classified according to their features and capabilities. These classifications are:

- range and endurance;
- size;
- weight;
- degree of autonomy;
- altitude; or
- a combination of these factors [47].

Table 1. UAV classification according to the US Department of Defence [47]

Category	**Size**	**Maximum gross take-off weight, lbs**	**Normal operating altitude, ft**	**Airspeed, knots**
Group 1	Small	0-20	<1,200 AGL	<100
Group 2	Medium	21-55	<3,500 AGL	<250
Group 3	Large	<1,320	<18,000 MSL	<250
Group 4	Larger	>1,320	<18,000 MSL	Any airspeed
Group 5	Largest	>1,320	>18,000 MSL	Any airspeed

Furthermore, the applications of UAVs in military and law enforcement include:

- reconnaissance, surveillance, and target acquisition;
- close air support;
- search and rescue;
- cargo deployment;
- firefighting; and
- other various operations.

"A quadcopter, also known as a quadrocopter, or quadrotor is a type of helicopter or multicopter equipped with four propellers" defined by [19]. This means that a quadcopter is a multi-helicopter driven by four rotors, positioned at the end of the symmetrical structure of the quadcopter in a square formation and distributed equally from the quadcopter's COM. The propellers of the quadcopters are employed with narrow chord airfoils to create a lift.

These rotors, which are positioned at the end of each arm of the quadcopter, allow a quadcopter to hover without the need for forward airspeed. This unique feature enables them to maintain their position over a specific point in the sky. Moreover, quadcopters are highly manoeuvrable and can quickly change direction, making them an excellent choice for utilisation in modern combat scenarios. Their manoeuvrability makes them well-suited for RTSA and attacking enemy targets.

Employing a quadcopter with a rocket launcher offers numerous advantages over using a helicopter for missions. Due to quadcopter's smaller size and reduced noise output, they are less noticeable than their conventional counterparts. Furthermore, their ability to operate in confined spaces makes them a practical choice for tactical combat scenarios.

Considering the current circumstances, delving into the depths of this study is important to gain a more comprehensive understanding of its aim and purpose.

The aim of this work is to create the mathematical model of a quadcopter during the launching of a rocket.

There have been no published on this subject, therefore, it is crucial to thoroughly analyse any potential irregularities of quadcopter's behaviour during the launch of a rocket and reach a clear and accurate understanding of the matter at hand.

This is achieved through the following tasks:
1. to analyse similar works of literature for quadcopter and rocket kinematics and dynamics, rocket physics, recoil effects, and mathematical model validation methods;
2. to determine the methodology of the work, covering mathematical model development for both pre- and post-launch;
3. to create a mathematical model of quadcopter behaviour before and after missile launch that includes both the quadcopter and the missile;
4. to validate the mathematical model by the dichotomy method;
5. to write the conclusions.

It is important to note that this study does not assess the stability, control systems, or trajectory movements of either the quadcopter or rocket in the pre- and post-launch environments.

1. Literature review

The study of a quadcopter's dynamic mathematical modelling forms the foundation for understanding the complexity of launching rockets from a quadcopter. While there are no studies specifically on launching a rocket from a quadcopter, considerable research exists on related topics, such as quadcopter dynamics, and rocket physics. These studies explain the fundamental features of mathematical modelling of quadcopter dynamics as well as recoil effects acting on the quadcopter while launching a rocket from it.

The further development of [14], [1] delves into a more detailed discussion on the modelling and control of the quadcopter. The literature specifically focuses on quadcopter altitude and attitude control. To achieve this, the literature derives the mathematical model from [14]. With this model, the authors are able to determine the best control methods for the quadcopter. The PD and PID control systems are discussed in this literature and compared for their effectiveness in controlling the quadcopter's altitude and attitude. Overall, the literature provides a comprehensive analysis of the modelling as well as quadcopter control systems. Although quadcopter control systems are not examined in the current study, this literature provides highly useful mathematical modelling and altitude and attitude examination.

[2] presents a complete analysis of structure movement characteristics in the event of launching a laser-guided short-range air defence rocket, analysing the impact of recoil, and resulting dumpings. To verify the structural model, a numerical simulation using MATLAB is conducted and the literature provides the vibrational characteristics of the system. Furthermore, the effects of a missile launch from a vehicle are presented. Overall, the literature provides a detailed insight into the recoil influence on a structure.

[3] presents a mathematical model of a quadcopter and examines its behaviour under different configurations. The model presented in this literature is defined with a coordinate system of the quadcopter and its kinematics and dynamics using the Newton-Euler method while taking into account the dynamics of the rigid 6 DOF body. The literature provides a coordinate system that presents the inertial body of the quadcopter, and the body coordinate system, which has its origin at the centre of the quadcopter. This hybrid coordinate system in the literature simplifies the model and enables the derivation of the quadcopter's dynamic model with respect to its coordinate. The derived equations are then implemented in a simulation to observe the behaviour of the quadcopter mathematical model in different conditions.

The authors highlight that due to the complexity of the structural model and the insignificance of prediction performance, some effects are not included in the model derivation Despite these facts, the literature provides a comprehensive mathematical model for quadcopter behaviour and showcases its applicability through simulation and graphical representation.

[4] presents the application of dynamic motion to analyse weapon and armament systems. The literature provides a depiction of a motion simulation that is conducted to study the movement of a shooter's shoulder when launching a weapon. Furthermore, the literature provides a detailed analysis of graphs that illustrate velocity versus damping coefficients for two specific shock absorbers, examined in the literature. This literature aims to create a model that could help in determining the ideal damper characteristics based on various ammunition parameters and weapon configurations.

Overall, this literature holds a fundamental understanding of the dynamic motion of a shoulder-fired weapon and the effects are studied to evaluate the post-launch rocket and quadcopter behaviours.

[5] presents challenges of obtaining continuous-time models for a small-size quadcopter's dynamics through a predictor-based subspace identification method. The control structures considered in this literature are the inputs and outputs of a standard quadcopter control architecture. Furthermore, the literature focuses on a continuous-time algorithm that utilises Laguerre filters as an identification procedure for deriving the necessary models. Although the literature does not further discuss the control structure of a standard quadcopter control, it emphasizes the continuous-time model work for small-scale quadcopter dynamics.

[6] presents a quadcopter mathematical model, considered as a second-order linear system. While the literature explicitly discusses the kinematics and dynamics of a quadcopter, it provides insight into two different methods that can be used to derive its dynamics: the Lagrangian equation and Newton's laws of motion. However, the literature emphasises the control and stabilization of the quadcopter rather than its detailed kinematics and dynamics. As a result, the literature is carefully analysed for the current study to understand the kinematics and dynamics of the quadcopter.

[7] presents modelling and control aspects of mini quadcopters. The literature primarily focuses on the design and control of a quadcopter and provides model analysis, considering the quadcopter's dynamics and aerodynamical variations that arise due to its motion. The literature also examines the control and stability analysis, which are essential for manoeuvring a quadcopter safely and efficiently in the air. Overall, the literature provides a detailed and comprehensive analysis of mini quadcopter modelling and control, however, the current study does not cover the control of a quadcopter, the modelling in this literature is studied to analyse the fundamentals of the quadcopter's dynamics.

[8] presents a comprehensive survey on the mathematical modelling of quadcopters, providing the identification of parameters using the models outlined in the survey. It encompasses various models of quadcopters, including nonlinear models with respect to their body and inertial frames. The discussed models encompass a range of parameters such as length of arm, sum of mass, inertial matrix, coefficient of friction, thrust, and drag. The literature highlights that the parameters can be obtained experimentally, by calculation, or through a combination of both methods.

Moreover, the literature discusses two different approaches to describe the dynamics of a quadcopter: Tait-Bryan angles and quaternions. It explains the advantages of using quaternions in terms of efficiency and singularity-free representation, which can significantly enhance the overall performance of the quadcopters. This literature offers a comprehensive overview of the mathematical modelling of quadcopters, beneficial for the current study.

Dynamic inversion is employed in [9] to develop a controller ensuring stability and tracking for the quadcopter, considering its roll-pitch-yaw dynamics. While the literature does not extensively delve into the quadcopter's kinematics and dynamics, it provides sufficient insight to comprehend the model structure for analysis in the current study.

In the field of quadcopters, two notable models demonstrate their dynamics. The first model uses the Euler-Lagrange method to derive translational dynamics, while the second model employs the Newton-Euler method for rotational dynamics. The critical difference between these models lies in their utilisation of rotational matrices, extensively studied through simulations. The authors conclude that while both models generate identical results for angular motions and elevation, they exhibit opposing behaviours in translational movements. This literature has been carefully examined to enhance the understanding of translational and rotational matrices in the current study [10].

[11] introduces the usage of the dichotomy (bisection) method. This methodology provides a numerical method for solving differential equations employing one variable. The literature describes an iterative process on how to determine the single parameter minimising root mean square error. This combination results in a highly accurate and efficient algorithm for solving differential equations.

In addition to the theoretical analysis of the dichotomy method, this literature provides a flow chart illustrating the steps involved in utilizing the method for root estimation. Overall, this literature offers a valuable resource for understanding the dichotomy method, which is a powerful approach to solving differential equations employing one variable.

[12] constructs a mathematical model for the quadcopter and presents a CAD model to estimate its physical properties such as mass and inertial properties. The literature utilises the 6 DOF quadcopter and incorporates the Newton-Euler approach for this purpose. However, it should be noted that a major limitation of this literature for the current study is its lack of comprehensive coverage of the quadcopter’s dynamics.

[13] presents parameters significantly impacting the performance of the recoil system in the tank gun. The mathematical model is thoroughly developed, providing a recoil cycle for each launching. This enables theoretical simulation, prediction, and evaluation of the recoil system’s performance using MATLAB/Simulink. The model in this literature offers a comprehensive understanding of the complexities of the recoil system, crucial for optimizing the system.

Additionally, the theoretical data is compared with actual data obtained from launching a tank gun in practice. This comparison enables a comprehensive analysis of both theoretical and real data, displaying a reliable match.

Moreover, the literature focuses on analysing the force along the recoil distance, concluding the importance of spring stiffness. This analysis is critical in understanding the behaviour of the recoil system and optimizing its performance. Overall, this literature provides valuable insights into the complexities of the recoil system, studied for implementation into the recoil of rockets in the current study.

[14] presents a quadcopter mathematical model, encompassing both its kinematics and dynamics. The quadcopter’s dynamic mathematical model is developed by considering thrust, moment, and the inertial matrix. The literature employs the Newton-Euler formulation to derive these elements. Before utilizing the quadcopter’s kinematics and dynamics, the literature introduces the axes of the inertial and body frame of the system. This approach aids in simulating the study to further analyse the quadcopter’s motion in space. Overall, this literature provides a detailed understanding of the quadcopter’s behaviour, studied in the current research to analyse and develop the quadcopter's state of motion.

[15] presents the underlying theory of flight systems of guided rockets, involving nonlinear motion equations, modelling dynamics of aerodynamics, actuator, and measurement. This literature aims to

linearize a nonlinear system dynamic model to assist in framing stability and flight control design. Overall, this literature covers aspects of rocket motion that contribute to describing the rocket's motion in the current study.

[16] presents a detailed approach to achieve a comprehensive understanding of the kinematics and dynamics of moving objects. The author thoroughly illustrates the fundamental principles of kinematics and dynamics, including Newton's laws of motion. This book showcases the author's deep understanding of the subject fundamentals and provides structural knowledge that can be employed to better understand the dynamics of quadcopters in motion. Overall, the book is a valuable source in the field of kinematics and dynamics, offering a solid foundation for understanding quadcopter dynamic structures.

[18] presents a dynamical model for a four-rotor VTOL quadcopter named X4-Flyer. The model considers the dynamics of the frame and motor, including the system aerodynamics and the rotational effects on the model. This literature also examines flight conditions, involving decoupling rigid body dynamics from motor dynamics and developing a backstepping separate control design for the coupled system.

Moreover, the literature covers calculations and validations ensuring the definiteness of certain matrices and bounds. The authors note that although these theoretical bounds can be loosened in real scenarios, the study highlights the importance of choosing appropriate values for certain parameters to ensure the positive definiteness of the incorporated matrices.

Overall, the literature provides a comprehensive analysis of the dynamic model proposed for the X4-Flyer, considering various factors affecting the system's performance. The fundamentals of this model are analysed to establish the current study.

[19] presents the complex issues of hovering a quadcopter and examines three key aerodynamic effects influencing a quadcopter's performance: its velocity, angle of attack, and frame design. The literature presents theoretical developments validated using trust test stand measurements and flight tests with the STARMAC quadcopter. It also covers modelling dynamics during the climb, including power and thrust calculations. This literature concludes that theoretical data exhibits a high degree of accuracy in relation to the experimental data, enhancing controller performance. This literature is analysed to understand the dynamics of the STARMAC quadcopter.

[20] presents an investigation of the angular velocity of a quadcopter and its correlation with the rates of change. The literature provides an explicit representation of the angular velocity, describing it in terms of the rates of change and the frame's moment of inertia.

The Lagrange-d'Alembert equations of the quadcopter's rotation and angular velocities are expressed in elemental form to delve deeper into its dynamics. Additionally, the literature utilises the conservation of linear and angular moments in a Newtonian setup to offer a better understanding of the quadcopter's dynamics.

Furthermore, the literature delves into Lie Group Theory and Differential Topology to develop a more advanced mathematical model of the quadcopter. This approach provides a more comprehensive understanding of the quadcopter's motion and behaviour under different conditions.

[22] presents an evaluation of an AA missile for guidance purposes. This guide is based on the target's location and velocity in relation to a guided object. The literature thoroughly examines a mathematical model and design system of a missile, focusing on its development phase. However, it's important to note that the study primarily focuses on the mathematical model evaluation of the missile. It doesn’t provide a comprehensive analysis of the mathematical methods necessary to ensure the feasibility and compatibility of the missile with system objectives. Nonetheless, it establishes a useful approach for explaining rocket physics, which is necessary to understand the behaviour of the rocket in the current study.

[24] presents a valuable resource for comprehending the frames of reference used in defining coordinate systems and frames for the quadcopter’s kinematics and dynamics. The body frame is a fixed reference frame linked to the physical structure of a quadcopter, while the inertial frame is fixed in relation to the Earth’s surface. Analysing the quadcopter's kinematics and dynamics using both the Euler-Lagrange and Newton-Euler methods, explores translational and rotational components that collectively represent the motion subsystem of the quadcopter.

While the study emphasizes the importance of quadcopter kinematics and dynamics, its primary focus remains on analyses of the control and stability. Overall, this literature on quadcopters provides a comprehensive overview of kinematics and dynamics, which are sufficient to comprehend the current study.

Effective modelling of the quadcopter’s kinematics and dynamics demands careful precision. Euler angles, involving three translational and three rotational axes, typically express rotations in 6 DOF. While this study primarily focuses on modelling quadcopter dynamics, it doesn't cover other aerodynamic effects, requiring additional experiments for comprehensive analysis. The literature linearises the nonlinear model specifically for hovering, employing a first-order Taylor expansion to approximate the nonlinear system. It offers sufficient information and understanding of the quadcopter's mathematical model for the current study [25].

[26] presents a study on the degrees of freedom in the modelling and simulation of quadcopters across various flight conditions. The authors developed a state space model encompassing longitudinal, lateral, and vertical operations, explaining quadcopter behaviour under diverse flight conditions. To ensure model accuracy, the literature employs the von Karman model for wind disturbances during flight, considering it more effective than the Dryden approach for wind gust modelling.

Additionally, the study creates a CAD model in SolidWorks and simulates the quadcopter using MATLAB/Simulink. These methods enable the testing and validation of the state space model's accuracy in depicting quadcopter behaviour under varying flight conditions. While offering insights into developing more efficient quadcopters for diverse applications, the current study doesn't cover such cases. However, this literature’s mathematical model is analysed for potential implementation in the current study.

[28] presents a comprehensive model utilising Euler-Newton and Lagrangian approaches to formulate the quadcopter's body dynamics. The model offers a highly nonlinear representation of the quadcopter’s system. However, the authors prioritize linear control methodologies, such as LQ and LQG methodologies, over exploring the system's kinematics and dynamics. While the control methodologies aren't studied in the current study, this literature provides a deep understanding of rigid body dynamics.

[29] presents a detailed mathematical model that explains the quadcopter dynamics. The model is constructed on Newton-Euler and Euler-Lagrange equations, which are used to study the linear and angular composition of the system independently. The literature also takes the drag force, which is generated by air resistance, into account to make the model concrete on a more realistic basis.

The literature highlights the importance of constructing a prototype of a quadcopter to accomplish more reliable and realistic outcomes. Overall, this literature provides invaluable insight into the complex structure of the dynamics of a quadcopter and lays a basis for the current study.

[30] presents the functions of dynamic simulation and experimental verification of short recoil-operated weapons with a vertical sliding wedge breechblock. The literature discusses the method for setting up the motion equation for the automatic launching system, in which the Lagrange equation of the second kind is utilised, and the model is compared with the experimental data. It provides the calculation steps and results of the dynamics calculation, which are then compared with the experimental data. Overall, this literature presents a comprehensive mathematical model and experimental verification for short recoil-operated weapons, highlighting the challenges and limitations in achieving accurate predictions.

[31] presents a guide to the subject of mathematical modelling, estimation, and control for quadcopter UAVs, emphasizing their manoeuvrability and the ability to move in 6 DOF. The literature further delves into the dynamic equations of the quadcopter's thrust, moment, and aerodynamics to present a detailed understanding of the underlying principles governing these systems. Moreover, it employs the linearisation of the quadcopter's dynamics to derive a linear controller. Additionally, it discusses the use of derivatives of the outputs, providing insight into calculating velocity, acceleration, and other dynamic variables essential to comprehend the dynamics of the studied system. Overall, this literature provides a comprehensive and detailed overview of the current study.

[32] presents a detailed design and analysis of an active rocket launcher intended for use in an attack helicopter. The literature focuses on the launcher's dynamic model and controller design, aimed at achieving accurate launch angles and reducing the workload of the pilot. Furthermore, it introduces drag and thrust disturbances to the rocket launcher. This literature covers various aspects of the launcher's design and analysis, including the mathematical models describing the launcher's dynamics, the control algorithms regulating the launcher's motion, and simulations testing and validating the launcher's performance. Overall, the literature provides comprehensive detail in the design and analysis of an active rocket launcher tailored to the needs of an attack helicopter. The system used in the literature is examined thoroughly, considering the dynamics of the rocket launcher studied in the current study.

[33] provides a comprehensive nonlinear dynamic model development of a quadcopter equipped with four rotors. The literature describes the model as a complex nonlinear system with differential equations in state space form, aiding in understanding the underlying dynamics of the quadcopter. The literature then describes the process of linearising the system around an equilibrium point to ensure the simplification of the nonlinear model.

Furthermore, the literature highlights the importance of controlling the stability and robustness of the system, which are crucial factors in ensuring the safe and efficient operation of the quadcopter. It provides a comprehensive analysis of these factors, emphasising the challenges and complexities involved in designing and implementing an effective control system for the quadcopter.

However, in the current study, the control, stability, and analysis of the robustness of the quadcopter and rocket are not included. A nonlinear dynamic model for a quadcopter equipped with four rotors is an essential aspect to consider in the current study.

[35] presents a mathematical model validation of a system. The literature introduces some fundamental elements of model validation to provide examples of planning, experiments, and validation comparisons. It emphasises that model validation must be carried out during the process of the model validation, rather than the actual validation criteria. The literature emphasises the importance of comparing results obtained from the model to experimentally obtained response measures to ensure the validity of the studied model. This process helps researchers to evaluate the accuracy of their models and make necessary modifications to improve their model's effectiveness. Proper model validation is important to confirm that a mathematical model can be relied upon to make predictions and decisions with confidence.

[36] presents the complexities of quadcopter dynamics and provides a quadcopter mathematical model. The primary focus of this literature is to determine the model parameters for the quadcopter and solve differential equations of the model. The values obtained through calculations are significant to ensure the desired aerodynamic properties and control of a quadcopter. This literature emphasises the importance of accurate calculation of parameters to ensure the optimal performance of the quadcopter. Overall, the literature presents a detailed analysis of quadcopter dynamics and provides valuable insights into the mathematical modelling of such systems.

[38] presents a dynamic model of an X4-Flyer quadcopter weighing four kilograms with one kilogram of payload. The authors of this literature highlight that while simple quadrotor dynamic models are a good starting point, they do not capture the complexity of behaviour exhibited by quadcopters in the real world. To elaborate, a quadcopter in the real world experiences blade flapping effects and variable propeller inflow velocities, which have never been discussed in simple models.

The authors argue that the development of quadcopter's dynamic mathematical modelling behaviour is essential for good control and better design analysis. The literature suggests that future studies should consider the flapping dynamics of quadcopters to create more accurate models. However, this literature does not cover aspects of the current study. It is worth noting the propeller's behaviour in the real world.

[40] presents the complexity of modelling a quadcopter, delving into the various challenges to accurately represent its dynamics. The quadcopter in this literature is characterised as an underactuated aircraft featuring fixed four-pitch angle rotors. The model is utilised using a simplified Lagrange equation, allowing for a detailed study of the quadcopter's simple dynamic features. The literature examines the use of rotors in the quadcopter to produce longitudinal, lateral, and yaw moments, along with thrust, moments, and inertia moments concerning the quadcopter's features. Furthermore, this literature further explores the design of a PID control system for the quadcopter. However, the control system is not part of the current study; this literature is used to deeply investigate quadcopter dynamics.

[42] presents a concept of DLSRRs released from a UAV at up to 20 km height and 700 m/s velocity. The literature examines theoretical and thermodynamic analysis, utilising various MATLAB codes to calculate the parameters and trajectories of the DLSRRs. Although this literature does not cover

the trajectory movement of both quadcopter and rocket in the post-launch environment, it explains the fundamentals of rocket physics.

[43] presents a comprehensive physical and mathematical model of an accelerator-equipped short recoil-operated firearm. The proposed model takes into account the recoil process stages of the system, enabling the simulation and evaluation of how different design parameters affect recoil velocities. The mathematical model is obtained using Eulerian integration and simulated in MATLAB. The literature provides a detailed and accurate representation of the firearm's action which helps to evaluate recoil force and stages acting on both quadcopter and rocket launcher in the current study.

[44] presents the quadcopter's kinematics utilising a mathematical model that considers variables such as velocity and its vector in the body frame. However, the quadcopter's dynamics are not examined explicitly in this literature. The primary focus of the study is on the development of a control algorithm and a self-stabilising control system for the quadcopter. Although the control algorithm and self-stabilising control system are not discussed in the current study, the fundamentals of kinematics through a mathematical model of the quadcopter are covered.

[45] presents a linear model for a quadcopter and examines it in 3 DOF and 6 DOF state space models utilising the basics of Newtonian setup to gain a better understanding of the quadcopter's dynamics. The literature emphasises the importance of state space models by simplifying the complexity of linearisation. This literature helps to understand the state space formation of quadcopter, examined in the current study.

[46] presents a quadcopter model equipped with four rotors and implements a dynamical model for this system without including the aerodynamic effects. The model provides the inertia of a quadcopter frame in roll, pitch, and yaw which are represented in a 3 by 3 matrix. It is essential to analyse the system's stability, especially concerning attitude stabilisation. The model's primary focus is on the stabilisation of a quadcopter in a defined attitude, and this is analysed using the Lyapunov function, a critical function for analysing the stability of dynamical systems. The Lyapunov function is utilised to study the system's properties by analysing the conservation of the energy of the system and its stability. In this case, the function is used to analyse the stability of a quadcopter in a defined attitude and to develop a control system that ensures the quadcopter's stability during flight. Although further development in the literature is done mainly in the control and stability of the quadcopter, it covers a comprehensive dynamical analysis of the quadcopter.

[48] presents a quadcopter's kinematic and dynamic mathematical model. The literature employs a PID controller and simulates the model using MATLAB to evaluate the performance of the designed model. Concluding the effectiveness of the simulation, the literature illustrates the efficacy of the model and ensures that the model meets the desired specifications.

[49] presents a focus on developing a quadcopter dynamic mathematical model to describe the behaviour of the quadcopter using the Newton-Euler method. The authors delve into the relationship between all variables involved and explain the kinematics of the system. They determine the position and orientation vectors as the elements of the inertial frame. The authors implement a Cartesian coordinate system and Euler angles to present these vectors. Additionally, they use rotational and transformation matrices to identify the correlation between linear and angular velocity, as these are the body frame elements.

Furthermore, the authors study the quadcopter dynamics and explain the relation between body forces, torques, and moments produced by each rotor. They then transform this fundamental state of the system into linear and nonlinear equations, which are essential for the system's control and further development. The quadcopter dynamics described in the literature are extremely nonlinear and closely coupled. As a result, in order to simplify the mathematical model and decouple its dynamics, the authors linearize the system around an equilibrium point.

The authors consider the translational equation of motion established from Newton's second law of motion to characterize the model's dynamics, which takes into account the quadcopter's total forces acting on its body. These fundamental features of the literature satisfy the vital aspects of the current study. Overall, this literature provides a comprehensive mathematical model for quadcopter behaviour and demonstrates its applicability through simulation and graphical representation.

[50] presents a comprehensive and detailed dynamical model of a small quadcopter. The dynamic model takes into account various variables, including position coordinates, angles, and moments of inertia. The literature further examines the stability and control method of a quadcopter, which is simulated using MATLAB/Simulink to model, simulate, and analyse the system dynamics. Through the simulation, the literature provides an analytical understanding of the behaviour of the quadcopter under various conditions, making the quadcopter possible to optimise its performance and control.

In conclusion, while there is a direct study on rocket launchers from a quadcopter is lacking, prior studies on quadcopter's kinematics and dynamics, as well as rocket physics, rocket motion, recoil studies, and related topics provide enough information to establish a solid base for the current study. Analysing findings from reviewed literature is critical in developing a comprehensive mathematical model to analyse the effects of a rocket launching on a quadcopter.

2. Methodology

This study aims to mathematically model a quadcopter in the event of launching a rocket from it. The methodology employed is constructed around the system's equations applying Newton-Euler equations and Newton's laws of motion and interpreting results to confirm the model using the dichotomy method.

Problem statement and objectives: This study centres on validating the mathematical model of a quadcopter describing its behaviour during a rocket launch. The primary objective of this study is to validate the accuracy of the developed model by employing the dichotomy method and interpreting the rocket's mass, acceleration, velocity, and other system characteristics to find the methodology's root values.

Mathematical model formulation: The model relies on Newton-Euler and Newton's laws of motion methods to derive its kinematic and dynamic equations, encompassing the quadcopter's motion. These equations determine the system's position, orientation, linear and angular velocities, thrust, torques, and accelerations in each axis, considering forces, moments, and inertial parameters.

Model validation with dichotomy method: The study assesses the quadcopter's behaviour by utilizing the dichotomy method in MATLAB. The code simulates the system's behaviour by solving nonlinear equations and generating graphs for analysis.

MATLAB implementation: The mathematical model is implemented into MATLAB, utilizing the dichotomy method to determine the quadcopter's velocities and accelerations under different rocket launch velocities. The code simulates the system's behaviour by solving nonlinear equations and generating graphs for analysis.

Analysis and interpretation: Generated graphs illustrate the relationship between the quadcopter's velocity and acceleration with varying rocket launch velocities. Interpretation involves analysing the plots for root approximation, interval contraction, convergence rate, and error reduction rate of the quadcopter's behaviour post-launch.

Model validation: The study's success is determined based on the root convergence calculated for each bound, confirming the model's validity. Conclusions drawn from the analysis assess the mathematical model's accuracy and its capability to simulate the quadcopter's behaviour during a rocket launch.

Conclusions: This methodology systematically validates the correctness of the mathematical model by employing methods throughout model creation and analysis. This section aims to verify the model's reliability in predicting the quadcopter's behaviour in the post-launch environment.

3. Mathematical model of the system

The system’s mathematical model offers a well-organized representation using mathematical equations, variables, and principles to describe the behaviour of a real-world or conceptual system. These models depict essential system characteristics, involving variables that represent its condition or parameters influencing system dynamics, often simplifying relationships into manageable representations.

Mathematical models come in various forms such as analytical, numerical, or empirical and can be found in applications across diverse fields such as engineering, economics, and biology. They help in predicting system behaviour, optimizing designs, and understanding complex system relationships. However, developing an accurate mathematical model that accurately captures real-world complexities is a challenging task and requires validation for reliability.

This study divides its modelling approach into two distinct sections. Subsection 3.1 examines the mathematical model of the pre-launch environment, treating the quadcopter and rocket as a unified structure, referred to as a system. It involves all factors affecting both the quadcopter and rocket prior to launch, considering their weight, linear and angular velocities, forces, torques, moments, etc. In this section, the aim is to create a comprehensive mathematical model that accurately predicts the system's behaviour pre-launch.

In contrast, Subsection 3.2 focuses on the mathematical model of the post-launch environment. Here, the quadcopter and rocket are observed as separate objects after launching. The goal is to construct a model accurately predicting the quadcopter's behaviour post-launch, accounting for factors influencing its behaviour such as direction, reaction forces, velocities, accelerations due to the launch's reaction, etc., along the X-axis. This study considers launching a rocket from the quadcopter on the positive X-axis. The objective is to create a comprehensive model predicting the quadcopter's behaviour in the post-launch environment.

3.1. Pre-launch mathematical model

The system framework maintains a perfectly balanced and rigid structure, established within a Cartesian coordinate system where the equilibrium point aligns with the COM of the system. The diagram below, obtained from [26], illustrates the system’s orientation in both inertial (shown on the left) and body frame (shown on the right) frames, axes, and their corresponding Euler angles.

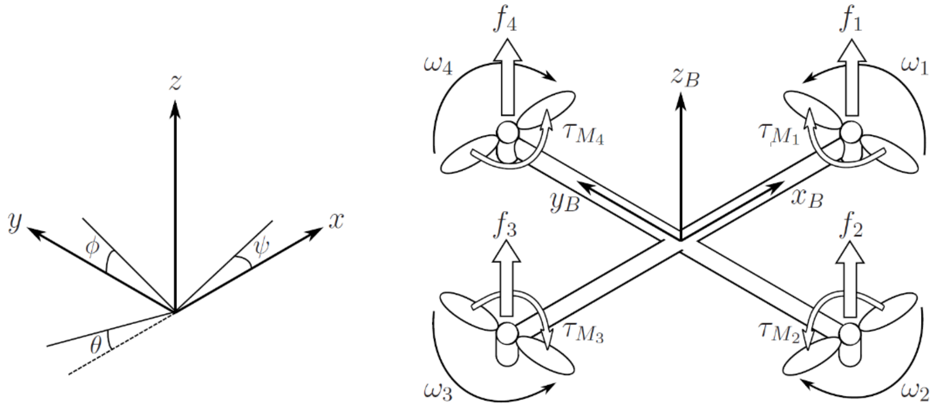

Fig. 1. The illustration of the system’s inertial and body frame respectively [26]

3.1.1. The system kinematics

The kinematics of a system refers to a fundamental aspect of mechanics that focuses on studying motion, excluding the forces that initiate it. In quadcopters, specific variables such as position, orientation, and linear and angular velocities play critical roles, significantly influencing the system's flight characteristics and performance.

3.1.1.1. The position

Position determination requires establishing two frames of reference: the inertial frame represented as E_I and the body frame represented as E_B. The inertial frame provides a fixed point of reference, describing the system's position and orientation as $E_I = [x_G \quad y_G \quad z_G]^T$. Conversely, the body frame is directly fixed to COM represented as $E_B = [x_B \quad y_B \quad z_B]^T$.

3.1.1.2. The position vector (ξ)

A vector is used to represent the position of the system as an element of the inertial frame:

$$\xi = [x \quad y \quad z]^T \in E_I \tag{1}$$

If: ξ – the position vector of the system being an element of the inertial frame; x, y, z – represent the longitudinal, lateral, and vertical position of the system respectively.

3.1.1.3. The orientation vector (η)

A vector is used to represent the orientation of the system using Euler angles as an element of the inertial frame:

$$\eta = [\phi \quad \theta \quad \psi]^T \in E_I \tag{2}$$

If: η – the orientation vector of the system is an element of the inertial frame; ϕ, θ, ψ – represent the system's roll, pitch, and yaw angles respectively.

The rotation matrix (R) from the body frame to the inertial frame using Euler angles is represented in Eq. (3).

$$R = R_z(\psi) \times R_y(\theta) \times R_x(\phi) \tag{3}$$

If: $R_z(\psi)$, $R_y(\theta)$, $R_x(\phi)$ – rotation matrices around the x, y, and z axes with angles of ϕ, θ, and ψ respectively.

3.1.1.4. The linear velocity (υ)

A vector is used to represent the linear velocity of the system as an element of the body frame:

$$\upsilon = [u \quad v \quad w]^T \in E_B \tag{4}$$

If: υ – the linear velocity of the system being an element of the body frame; u, v, w – the linear velocity along the x, y, and z axes respectively.

3.1.1.5. The angular velocity (ω)

A vector is used to represent the angular velocity of the system as an element of the body frame:

$$\omega = [p \quad q \quad r]^T \in E_B \tag{5}$$

If: ω – the angular velocity of the system being an element of the body frame; p, q, r – the angular velocity around the x, y, and z axes respectively.

The kinematic relationships between the derivative of position and linear velocity, and the derivative of orientation and angular velocity are represented in Eq. (6) and Eq. (7) respectively.

$$\dot{\xi} = \boldsymbol{Rv} \in E_I \tag{6}$$

$$\dot{\eta} = \boldsymbol{J\omega} \in E_I \tag{7}$$

If: $\dot{\xi}$ – the derivative of the position vector (ξ), representing the linear velocity of the system in the x, y, and z axes; $\dot{\eta}$ – the derivative of the orientation vector (η), representing the angular velocity of the system in the ϕ, θ, and ψ angles.

The transformation matrix of the system links the orientation vector (for roll and pitch angles) to the transformation matrix, showing how alterations in the roll and pitch angles impact the transformation between the orientation and transformation matrices for the system. Thus, the representation of the system's transformation matrix as in Eq. (8).

$$J = \begin{bmatrix} 1 & 0 & -sin\theta \\ 0 & cos\phi & sin\phi cos\theta \\ 0 & -sin\phi & cos\phi cos\theta \end{bmatrix} \tag{8}$$

The system's rotation matrix uses Euler angles to describe the orientation of the body frame in relation to the inertial frame. It displays the system's conversion from body coordinates to inertial coordinates. The representation of the system's rotation matrix is in Eq. (9).

$$R = \begin{bmatrix} \cos\theta\cos\psi & \cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi & \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi \\ \cos\phi\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{9}$$

3.1.2. The system dynamics

In a simplified scenario where aerodynamic effects are ignored and the system's body frame is symmetric and rigid, the primary source of the lifting force and moments generated by the quadcopter's four propellers is the thrust produced. This means that the propellers' rotational motion generates a force that propels the air downward, subsequently creating an upward force on the system. Moreover, the rotation of the propellers generates moments that contribute to the system's stability in the air.

3.1.2.1. Total thrust ($\boldsymbol{F_{thrust}}$)

A rotor's force is proportional to the square of its rotating speed, also known as angular velocity. This relationship is based on fundamental aerodynamic principles and is typically expressed using a mathematical equation, shown in Eq. (10).

$$F_{thrust} = k_a \times \omega_i^2 \tag{10}$$

If: F_{thrust} – the total thrust generated by four rotors of the quadcopter; k_a – the aerodynamic force constant; ω_i – the rotational speed generated by each propeller.

The total thrust (F_{thrust}) generated by all rotors can be expressed as in Eq. (11).

$$F_{thrust} = k_a \times (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \tag{11}$$

3.1.2.2. Total moment (M_{total})

When the propellers generate moments, they produce torque, which influences the system's rotation. Propellers create moments that are proportional to the square of their rotating speed, also known as angular velocity. This relationship is expressed as in Eq. (12).

$$M_{total} = k_m \times \omega_i^2 \tag{12}$$

If: M_{total} – the total moment generated by four propellers of the quadcopter; k_m the moment constant.

The total moment can be expressed as in Eq. (13).

$$M_{total} = k_m \times (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \tag{13}$$

3.1.2.3. Torques (τ)

The torques generated by propellers of the quadcopter are expressed in Eq. (14-16).

$$\tau_\phi = L \times k_a \times [(\omega_2^2 + \omega_3^2) - (\omega_1^2 + \omega_4^2)] \tag{14}$$

$$\tau_\theta = L \times k_a \times [(\omega_1^2 + \omega_2^2) - (\omega_3^2 + \omega_4^2)] \tag{15}$$

$$\tau_\psi = k_m \times [(\omega_1^2 + \omega_3^2) - (\omega_2^2 + \omega_4^2)] \tag{16}$$

If: τ_ϕ, τ_θ, τ_ψ – the torques generated on roll, pitch, and yaw respectively; L – the distance between the centre of the system and the quadcopter's motor.

3.1.2.4. The rotational motion

The rotational motion equations are expressed in Eq. (17) by establishing the Newton-Euler method.

$$I \times \ddot{\eta} + I \times \dot{\eta} = [\tau_\phi \quad \tau_\theta \quad \tau_\psi]^T \tag{17}$$

If: $I \times \ddot{\eta}$ – refers to the torque of the system; $I \times \dot{\eta}$ – refers to the angular momentum of the system.

3.1.2.5. The moment of inertia (τ)

The quadcopter's propellers are arranged in a square formation, resulting in a symmetrical structure. Because of this structural type, the system's moment of inertia is expressed in Eq. (18).

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{18}$$

If: I_{xx}, I_{yy}, I_{zz} – the moments of inertia around the x, y, and z axes respectively.

The following equation can be obtained by implementing Eq. (18) into Eq. (17).

$$\begin{bmatrix} I_{xx}\ddot{\phi} \\ I_{yy}\ddot{\theta} \\ I_{zz}\ddot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\theta}I_{zz}\dot{\psi} - \dot{\psi}I_{yy}\dot{\theta} \\ \dot{\psi}I_{xx}\dot{\phi} - \dot{\phi}I_{zz}\dot{\psi} \\ \dot{\phi}I_{yy}\dot{\theta} - \dot{\theta}I_{xx}\dot{\phi} \end{bmatrix} = \begin{bmatrix} \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} \tag{19}$$

3.1.2.6. The rotational acceleration ($\ddot{\phi}$, $\ddot{\theta}$, $\ddot{\psi}$)

The rotational accelerations affecting roll, pitch, and yaw can be obtained by considering the relationship demonstrated in Eq. (19).

$$\ddot{\phi} = \frac{\tau_{\phi}}{I_{xx}} + \frac{I_{yy}}{I_{xx}} \times \dot{\theta}\dot{\psi} - \frac{I_{zz}}{I_{xx}} \times \dot{\theta}\dot{\psi} \tag{20}$$

$$\ddot{\theta} = \frac{\tau_{\theta}}{I_{yy}} + \frac{I_{zz}}{I_{yy}} \times \dot{\phi}\dot{\psi} - \frac{I_{xx}}{I_{yy}} \times \dot{\phi}\dot{\psi} \tag{21}$$

$$\ddot{\psi} = \frac{\tau_{\psi}}{I_{zz}} + \frac{I_{xx}}{I_{zz}} \times \dot{\phi}\dot{\theta} - \frac{I_{yy}}{I_{zz}} \times \dot{\phi}\dot{\theta} \tag{22}$$

3.1.2.7. The translational acceleration ($\ddot{x}$, $\ddot{y}$, $\ddot{z}$)

The dynamics of the system have a direct relationship with the acceleration of the rotational and translational motions, meaning that the dynamic equations must be in acceleration form. Therefore, Newton's second law of motion, which refers to the acceleration of the system, can be used to derive the translational motion equation of the system.

$$m_s\ddot{\xi} = R \times F_{total} + [0 \quad 0 \quad -m_s g]^T \tag{23}$$

$$F_{total} = [0 \quad 0 \quad F_{thrust}]^T \tag{24}$$

If: m_s – the mass of the system; $\ddot{\xi}$ – translational acceleration; g – the gravitational acceleration; F_{total} – total force.

The translational accelerations of the system can be obtained by implementing Eq. (24) into Eq. (23).

$$\ddot{x} = \frac{F_{thrust}}{m_s} \times (\cos\phi \cos\psi \sin\theta + \sin\phi \sin\psi) \tag{25}$$

$$\ddot{y} = \frac{F_{thrust}}{m_s} \times (\cos\phi \sin\theta \sin\psi - \cos\psi \sin\phi) \tag{26}$$

$$\ddot{z} = \frac{F_{thrust}}{m_s} \times (\cos\phi \cos\theta) - g \tag{27}$$

3.1.2.8. Linearisation

Because the observed dynamic state of the system is very nonlinear, the equations must be linearized at an equilibrium point where the system's 6 DOF state is stable. The linearisation of nonlinear equations contributes to the simplification and decoupling of the mathematical model's dynamics.

The rotational acceleration of the system therefore can be expressed in Eq. (28-30).

$$\ddot{\phi} = \frac{\tau_\phi}{I_{xx}} \tag{28}$$

$$\ddot{\theta} = \frac{\tau_\theta}{I_{yy}} \tag{29}$$

$$\ddot{\psi} = \frac{\tau_\psi}{I_{zz}} \tag{30}$$

The translational acceleration of the system furthermore can be expressed in Eq. (31-33).

$$\ddot{x} = -g\theta \tag{31}$$

$$\ddot{y} = g\phi \tag{32}$$

$$\ddot{z} = g - \frac{F_{thrust}}{m_s} \tag{33}$$

Furthermore, the representation of the inertial state of the system is as follows:

$$Q = [x \quad \dot{x} \quad \phi \quad \dot{\phi} \quad y \quad \dot{y} \quad \theta \quad \dot{\theta} \quad z \quad \dot{z} \quad \psi \quad \dot{\psi}]^T \tag{34}$$

If: Q – the state vector of the system; x, $\dot{x}$ – the longitudinal position and velocity of the system along the X-axis respectively; y, $\dot{y}$ –the lateral position and velocity of the system along the Y-axis respectively; z, $\dot{z}$ –the vertical position and velocity of the system along the Z-axis respectively; ϕ, $\dot{\phi}$ –the roll angle and angular velocity of the system about the longitudinal axis respectively; θ, $\dot{\theta}$ –the pitch angle and angular velocity of the system about the lateral axis respectively; ψ, $\dot{\psi}$ –the yaw angle and angular velocity of the system about the vertical axis respectively.

The linear dynamic model's state space is represented in Eq. (35).

$$\dot{x} = Ax + Bu \tag{35}$$

If: $\dot{x}$ – the first derivative of the state vector; A – system matrix; x –the state vector; B – input vector; u – input or control vector.

Therefore, the state space form can be represented as:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \\ \dot{y} \\ \ddot{y} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{z} \\ \ddot{z} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \\ y \\ \dot{y} \\ \theta \\ \dot{\theta} \\ z \\ \dot{z} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_{xx}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} \begin{bmatrix} F_{thrust} \\ \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix}$$

3.2. Post-launch mathematical model

Understanding the dynamics of the quadcopter and rocket system in the post-launch environment requires a thorough consideration of Newton’s third law of motion, which entitles “for every action (force) in nature there is an equal and opposite reaction” defined by [17]. This law plays a critical role in this section, as it directly influences the conservation of energy within the system.

As highlighted in Section 3, in this section, the quadcopter and rocket are no longer considered as a single structure because they are no longer attached. Instead, they are analysed as distinct entities, aiming to create a mathematical model that accurately predicts the behaviour of the quadcopter in the post-launch environment.

The initial state of the post-launch environment can be described as a conservation of energy.

$$\varepsilon_q = \varepsilon_r \tag{36}$$

If: ε_q – the total energy of the quadcopter in the post-launch environment; ε_r – the total energy of the rocket in the post-launch environment.

3.2.1. Newton’s conservation of energy

The rocket initiates its thrust instantaneously in the first stage of launching. In this stage, the drag loss is negligible. The velocity gained from the rocket’s propulsion is neglected. Therefore, the total kinetic and potential energy of both quadcopter and rocket are represented in Eq. (37), employed from Eq. (36).

$$m_q \times \left(gh_0 + \frac{\left(v_{0_{quadcopter}}\right)^2}{2} \right) = m_r \times \left(gh_0 + \frac{\left(v_{0_{rocket}}\right)^2}{2} \right) \tag{37}$$

If: m_q – the quadcopter mass; h_0 – launch altitude; $v_{0_{quadcopter}}$ – the initial velocity of the quadcopter initiated by rocket launch; m_r – the rocket mass; $v_{0_{rocket}}$ – the launch velocity of the rocket.

The initial velocity of the quadcopter therefore is employed from Eq. (37).

$$v_{0_{quadcopter}} = \sqrt{\frac{m_r}{m_q}\left(2gh_0 + v_{0_{rocket}}^2\right) - 2gh_0} \quad (38)$$

3.2.2. Quadcopter kinematics

Due to the impact exerted on the quadcopter triggered by the rocket launch, the quadcopter's kinematics must be redefined. Therefore, the new linear (v') and angular (ω') velocities, and kinematic relationships ($\dot{\xi}'$, $\dot{\eta}'$) are represented in the following equations.

$$v' = [u' \quad v' \quad w']^T \in E_B \quad (39)$$

$$\omega' = [p' \quad q' \quad r']^T \in E_B \quad (40)$$

$$\dot{\xi}' = \boldsymbol{R}\boldsymbol{v}' \in E_I, \qquad \begin{bmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{bmatrix} = \boldsymbol{R} \times \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} \quad (41)$$

$$\dot{\eta}' = \boldsymbol{J}\boldsymbol{\omega}' \in E_I, \qquad \begin{bmatrix} \dot{\phi}' \\ \dot{\theta}' \\ \dot{\psi}' \end{bmatrix} = \boldsymbol{J} \times \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix} \quad (42)$$

As discussed in Section 3, this study assumes the rocket is launched from the quadcopter along the positive X-axis and focuses on analysing the quadcopter's behaviour specifically along this axis. The first assumption applies to the direction of the rocket, presumed to be launched along the positive X-axis. Consequently, the reaction force exerted on the quadcopter is expected to be in the negative X-axis direction. The second assumption considers the reaction force moment experienced by the quadcopter during the rocket's launch, resulting in a downward pitch moment affecting the quadcopter.

3.2.3. The rotational acceleration ($\ddot{\phi}'$, $\ddot{\theta}'$, $\ddot{\psi}'$)

The new rotational accelerations ($\ddot{\phi}'$, $\ddot{\theta}'$, $\ddot{\psi}'$) acting on roll, pitch, and yaw can be obtained considering the relationships in Eq. (20-22).

$$\ddot{\phi}' = \frac{{\tau_\phi}'}{I_{xx}} + \frac{I_{yy}}{I_{xx}} \times \dot{\theta}'\dot{\psi}' - \frac{I_{zz}}{I_{xx}} \times \dot{\theta}'\dot{\psi}' \quad (43)$$

$$\ddot{\theta}' = \frac{{\tau_\theta}'}{I_{yy}} + \frac{I_{zz}}{I_{yy}} \times \dot{\phi}'\dot{\psi}' - \frac{I_{xx}}{I_{yy}} \times \dot{\phi}'\dot{\psi}' \quad (44)$$

$$\ddot{\psi}' = \frac{{\tau_\psi}'}{I_{zz}} + \frac{I_{xx}}{I_{zz}} \times \dot{\phi}'\dot{\theta}' - \frac{I_{yy}}{I_{zz}} \times \dot{\phi}'\dot{\theta}' \quad (45)$$

3.2.4. The translational acceleration ($\ddot{x}'$, $\ddot{y}'$, $\ddot{z}'$)

The state of the total force in Eq. 24 should be redefined since we have obtained a new force, named reaction force ($F_{reaction}$) resulting negative direction in the X-axis of the quadcopter. Therefore, Newton's second law of motion can be redefined in Eq. (43).

$$m_q\ddot{\xi}' = R \times F_{total}' + [0 \quad 0 \quad -m_q g]^T \tag{46}$$

$$F_{total}' = [-F_{reaction} \quad 0 \quad F_{thrust}]^T \tag{47}$$

The new translational accelerations ($\ddot{x}'$, $\ddot{y}'$, $\ddot{z}'$) of the quadcopter can be obtained by implementing Eq. (47) into Eq. (46).

$$\ddot{x}' = \frac{1}{m_q} \times [F_{thrust}(\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi) - F_{reaction}(\cos\theta\cos\psi)] \tag{48}$$

$$\ddot{y}' = \frac{1}{m_q} \times [F_{thrust}(\cos\phi\sin\theta\sin\psi - \cos\psi\sin\phi) - F_{reaction}(\cos\phi\sin\psi)] \tag{49}$$

$$\ddot{z}' = \frac{1}{m_q} \times [F_{thrust}(\cos\phi\cos\theta) + F_{reaction}(\sin\theta)] - g \tag{50}$$

3.3. General boundary conditions

The created model is validated under the following boundary conditions shown in Table 2. The model does not evaluate other variables such as aerodynamic effects and control systems in these specific analyses.

Table 2. The system's boundary conditions

Parameters	**Values**	**Units**
Air pressure	101.325	kPa
Temperature	20	°C
Air density	1.204	kg/m^3
Gravitational acceleration	9.81	m/s^2
Wind speed	0	m/s

4. Calculations

Through the application of Newton's third law of motion, the model is able to capture the complexity of the post-launch model between the quadcopter and rocket. It is important to note that the parameters for this numerical experiment were chosen freely, and there might be different results if these parameters were chosen differently. In the specific scenario where the quadcopter hovers 50 metres AGL during the launch of the rocket, Newton's third law enables to identification of the reaction forces, ultimately contributing to an accurate portrayal of the quadcopter's behaviour in the post-launch environment.

4.1. Dependent variables

The system's parameter assumptions are shown in Table 3.

Table 3. The system's parameters during the launch

Parameters	Values	Units
Quadcopter's mass, m_q	4	kg
Rocket's mass, m_r	1	kg
System's mass, m_s	5	kg
Gravitational acceleration, g	9.81	m/s²
Launch altitude, h_0	50	m

When a rocket is launched from a quadcopter at an altitude of 50 metres, the reaction force of the rocket's propulsion generates an equal and opposite reaction exerted upon the quadcopter. Consequently, the quadcopter experiences a momentary downward tilt or rotation about its axis. This sudden change in force distribution and the COM results in a downward pitch angle (θ) set at 30°.

After determining the initial conditions and parameters of the quadcopter and rocket system, the position and orientation as well as values of kinematics of the quadcopter in the post-launch environment can be determined using Eq. (1-2) and Eq. (39-42) respectively.

$$\xi' = [0 \quad 0 \quad 50]^T (m) \tag{51}$$

$$\eta' = [0 \quad -30 \quad 0]^T (°) \tag{52}$$

$$v' = \left[\sqrt{\frac{v_{0_{rocket}}^2}{4} - 735.75} \quad 0 \quad 0\right]^T (m/s) \tag{53}$$

$$\omega' = [0 \quad 0 \quad 0]^T (rad/s) \tag{54}$$

$$\dot{\xi}' = \begin{bmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{bmatrix} = \begin{bmatrix} 0.87 & 0 & -0.50 \\ 0 & 1 & 0 \\ 0.50 & 0 & 0.87 \end{bmatrix} \times \begin{bmatrix} \sqrt{\frac{v_{0_{rocket}}^2}{4} - 735.75} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.87 \times \sqrt{\frac{v_{0_{rocket}}^2}{4} - 735.75} \\ 0 \\ 0.50 \times \sqrt{\frac{v_{0_{rocket}}^2}{4} - 735.75} \end{bmatrix} (m/s) \tag{55}$$

$$\dot{\eta}' = \begin{bmatrix} \dot{\phi}' \\ \dot{\theta}' \\ \dot{\psi}' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.50 \\ 0 & 1 & 0 \\ 0 & 0 & 0.87 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} (rad/s) \tag{56}$$

It should be noted that the study does not check the specifications of the quadcopter's motors, thus the angular velocities (ω') are excluded.

Because the quadcopter is hovering at an altitude of 50 meters AGL, the thrust force created by each motor equals gravitational force (F_G), shown in Eq (57).

$$F_G = F_{thrust} \tag{57}$$

Therefore, the total force matrix (F'_{total}) is represented in Eq. (58)

$$F'_{total} = [-m_q \dot{x}' \quad 0 \quad m_q g]^T = \left[-3.48 \times \sqrt{\frac{\upsilon_{0_{rocket}}^2}{4} - 735.75} \quad 0 \quad 39.24 \right]^T (N) \tag{58}$$

The quadcopter possesses a square formation of rotors, therefore it has a symmetrical structure, as previously stated. Due to its structure type, it results in symmetrical values for three axes. Hence, the moment of inertia is presented in Eq. (59).

$$I_{xx} = I_{yy} = I_{zz} = \frac{m_q}{12} \times (a^2 + b^2) \tag{59}$$

The quadcopter is assumed to have a 0.3 by 0.3 metre formation. As a result, the (a) and (b) in Eq. (59) can be redefined as 0.3 metres and 0.3 metres respectively.

$$I = \begin{bmatrix} 0.06 & 0 & 0 \\ 0 & 0.06 & 0 \\ 0 & 0 & 0.06 \end{bmatrix} (kg \cdot m^2) \tag{60}$$

The new rotational accelerations of the quadcopter values cannot be obtained due to the unknown angular velocity of each motor.

The new translational accelerations of the quadcopter values can be obtained by utilising Eq. (48-50).

$$\ddot{x}' = -4.91 - 0.75 \times \sqrt{\frac{\upsilon_{0_{rocket}}^2}{4} - 735.75} \; (m/s^2) \tag{61}$$

$$\ddot{y}' = 0 \; (m/s^2) \tag{62}$$

$$\ddot{z}' = 0.46 \times \sqrt{\frac{\upsilon_{0_{rocket}}^2}{4} - 735.75} - 1.31 \; (m/s^2) \tag{63}$$

4.2. Independent variables

Table 3 shows the independent variables, not dependent on any other factors to determine their value. In addition to these variables, the launch velocity of the rocket ($\upsilon_{0_{rocket}}$) can be defined as an independent variable.

Determining system boundary conditions is crucial for determining the values where the study can be examined. To compute the minimum launch velocity of the rocket, Eq. (38) is set to 0.

$$\sqrt{\frac{1}{4}\left(2 \times 9.81 \times 50 + v_{0_{rocket}}^2\right) - 2 \times 9.81 \times 50} = 0, \qquad v_{0_{rocket\_min}} \approx 55\ m/s$$

Consequently, the following statements can be made:
1. the minimum launch velocity of the rocket is not less than 55 m/s;
2. the gross weight of the system is 5 kg;
3. the rocket weight is 1 kg.

The rocket used in this study has been designed to maintain a lightweight, weighing a kilogram. This has been done to minimize the complexity of system calculations, primarily due to hardware constraints.

5. Dichotomy method

The dichotomy method, also known as the bisection method [37], is a simple numerical method for finding roots or solutions to nonlinear equations. In the context of this study, the method is applied to solve specific equations related to the behaviour and dynamics of the quadcopter in the post-launch environment.

The dichotomy method is one of numerous numerical approaches used to solve nonlinear equations, including the Golden Ratio method, the Fibonacci method, and Nonlinear Regression. Despite its simplicity, the dichotomy approach can deliver correct findings for a wide range of nonlinear systems.

The principle of this method is to narrow down a range of a nonlinear equation where a root or solution might exist to determine an approximate solution to a function $f(x) = 0$. It starts with two initial points $[a, b]$, a lower and upper bound, where the root is expected to lie. At least one real root or solution x_0 exists within the lower and upper bound when the function is equal to zero, $f(x) = 0$. Until the $f(x_0) = 0$, the method divides the intervals $c_1, c_2, \dots, c_n$ into halves to determine which subinterval holds the root.

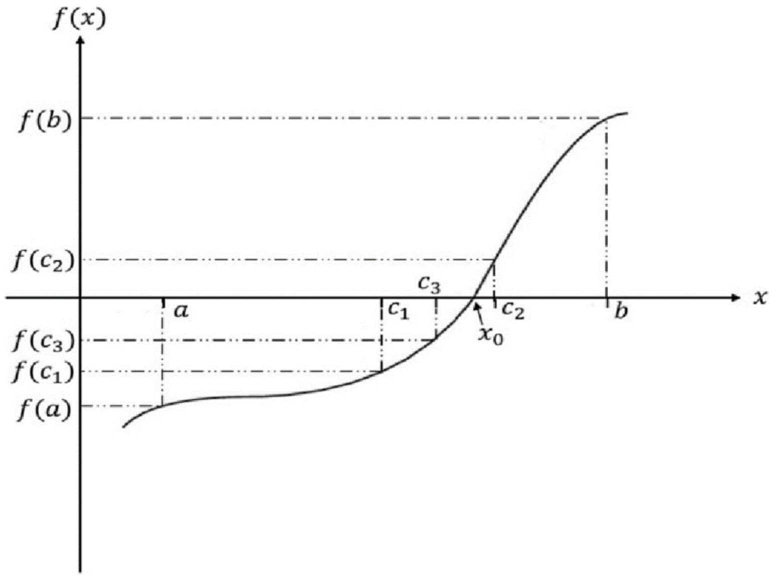

Fig. 2. Dichotomy method concept [11]

By iterating through the method and determining the bounds, a root or solution to a function can be concluded. In this study, the method is employed to determine the root value of the launch velocity of the rocket, to determine the behaviour of the quadcopter in the post-launch environment, allowing us to understand how velocities and accelerations of the quadcopter changes under each rocket velocities along the X-axis.

This method requires setting initial bounds and a tolerance level for convergence, ensuring the accuracy of the solution of the function. It is a fundamental tool in numerical analysis, allowing us to understand the system's behaviour through computational approaches.

In this case, the tolerance level is set to one millionth (0.000001), and the initial intervals $[a, b]$, which is the rocket's initial velocity, is set to be 50 and 60 m/s. In Subsection 4.2, the minimum launch velocity of the rocket has been calculated and the value is $v_{0_{rocket\_min}} \approx$ 55 m/s, however, the model is tested between 50 and 60 m/s with increments of 0.5 m/s to determine if the dichotomy approach is successful for this model.

5.1. The velocity function of the quadcopter

The velocity function of the quadcopter is employed from Eq. 55. Therefore, the following function delivers the quadcopter's velocity function of time.

$$f(\dot{x'}) = 0.87 \times \sqrt{\frac{v_{0_{rocket}}^2}{4} - 735.75}\ (m/s) \tag{64}$$

The method has converged to $x_0 = 59.99$ m/s within the specified tolerance and intervals. Since the dichotomy method aims to find a value of x_0 such that the function $f(x_0) \cong 0$, the results suggest that the function evaluated at $x_0 = 59.99$ m/s is approximately equal to 0 based on the specified tolerance. The method has succeeded in providing an approximate solution within the interval and tolerance within 24 iterations.

However, the function's convergence rate is 0, which typically indicates the method is not converging or converging extremely slow. Several factors might affect the convergence rate such as interval selection, function behaviour, algorithm sensitivity, or method limitations.

Examining these factors can provide following assumptions:

- the method has converged to a root value, which means determined bounds were established correctly. This eliminates the interval selection factor;
- the function might have a complex behaviour, such as high-order roots, which might pose challenges for convergence. The number of iterations can be increased;
- algorithm parameters and the method limitation might affect the convergence rate of the function.

These assumptions could be thoroughly examined by adjusting method intervals or the algorithm of the method to converge the function to a better conclusion; however, due to the hardware utilised to calculate these functions, the additional studies could not be computed.

The graph of root approximation against iteration in Fig. 3 illustrates how the approximation of the root changes with each iteration. As the iteration continues, the value approaches a true root value, which, in this case, is $x_0 = 59.99$ m/s.

The graph of interval contraction in Fig. 4 illustrates how the size of the interval changes with each iteration. The interval size drops towards the root, which is ideal.

The graph of convergence rate against iteration in Fig. 5 illustrates how fast the method converges towards the root. Ideally, the convergence rate should approach a value close to 0.5 for effective convergence.

The graph of the error reduction rate against iteration in Fig. 6 illustrates how the error between the estimated root and actual root reduces with each iteration. A decreasing trend indicates the method is converging.

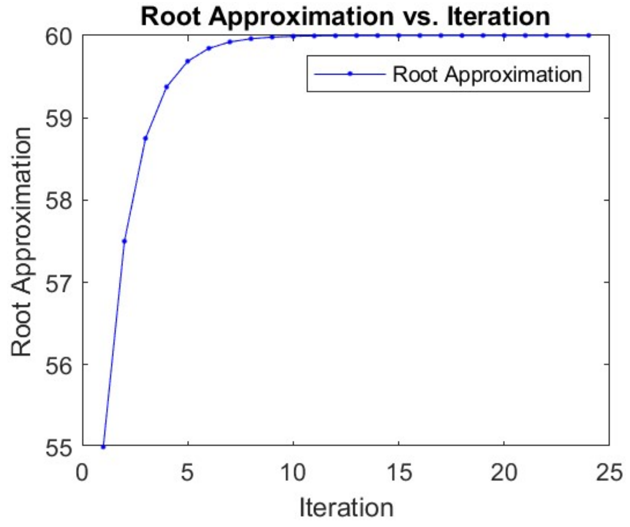

Fig. 3. Root approximation vs. iteration (velocity)

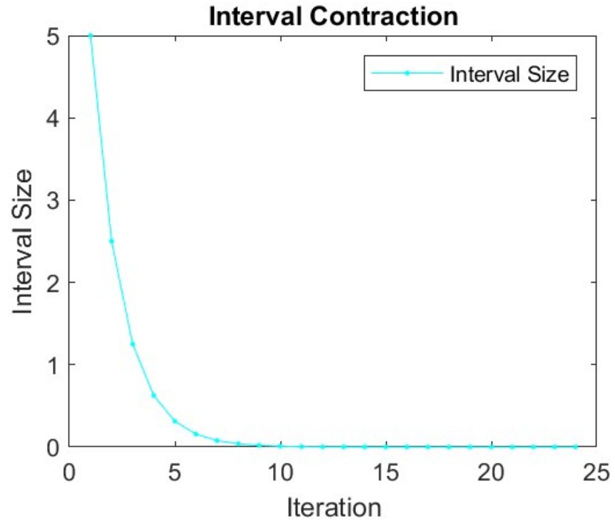

Fig. 4. Interval contraction (velocity)

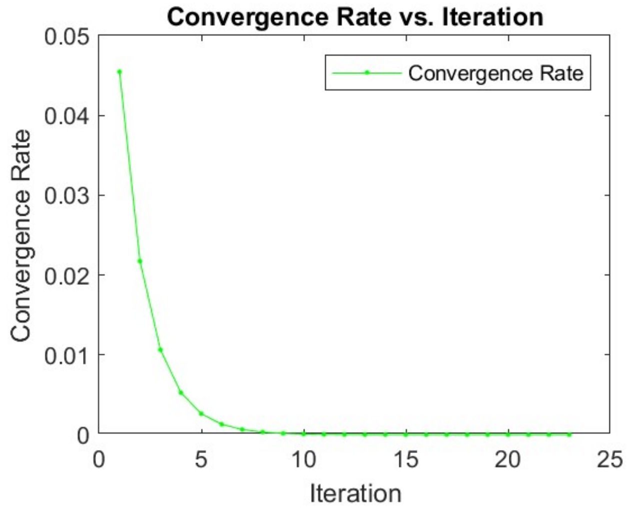

Fig. 5. Convergence rate vs. iteration (velocity)

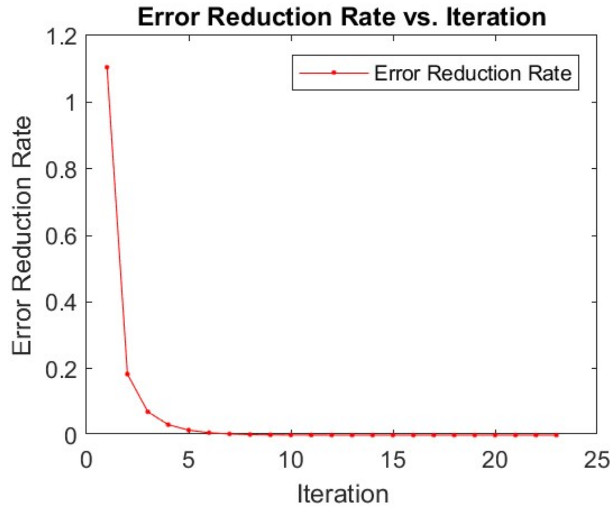

Fig. 6. Error reduction rate vs. iteration (velocity)

Furthermore, the trend between the quadcopter velocity and the rocket launch velocity is illustrated in Fig. 7, and values are shown in Table 4.

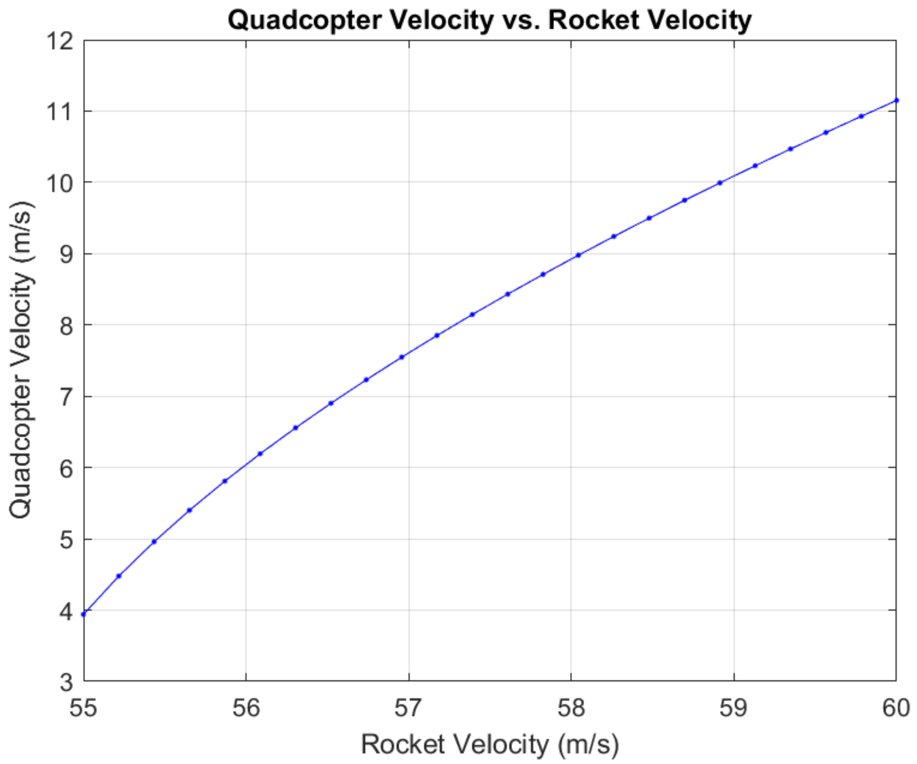

Fig. 7. Quadcopter velocity against rocket launch velocity in m/s

Table 4. Corresponding rocket velocities to quadcopter velocities in each iteration

Iteration no.	Rocket launch velocity, m/s	Quadcopter velocity, m/s
1	55.00000	3.93909
2	57.50000	8.29072
3	58.75000	9.80983
4	59.37500	10.49776
5	59.68750	10.82790
6	59.84375	10.98988
7	59.92188	11.07014
8	59.96094	11.11009
9	59.98047	11.13002
10	59.99023	11.13998
11	59.99512	11.14495
12	59.99756	11.14744
13	59.99878	11.14868
14	59.99939	11.14930
15	59.99969	11.14961
16	59.99985	11.14977
17	59.99992	11.14985
18	59.99996	11.14989

19	59.99998	11.14991
20	59.99999	11.14992
21	60.00000	11.14992
22	60.00000	11.14992
23	60.00000	11.14992
24	60.00000	11.14992

As the values shown in Table 4, rocket launch velocity has a significant impact on the quadcopter's velocity. The quadcopter's velocity is directly affected by the rocket's velocity, presenting a relation between these two variables. The quadcopter's velocity furthermore exhibits an increase corresponding to higher rocket launch velocity.

The following illustration presents the quadcopter's displacement over time for different rocket velocities and values are shown in Table 5.

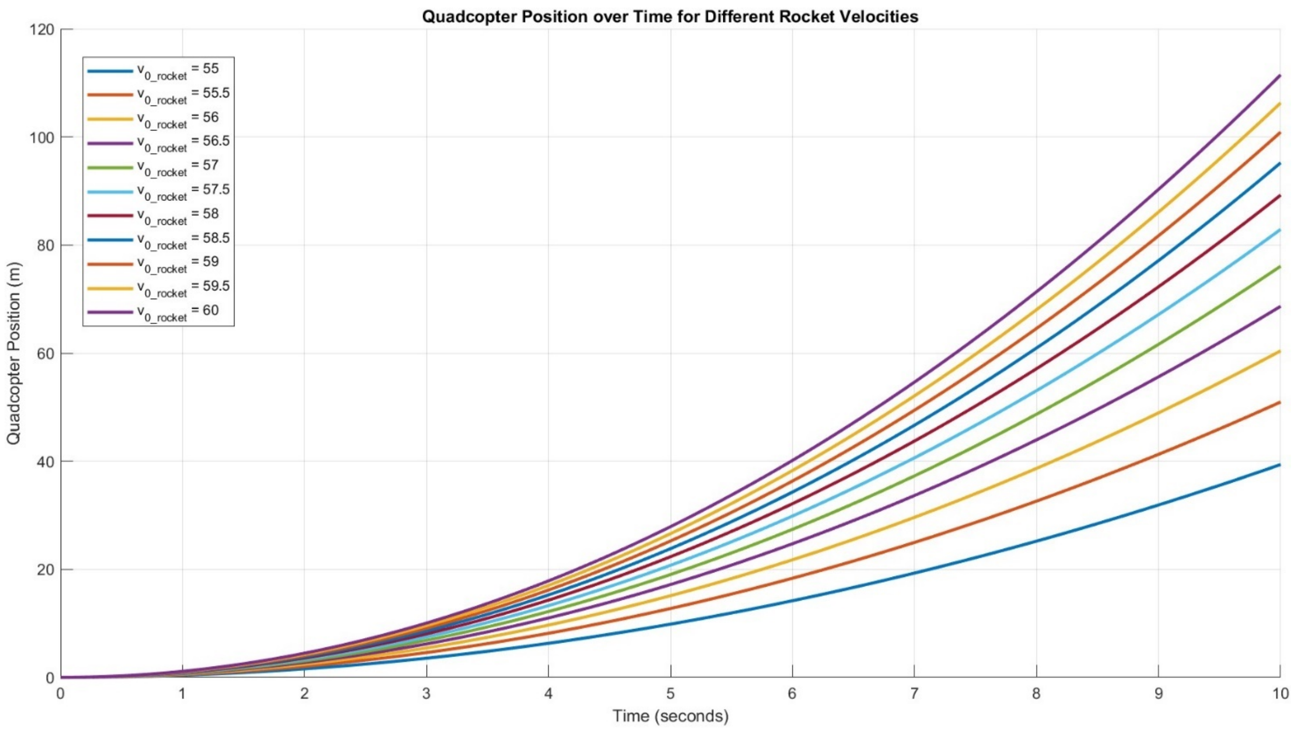

Fig. 8. Quadcopter position over time for different rocket velocities

Table 5. Quadcopter displacement (δ_x) values of each rocket launch velocity at specific times

Time, s	**Quadcopter displacement (δ_x), m**										
$v_{0_{rocket}}$, **m/s**	**55.0**	**55.5**	**56.0**	**56.5**	**57.0**	**57.5**	**58.0**	**58.5**	**59.0**	**59.5**	**60.0**
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.4	0.5	0.6	0.7	0.8	0.8	0.9	0.9	1.0	1.1	1.1
2	1.6	2.0	2.4	2.7	3.0	3.3	3.6	3.8	4.0	4.2	4.4
3	3.5	4.6	5.4	6.2	6.8	7.4	8.0	8.6	9.1	9.5	10.0
4	6.3	8.1	9.7	11.0	12.2	13.2	14.3	15.2	16.1	17.0	17.8

5	9.8	12.7	15.1	17.2	19.0	20.7	22.3	23.8	25.2	26.6	27.8
6	14.2	18.3	21.7	24.7	27.4	29.8	32.1	34.3	36.3	38.2	40.1
7	19.3	25.0	29.6	33.6	37.3	40.6	43.7	46.6	49.4	52.1	54.6
8	25.2	32.6	38.7	43.9	48.7	53.0	57.1	60.9	64.6	68.0	71.3
9	31.9	41.3	48.9	55.6	61.6	67.1	72.3	77.1	81.7	86.1	90.3
10	39.4	51.0	60.4	68.7	76.1	82.9	89.3	95.2	100.9	106.3	111.5

Fig. 8 illustrates the trajectory of the quadcopter over time under different rocket launch velocities at specific time ranges. This analysis helps us to depict how the quadcopter moves through space when it is subjected to different rocket launch velocity values. There is a direct impact of changing the rocket velocities on the quadcopter's displacement, showcasing the quadcopter's movement and position.

Understanding this behaviour assists in designing navigation algorithms or control systems. The illustration of displacement and values shown in Table 5 provide insights into managing and predicting the quadcopter's position influenced by different rocket launch velocities.

The following illustration presents the quadcopter velocity over time for different rocket launch velocities and values are shown in Table 6.

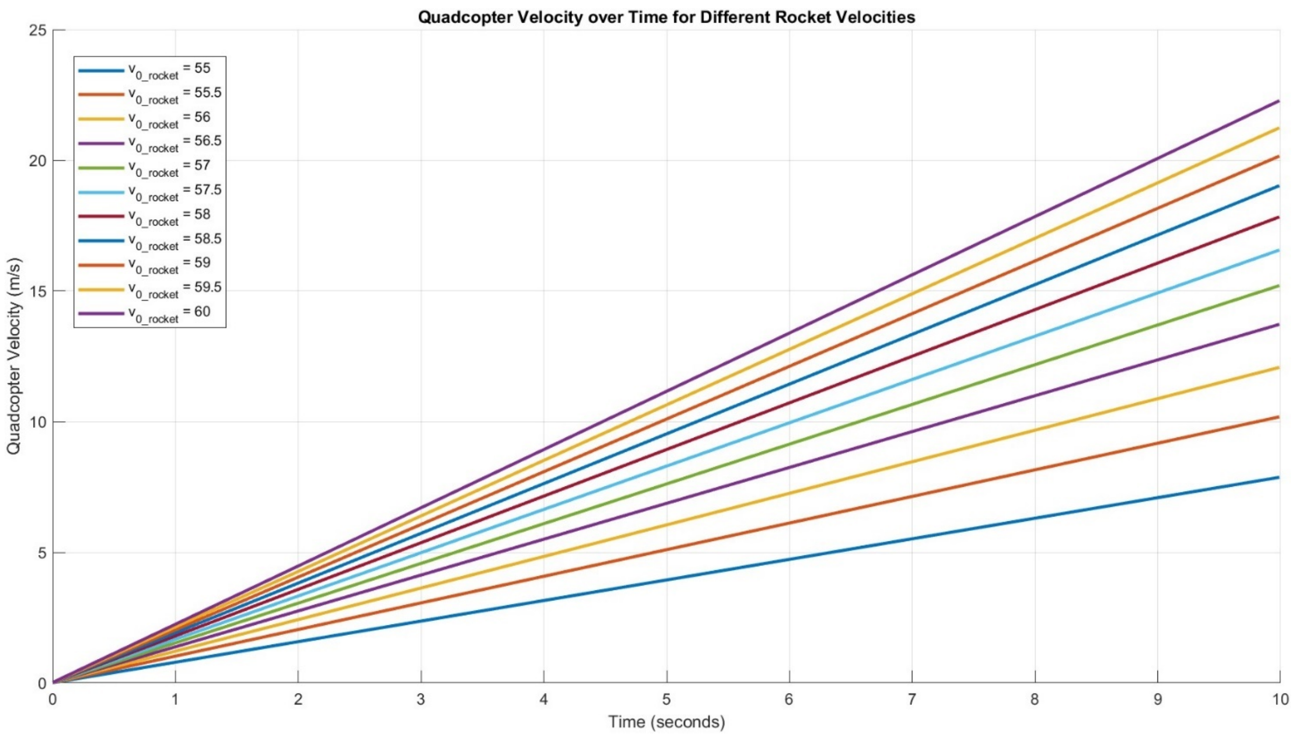

Fig. 9. Quadcopter velocity over time for different rocket velocities

Table 6. Quadcopter velocity $\left(v_{0_{quadcopter_x}}\right)$ values of each rocket launch velocity at specific times

Time, s	**Quadcopter velocity $\left(v_{0_{quadcopter_x}}\right)$, m/s**										
$v_{0_{rocket}}$, **m/s**	**55.0**	**55.5**	**56.0**	**56.5**	**57.0**	**57.5**	**58.0**	**58.5**	**59.0**	**59.5**	**60.0**

0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
2	1.6	2.0	2.4	2.7	3.0	3.3	3.6	3.8	4.0	4.2	4.5
3	2.4	3.1	3.6	4.1	4.6	5.0	5.3	5.7	6.0	6.4	6.7
4	3.1	4.1	4.8	5.5	6.1	6.6	7.1	7.6	8.1	8.5	8.9
5	3.9	5.1	6.0	6.9	7.6	8.3	8.9	9.5	10.1	10.6	11.1
6	4.7	6.1	7.2	8.2	9.1	9.9	10.7	11.4	12.1	12.7	13.4
7	5.5	7.1	8.5	9.6	10.6	11.6	12.5	13.3	14.1	14.9	15.6
8	6.3	8.1	9.7	11.0	12.2	13.3	14.3	15.2	16.1	17.0	17.8
9	7.1	9.2	10.9	12.3	13.7	14.9	16.0	17.1	18.1	19.1	20.0
10	7.9	10.2	12.1	13.7	15.2	16.6	17.8	19.0	20.2	21.2	22.3

Fig. 9 illustrates how the quadcopter's velocity changes over time when it is subjected to different rocket velocities, providing a comprehensive understanding of the dynamic behaviour of the quadcopter under different rocket launch velocities.

An increasing trend in the quadcopter's velocity against a specific time range can be observed in Table 6. This allows us to examine the velocity changes corresponding to the rocket launch velocities with respect to a specific time range, concluding the effect in the change of quadcopter's motion.

The following illustration presents the quadcopter's total energy over time for different rocket velocities and values are shown in Table 7.

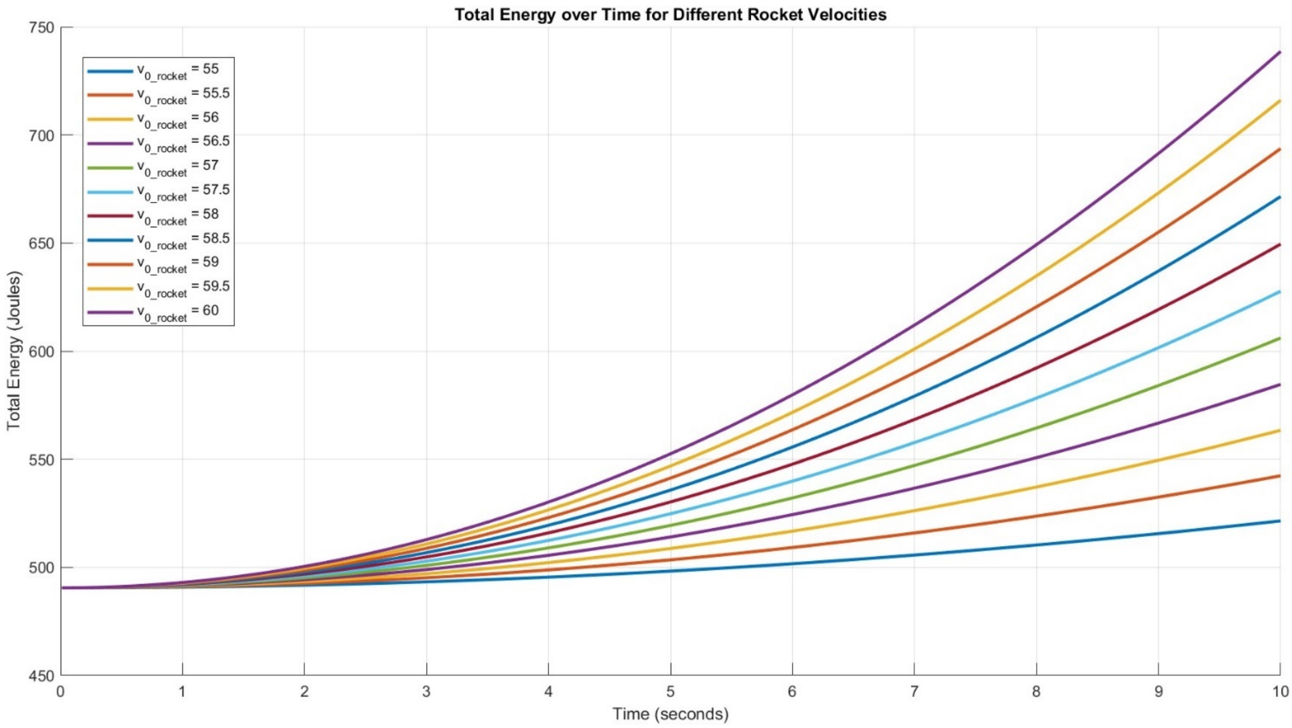

Fig. 10. Total energy over time for different rocket velocities

Table 7. Total energy (ε_{q_x}) values of each rocket launch velocity at specific times

Time, s	**Energy (ε_{q_x}), joules**										
$v_{0_{rocket}}$, **m/s**	**55.0**	**55.5**	**56.0**	**56.5**	**57.0**	**57.5**	**58.0**	**58.5**	**59.0**	**59.5**	**60.0**
0	490.5	490.5	490.5	490.5	490.5	490.5	490.5	490.5	490.5	490.5	490.5
1	490.8	491.0	491.2	491.4	491.7	491.9	492.1	492.3	492.5	492.8	493.0
2	491.7	492.6	493.4	494.3	495.1	496.0	496.9	497.7	498.6	499.5	500.4
3	493.3	495.2	497.1	499.0	500.9	502.8	504.8	506.8	508.8	510.8	512.8
4	495.5	498.8	502.2	505.6	509.0	512.5	515.9	519.5	523.0	526.6	530.2
5	498.2	503.5	508.7	514.0	519.4	524.8	530.3	535.8	541.3	546.9	552.5
6	501.6	509.2	516.7	524.4	532.1	539.9	547.7	555.7	563.7	571.7	579.8
7	508.9	521.2	533.7	546.3	559.0	571.8	584.8	597.8	611.0	624.2	637.6
8	510.3	523.7	537.2	550.7	564.5	578.3	592.3	606.3	620.5	634.9	649.3
9	515.6	532.5	549.5	566.8	584.1	601.6	619.3	637.1	655.1	673.2	691.5
10	521.5	542.3	563.4	584.6	606.1	627.7	649.5	671.5	693.7	716.1	738.6

Total energy changes are estimated from the quadcopter's kinetic and potential energies while hovering at an altitude of 50 meters AGL. Fig. 10 illustrates how the total energy of the quadcopter changes over time under different rocket launch velocities at specific time ranges. This analysis helps us understand the overall energy state of the quadcopter during flight which is crucial for energy efficient design or control systems.

Understanding energy changes in the quadcopter is crucial for evaluating system performance, efficiency, and potential limitations under different operational conditions.

Other variables such as aerodynamic effects and control systems are not considered in these specific analyses. These effects can significantly influence the observed relationship between the quadcopter and the rocket. More comprehensive analyses considering these variables, simulations, or empirical studies might be necessary to fully understand the complex relationship between rocket launch velocity and quadcopter behaviour.

5.2. The acceleration function of the quadcopter

The translational acceleration function of the quadcopter is employed from Eq. 61. Therefore, the following function delivers the quadcopter's acceleration function of time.

$$f(\ddot{x}') = -4.91 - 0.75 \times \sqrt{\frac{v_{0_{rocket}}^2}{4} - 735.75}\ (m/s^2) \tag{65}$$

The method has converged to $x_0 = 59.99$ m/s within the specified tolerance and intervals. The method has succeeded in providing an approximate solution within the interval and tolerance within 24 iterations.

However, this function's convergence rate is also 0, which, as mentioned in Subsection 5.1, typically indicates the method is neither converging nor converging extremely slow. The assumptions could be thoroughly examined by adjusting method intervals or the algorithm of the method to converge the function to a better conclusion; however, due to the hardware utilised to calculate these functions, the additional studies could not be computed.

The graph of root approximation against iteration in Fig. 11 illustrates how the approximation of the root changes with each iteration. As the iteration continues, the value approaches a true root value, which, in this case, is $x_0 = 59.99$ m/s.

The graph of interval contraction in Fig. 12 illustrates how the size of the interval changes with each iteration. The interval size drops towards the root, which is ideal.

The graph of convergence rate against iteration in Fig. 13 illustrates how fast the method converges towards the root. Ideally, the convergence rate should approach a value close to 0.5 for effective convergence.

The graph of the error reduction rate against iteration in Fig. 14 illustrates how the error between the estimated root and actual root reduces with each iteration. A decreasing trend indicates the method is converging.

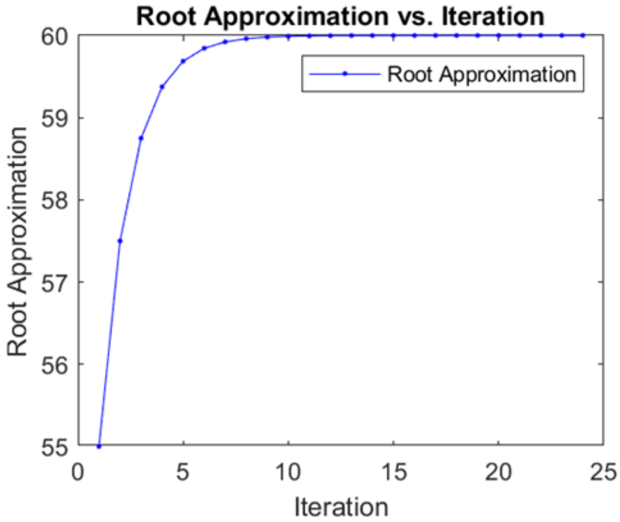

Fig. 11. Root approximation vs. iteration (acceleration)

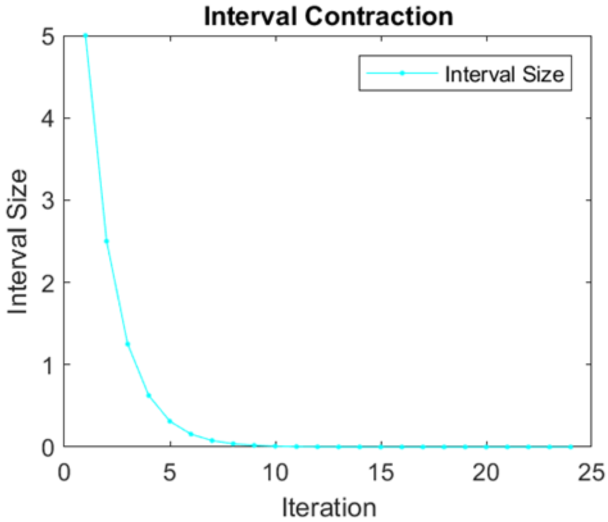

Fig. 12. Interval contraction (acceleration)

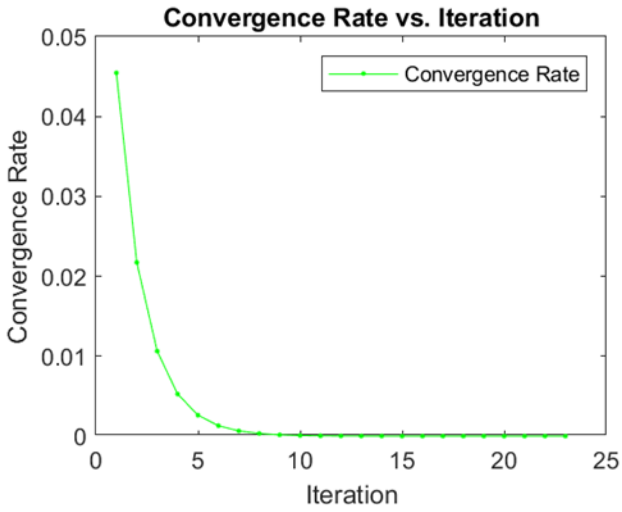

Fig. 13. Convergence rate vs. iteration (acceleration)

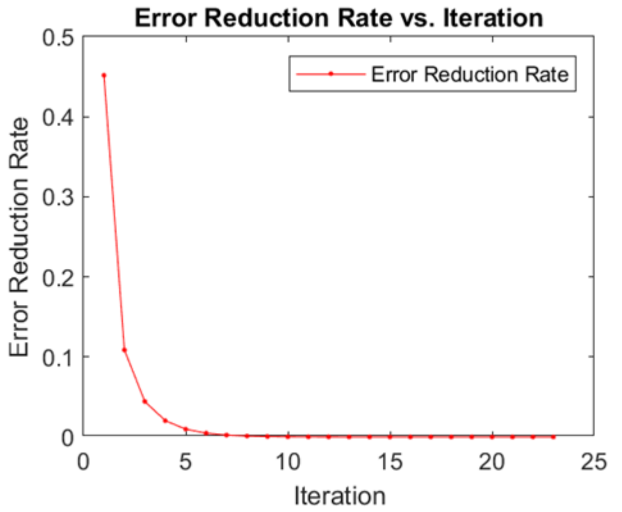

Fig. 14. Error reduction rate vs. iteration (acceleration)

Furthermore, the trend between the quadcopter's acceleration and the rocket's velocity is illustrated in Fig. 15, and values are shown in Table 8.

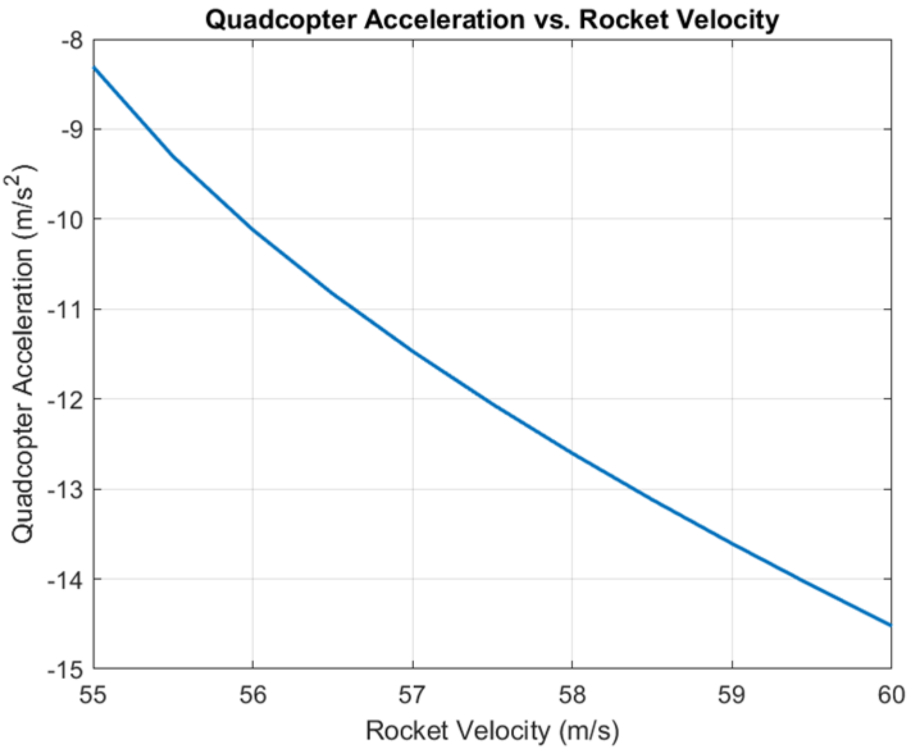

Fig. 15. Quadcopter acceleration against rocket launch velocity in m/s^2

Table 8. Corresponding rocket velocities to quadcopter accelerations in each iteration

Iteration no.	**Rocket launch velocity, m/s**	**Quadcopter accelerations, m/s^2**
1	55.00000	-8.30577
2	57.50000	-12.05717
3	58.75000	-13.36675

4	59.37500	-13.95979
5	59.68750	-14.24439
6	59.84375	-14.38404
7	59.92188	-14.45322
8	59.96094	-14.48766
9	59.98047	-14.50485
10	59.99023	-14.51343
11	59.99512	-14.51772
12	59.99756	-14.51986
13	59.99878	-14.52093
14	59.99939	-14.52147
15	59.99969	-14.52174
16	59.99985	-14.52187
17	59.99992	-14.52194
18	59.99996	-14.52197
19	59.99998	-14.52199
20	59.99999	-14.52200
21	60.00000	-14.52200
22	60.00000	-14.52200
23	60.00000	-14.52200
24	60.00000	-14.52200

A linear and steady acceleration is shown in Fig. 16 and Table 9. The result is that the quadcopter is subjected to a constant increase in velocity, indicating a constant acceleration. The quadcopter's acceleration is calculated using Eq. (65). This equation suggests that the quadcopter's acceleration remains constant regardless of changes in the rocket launch velocities.

Other variables such as aerodynamic effects and control systems are not considered in these specific analyses. These effects can significantly influence the observed relationship between the quadcopter and the rocket. More comprehensive analyses considering these variables, simulations, or empirical studies might be necessary to fully understand the complex relationship between rocket launch velocity and quadcopter behaviour.

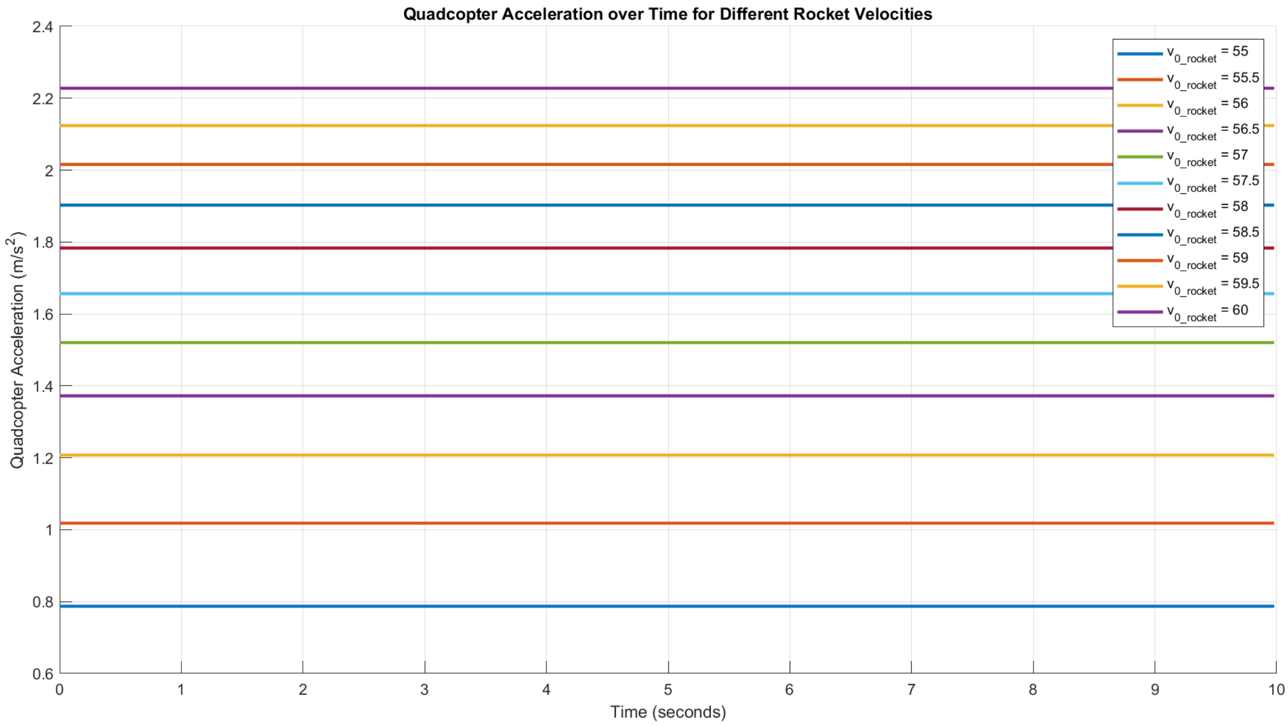

Fig. 16. Quadcopter acceleration over time for different rocket velocities

Table 9. Acceleration $\left(a_{q_x}\right)$ values of each rocket launch velocity at specific times

Time, s	**Acceleration $\left(a_{q_x}\right)$, m/s^2**										
$v_{0_{rocket}}$, m/s	**55.0**	**55.5**	**56.0**	**56.5**	**57.0**	**57.5**	**58.0**	**58.5**	**59.0**	**59.5**	**60.0**
0	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
1	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
2	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
3	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
4	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
5	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
6	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
7	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
8	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
9	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2
10	0.8	1.0	1.2	1.4	1.5	1.7	1.8	1.9	2.0	2.1	2.2

Conclusions

1. A mathematical model of quadcopter in pre- and post-launch environments was created. The mathematical model includes equations defining the system's position and orientation in space, linear and angular velocity, total thrust, total moment, torque, rotational and translational acceleration, and recoil evaluation, all linearized and presented in state space form.
2. The developed mathematical model was verified by assuming that the quadcopter flying at an altitude of 50 metres AGL, with a downward pitch angle of 30°. Dependent variables and independent variables were presented. Dependent variables include the quadcopter's reaction force, velocity, and acceleration, while independent variables involve the quadcopter and rocket mass, system mass, gravitational acceleration, rocket launch altitude, and rocket launch velocities.
3. The model's validity was affirmed by successfully determining the convergence of calculated velocities when $\left[f\left(v_{0_{rocket}}\right), v_{0_{rocket}}\right] = [0,\ \ 59.99\ m/s]$.
4. The model analysis allows us to determine the mathematical model's accuracy and capability to simulate the quadcopter's behaviour during a rocket launch.
5. The model simulation illustrated the quadcopter's position, energy, velocity, and acceleration over time for different rocket launch velocities, and the numerical values of such simulation were presented. Understanding the behaviour of the quadcopter for different rocket launch velocities helps to determine the navigation algorithms and/or control systems for the quadcopter.
6. To ensure a systematic validation of the system's mathematical model the dichotomy methodology was used. The success is determined by root convergence for each bound. The generated MATLAB graphs illustrate the relationship between quadcopter velocity and acceleration for specific timeframes across different rocket launch velocities in the post-launch scenario.
7. Other variables such as aerodynamic effects and control systems are not considered in these specific analyses. These effects can significantly influence the observed relationship between the quadcopter and the rocket. More comprehensive analyses considering these variables, simulations, or empirical studies might be necessary to fully understand the complex relationship between rocket launch velocity and quadcopter behaviour.
8. Real experiments should be conducted to verify the accuracy of the model so that the model can be trusted and based on it, real drones that shoot rockets can be created.

List of references

1. AHMAD, Faraz, P. KUMAR and P. P. PATIL. Modeling and Simulation of a Quadcopter with Altitude and Attitude Control. *Nonlinear Studies* [online]. 2018. Vol. 25, no. 2, pp. 287–299 [viewed 21 November 2023]. Available from: https://www.researchgate.net/publication/325627021_Modeling_and_simulation_of_a_quadcopter_with_altitude_and_attitude_control
2. BARAUSKAS, Rimantas, A. FEDARAVIČIUS, and K. JASAS. Structural Model Analysis for the Laser Guided Mobile Short-Range Air Defence System. *Problemy Mechatroniki* [online]. 31 March 2023. Vol. 14, no. 1, pp. 9–22 [viewed 30 November 2023]. DOI: 10.5604/01.3001.0016.2957
3. BENIC, Zoran, P. PILJEK and D. KOTARSKI. Mathematical Modelling of Unmanned Aerial Vehicles with Four Rotors. *Interdisciplinary Description of Complex Systems* [online]. 2016. Vol. 14, no. 1, pp. 88–100 [viewed 21 November 2023]. DOI: 10.7906/indecs.14.1.9.
4. BENZKOFER, Philip D. and ARMY ARMAMENT RESEARCH DEVELOPMENT AND ENGINEERING CENTER PICATINNY ARSENAL NJ. *Dynamic Analysis of Shoulder-Fired Weapons* [online]. Army Armament Research, Development and Engineering Center, 1993 [viewed 21 November 2023]. Available from: https://apps.dtic.mil/sti/citations/ADP009073
5. BERGAMASCO, Marco and M. LOVERA. Identification of Linear Models for the Dynamics of a Hovering Quadrotor. *IEEE Transactions on Control Systems and Technology* [online]. 6 February 2014. Vol. 22, no. 5, pp. 1696–1707 [viewed 21 November 2023]. DOI: 10.1109/tcst.2014.2299555
6. BOLANDI, Hossein, M. REZAEI, R. MOHSENIPOUR, H. NEMATI and S. M. SMAILZADEH. Attitude Control of a Quadrotor with Optimized PID Controller. *Intelligent Control and Automation* [online]. 2013. Vol. 04, no. 03, pp. 335–342 [viewed 21 November 2023]. DOI: 10.4236/ica.2013.43039.
7. BOUABDALLAH, Samir and R. SIEGWART. Full Control of a Quadrotor. *Intelligent Robots and Systems* [online]. 10 December 2007 [viewed 21 November 2023]. DOI: 10.1109/iros.2007.4399042
8. CHOVANCOVÁ, Anežka, T. FICO, Ľ. CHOVANEC and P. HUBINSK. Mathematical Modelling and Parameter Identification of Quadrotor (A Survey). *Procedia Engineering* [online]. 2014. Vol. 96, pp. 172–181 [viewed 21 November 2023]. DOI: 10.1016/j.proeng.2014.12.139
9. DAS, Abhijit, F. LEWIS and K. SUBBARAO. Dynamic Inversion with Zero-Dynamics Stabilisation for Quadrotor Control. *IET Control Theory & Applications* [online]. 1 March 2009. Vol. 3, no. 3, pp. 303–314 [viewed 21 November 2023]. DOI: 10.1049/iet-cta:20080002.
10. DE JESUS RUBIO, Jose, J. HUMBERTO PEREZ CRUZ, Z. ZAMUDIO and A. J. SALINAS. Comparison of Two Quadrotor Dynamic Models. *IEEE Latin America Transactions* [online]. June 2014. Vol. 12, no. 4, pp. 531–537 [viewed 21 November 2023]. DOI: 10.1109/tla.2014.6868851.
11. ELHAMMOUDY, Abdelfattah, M. ELYAQOUTI, D. B. HMAMOU, E. H. ARJDAL, D. SAADAOUI, S. LIDAIGHBI and I. A. CHOULLI. Novel Numerical Method for Estimation the Photovoltaic Cells/Modules Parameters Based on Dichotomy Method. *Results in Optics* [online]. 1 July 2023. Vol. 12, pp. 100445–100445 [viewed 30 November 2023]. DOI: 10.1016/j.rio.2023.100445

12. ELRUBY, Ahmed Y., M. EL-KHATIB, N. EL-AMARY and A. HASHAD. Dynamic Modeling and Control of Quadrotor Vehicle. *The International Conference on Applied of Mechanics and Mechanical Engineering* [online]. 1 May 2012. Vol. 15, no. 15, pp. 1–10 [viewed 21 November 2023]. DOI: 10.21608/amme.2012.37092.
13. EL-SAADY, W. M., A. A. ABD ALLAH, A. Z. IBRAHIM and A. HUSSIEN. H. Dynamic Simulation of Tank Gun Recoil Cycle. *The International Conference on Applied Mechanics and Mechanical Engineering (Print)* [online]. 1 May 2014. Vol. 16, no. 16, pp. 1–11 [viewed 30 November 2023]. DOI: 10.21608/amme.2014.35481
14. FARAZ, Ahmed, P. KUMAR and P. P. PATIL. Modeling and Simulation of a Quadcopter UAV. *Nonlinear Studies* [online]. 2016. Vol. 24, no. 4, pp. 553–561 [viewed 21 November 2023]. Available from: https://www.researchgate.net/publication/316688075_Modeling_and_simulation_of_a_quadcopter_UAV
15. FRESCONI, Frank, I. CELMINS, S. SILTON and M. COSTELLO. High Maneuverability Projectile Flight Using Low Cost Components. *Aerospace Science and Technology* [online]. February 2015. Vol. 41, pp. 175–188 [viewed 21 November 2023]. DOI: 10.1016/j.ast.2014.12.007
16. GREENWOOD, Donald T. *Principles of Dynamics*. Prentice Hall, 1988. ISBN 9781483139081, 1483139085.
17. HALL, Nancy. Newton's Laws of Motion. *Glenn Research Center, NASA* [online]. 27 October 2022 [viewed 5 December 2023]. Available from: https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/newtons-laws-of-motion/#:~:text=His%20third%20law%20states%20that
18. HAMEL, Tarek, R. MAHONY, R. LOZANO and J. OSTROWSKI. Dynamic Modelling and Configuration Stabilization for an X4-Flyer. *IFAC Proceedings Volumes* [online]. 1 January 2002. Vol. 35, no. 1, pp. 217–222 [viewed 21 November 2023]. DOI: 10.3182/20020721-6-es-1901.00848.
19. HOFFMANN, Gabriel, H. HUANG, S. WASLANDER and C. TOMLIN. Quadrotor Helicopter Flight Dynamics and Control: Theory and Experiment. *AIAA Guidance, Navigation and Control Conference and Exhibit* [online]. 15 June 2007 [viewed 1 December 2023]. Available from: DOI: 10.2514/6.2007-6461.
20. HUANG, Qikai and B. WINGO. Mathematical Modeling of Quadcopter Dynamics Mathematical Modeling of Quadcopter Dynamics. *Rose-Hulman Institute of Technology* [online]. 30 May 2016. [viewed 21 November 2023]. Available from: https://scholar.rose-hulman.edu/math_mstr/158/
21. IMPERIAL WAR MUSEUMS. A Brief History of Drones. *Imperial War Museums* [online]. 2018 [viewed 5 December 2023]. Available from: https://www.iwm.org.uk/history/a-brief-history-of-drones
22. ISTVÁN, Papp. Missile Mathematical Model and System Design. *Academic and Applied Research in Military and Public Management Science* [online]. 30 April 2017. Vol. 16, no. 1, pp. 29–35 [viewed 30 December 2023]. DOI: 10.32565/aarms.2017.1.3
23. JACOBSEN, Mark. Ukraine's Drone Strikes Are a Window into the Future of Warfare. *Atlantic Council* [online]. 14 September 2023 [viewed 5 December 2023]. Available from: https://www.atlanticcouncil.org/blogs/new-atlanticist/ukraines-drone-strikes-are-a-window-into-the-future-of-warfare/

24. KATIAR, Asma, R. RASHDI, Z. ALI and U. BAIG. Control and Stability Analysis of Quadcopter. *IEEE Xplore* [online]. 1 March 2018 [viewed 17 December 2023]. Available from: DOI: 10.1109/icomet.2018.8346419.
25. KHAN, M. Rehan and S. MALIK. Conversion of Nonlinear Quadcopter Mathematical Model to Linear Simplified Model. *International Journal of Science and Research (IJSR)* [online]. 5 June 2016. Vol. 5, no. 6, pp. 1954–1956 [viewed 21 November 2023]. DOI: 10.21275/v5i6.nov164635.
26. KOSE, Oguz and T. OKTAY. Dynamic Modeling and Simulation of Quadrotor for Different Flight Conditions. *European Journal of Science and Technology* [online]. 2 March 2019. No. 15, pp. 132–142 [viewed 21 November 2023]. DOI: 10.31590/ejosat.507222.
27. KULLAB, Samya. Ukraine is Building an Advanced Army of Drones. For Now, Pilots Improvise with Duct Tape and Bombs. *AP News* [online]. 25 September 2023 [viewed 5 December 2023]. Available from: https://apnews.com/article/drones-ukraine-war-russia-innovation-technology-589f1fc0e0db007ea6d344b197207212
28. KURAK, Sevkuthan and M. HODZIC. Control and Estimation of a Quadcopter Dynamical Model. *Periodicals of Engineering and Natural Sciences (PEN)* [online]. 19 March 2018. Vol. 6, no. 1, pp. 63 [viewed 21 November 2023]. DOI:10.21533/pen.v6i1.164.
29. LUUKKONEN, Teppo. Modelling and Control of Quadcopter. *Aalto University School of Science* [online]. 22 August 2021 [viewed 30 December 2023]. Available from: https://sal.aalto.fi/publications/pdf-files/eluu11_public.pdf
30. MACKO, Martin, B. V. VO and Q. T. ANH. Dynamics of Short Recoil-Operated Weapon. *Problems of Mechatronics Armament Aviation Safety Engineering* [online]. 30 September 2021. Vol. 12, no. 3, pp. 9–26 [viewed 30 November 2023]. DOI: 10.5604/01.3001.0015.2432
31. MAHONY, Robert, V. KUMAR and P. CORKE. Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor. *IEEE Robotics & Automation Magazine* [online]. September 2012. Vol. 19, no. 3, pp. 20–32 [viewed 21 November 2023]. DOI: 10.1109/mra.2012.2206474
32. MERIÇ, Tunç B. An Active Rocket Launcher Design for an Attack Helicopter. *Middle East Technical University* [online]. 2018 [viewed 21 November 2023]. Available from: https://open.metu.edu.tr/handle/11511/27128
33. MISTLER, V., A. BENALLEGUE, and N M'SIRDI. Exact Linearization and Noninteracting Control of a 4 Rotors Helicopter via Dynamic Feedback. *Robot and Human Interactive Communication* [online]. 18 September 2001. Pp. 586–593. [viewed 21 November 2023]. DOI: 10.1109/roman.2001.981968.
34. NEWMAN, Richard J. The Little Predator That Could. *Air & Space Forces Magazine* [online]. 1 March 2002 [viewed 5 December 2023]. Available from: https://www.airandspaceforces.com/article/0302predator/
35. PAEZ, Thomas. Introduction to Model Validation. *Sandia National Laboratories* [online]. 2008 [viewed 30 December 2023]. Available from: https://www.osti.gov/servlets/purl/1142730
36. PASHAYEV, Adalat and E. SABZIEV. Refinement of the Parameters of a Mathematical Model of Quadcopter Dynamics. *Scientific Journal of Silesian University of Technology. Series Transport* [online]. 1 December 2020. Vol. 109, pp. 141–151 [viewed 21 November 2023]. DOI: 10.20858/sjsutst.2020.109.13

37. POTAPOV, M.M. Dichotomy Method - Encyclopedia of Mathematics. *Encyclopedia of Mathematics* [online]. 5 June 2020 [viewed 12 May 2023]. Available from: https://encyclopediaofmath.org/wiki/Dichotomy_method
38. POUNDS, P., R. MAHONY and P. CORKE. Modelling and Control of a Large Quadrotor Robot. *Control Engineering Practice* [online]. July 2010. Vol. 18, no. 7, pp. 691–699 [viewed 21 November 2023]. DOI: 10.1016/j.conengprac.2010.02.008
39. READE, Leslie. Bombs over Venice. *History Today* [online]. June 1958. Vol. 8, no. 6 [viewed 5 December 2023]. Available from: https://www.historytoday.com/archive/bombs-over-venice
40. SALIH, Atheer L., M. MOGHAVVEMI and K. GAEID. Flight PID Controller Design for a UAV Quadrotor. *Scientific Research and Essays* [online]. 4 December 2010. Vol. 5, no. 23, pp. 3660–3667. [viewed 21 November 2023]. Available from: https://www.researchgate.net/publication/230633819_Flight_PID_Controller_Design_for_a_UAV_Quadrotor
41. SHERMAN, Jason. Drone-on-Drone Combat in Ukraine Marks a New Era of Aerial Warfare. *Scientific American* [online]. 3 April 2023 [viewed 5 December 2023]. Available from: https://www.scientificamerican.com/article/drone-on-drone-combat-in-ukraine-marks-a-new-era-of-aerial-warfare/
42. SHUBOV, Mikhail V. Drone Launched Short Range Rockets. *Aerospace* [online]. 8 June 2020. Vol. 7, no. 6, p. 76 [viewed 21 November 2023]. DOI: 10.3390/aerospace7060076
43. SZUPIEŃKO, Damian and R. WOŹNIAK. Preliminary Physical and Mathematical Model of the Recoil Operated Firearm within the Bolt Recoil Period. *Problemy Mechatroniki* [online]. 31 March 2021. Vol. 12, no. 1, pp. 59–74 [viewed 30 November 2023]. DOI: 10.5604/01.3001.0014.7851
44. TAGAY, Adilet, A. OMAR and H. ALI. Development of Control Algorithm for a Quadcopter. *Procedia Computer Science* [online]. 1 January 2021. Vol. 179, pp. 242–251 [viewed 21 November 2023]. DOI: 10.1016/j.procs.2021.01.003.
45. TAHIR, Zaid. State Space System Modeling of a Quad Copter UAV. *Indian Journal of Science and Technology* [online]. 20 January 2015. Vol. 8, no. 1, p. 1–5 [viewed 12 May 2023]. DOI: 10.17485/ijst/2016/v9i27/95239
46. TAYEBI, Abdelhamid and S. MCGILVRAY. Attitude Stabilization of a VTOL Quadrotor Aircraft. *IEEE Transactions on Control Systems Technology* [online]. May 2006. Vol. 14, no. 3, pp. 562–571 [viewed 21 November 2023]. DOI: 10.1109/tcst.2006.872519
47. U.S. ARMY UNMANNED AIRCRAFT SYSTEMS CENTER OF EXCELLENCE. *U.S. Army Unmanned Aircraft Systems Roadmap 2010-2035: Eyes of the Army* [online]. U.S. Army Unmanned Aircraft Systems Center of Excellence, 2010, pp. 12-13 [viewed 2 December 2023]. Available from: https://books.google.lt/books?id=hb9NnQAACAAJ
48. USMAN, Muhammad. Quadcopter Modelling and Control with MATLAB/Simulink Implementation. *LAB University of Applied Sciences* [online]. 2020 [viewed 21 November 2023]. Available from: https://www.theseus.fi/handle/10024/333992
49. WANG, Pengcheng, Z. MAN, Z. CAO, J. ZHENG and Y. ZHAO. Dynamics Modelling and Linear Control of Quadcopter. *IEEE Xplore* [online]. 1 November 2016. Pp. 498–503 [viewed 21 November 2023]. DOI: 10.1109/ICAMechS.2016.7813499.

50. XIONG, Jing-Jing and E. ZHENG. Position and Attitude Tracking Control for a Quadrotor UAV. *ISA Transactions* [online]. May 2014. Vol. 53, no. 3, pp. 725–731 [viewed 21 November 2023]. DOI: 10.1016/j.isatra.2014.01.004