

Application of the Seasonal Holt-Winters Model to Study Exchange Rate Volatility

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The paper proposes a new approach to investigating the dynamics of hourly exchange rates of two currencies – the Euro (EUR) and US dollar (USD). The dynamics of foreign exchange (forex) rate is a complex process that can be better understood through a study of its characteristics, such as for instance volatility. In this article the exchange rate fluctuation is analysed by calculating the sum of absolute differences (SAD) of a time series per hour. It has been shown empirically that a new time series constructed from SAD values is more suitable for predicting exchange rate volatility if it takes into account only the magnitude of the exchange rate fluctuation and ignores its direction. The analysis of EUR/USD exchange rate data at major financial centers has revealed that both exchange rates undergo periodical intraday variations; therefore, both additive and multiplicative versions of Holt-Winters exponential smoothing techniques have been applied in the analysis and predict of exchange rate volatility. These methods are appropriate for a time series with a linear trend and seasonal variations. Since a time series of SAD values does not have a clear trend, simplified versions (without changes in trend) of the Holt-Winters model were applied. Two different statistics - mean absolute error (MAE) and root mean squared error (RMSE) - were applied to select the optimal parameters of four versions of the Holt-Winters model. The study showed that volatility is best predicted by a simplified version of the multiplicative Holt-Winters model.

Keywords: *Holt-Winters Model, Volatility of Exchange Rate, Prediction, Optimal Model, Mean Absolute Error.*

Introduction

The prediction of exchange rate volatility is a challenging problem in financial market studies. It is widely believed that increased exchange rate volatility inhibits the growth of foreign trade. Various econometrical models are widely used in predicting foreign exchange rates. The construction of an optimal model is a complicated issue due to a number of factors which can influence the volatility of foreign exchange rates. The value and factor effects on the exchange rate change over time; therefore investors have to be cautious, as there can always be a certain level of risk in investing foreign currencies.

A foreign exchange rate between two currencies is the rate at which one currency will be exchanged for another, namely the ratio of currency prices. The exchange rate dynamics is a complex process that can be better understood through the investigation of its characteristics. Usually, models of exchange rate volatility forecasting are based on history in time series. A wide range of methods are devoted to predicting a currency exchange rate (Andersen *et al.*, 2001, 2003; Baillie *et al.*, 1989; Bollerslev, 1986, 2010; Giacomini *et al.*, 2006; Meese *et al.*, 1983; Visser, 2010; Yoon *et al.*, 2008; Zhang *et al.*, 2004; Diebold *et al.*, 1995). Some authors investigated the intraday exchange rate volatility pattern in the 24-hour exchange rate markets (Baillie *et al.*, 1990; Admati *et al.*, 1988; Andersen *et al.*, 1998; Edirington *et al.*, 2001; Rhee *et al.*, 1992; Cyree *et al.*, 2004). For example (Bailler, 1990 & Andersen, 1998) determined that the dynamics of exchange rate intraday volatility follows evident regularities within the twenty four hour period. Other authors identified that foreign exchange

rate market fluctuations are best analyzed when a number of changes are taken into consideration at the same time (Ane *et al.*, 2000; Oomen, 2006; Griffin *et al.*, 2008). It has been shown in several studies that suitable information for volatility forecasting can be obtained from the study of exchange rate option prices (Jorion, 2005; Xu *et al.*, 1995) and realized volatility (Andersen *et al.*, 1998). Empirical studies of realized volatility models based on long memory forecasts revealed that models based on short memory applied to high frequency data are more accurate in predicting exchange rate volatility (Gallantat *et al.*, 1999; Alizadeh *et al.*, 2002).

Prior research, however, has not analysed the sums of absolute differences of rates per second in predicting exchange volatility with specific versions of the seasonal Holt-Winters model. Hence, this paper mainly follows this approach and looks into the reasons of the volatility seasonality of intraday EUR/USD exchange rates. We studied exchange rates every second a day. Since in the observed data seasonality per day was identified, the Holt-Winters exponential smoothing model was used for research purposes. The time series of SAD values takes into account only the magnitude of exchange rate fluctuation and ignores its direction. The empirical analysis of the time series of SAD values revealed that there is no clear trend, so two reduced versions (without changes in trend) of the Holt-Winters model were studied too. The paper proposes a new approach in applying these models. The simplified version of the Holt-Winters model allows reducing extreme volatilities. We investigated the applicability and evaluated the accuracy of four versions of Holt-Winters model in predicting the exchange volatility to the EUR/USD currency

pair, i.e. a multiplicative seasonal model, an additive seasonal model and two simplified versions of these models.

The aim of the research is to study the possibility of applying four versions of the Holt-Winters model in predicting the volatility of foreign exchange rates.

Objectives of the research:

- To construct a new time series from historical data of exchange rates.
- To apply the original and slightly modified versions of the Holt–Winters model in predicting exchange rate volatility.
- Estimate prediction errors and optimal parameters of the models.
- To test the models and suggest the methodology for selecting an appropriate model.

Methods used for research are methods applied in statistics and financial time series.

Intraday volatility data

For our research we used the ratio between the US Dollar and Euro each second, from 10:00PM of 31 January, 2009 to 10:00PM of 24 December, 2012. The initial data (Gain Capital, 2013) was downloaded in the form of a triple (t_i, ask_i, bid_i) , where t_i denotes a time moment of ask and bid prices: ask_i, bid_i . By averaging these prices we gained pairs of variables (t_i, x_i) , where $x_i = \frac{(ask_i + bid_i)}{2}$. Every SAD value $y_j, j = 1, 2, \dots$ was calculated per each hour j according to the following formula:

$$y_j = \sum_{i=k_{j-1}}^{k_j-1} |x_{i+1} - x_i|, \quad (1)$$

where k_j denotes the index of the last value of x_i in j^{th} hour.

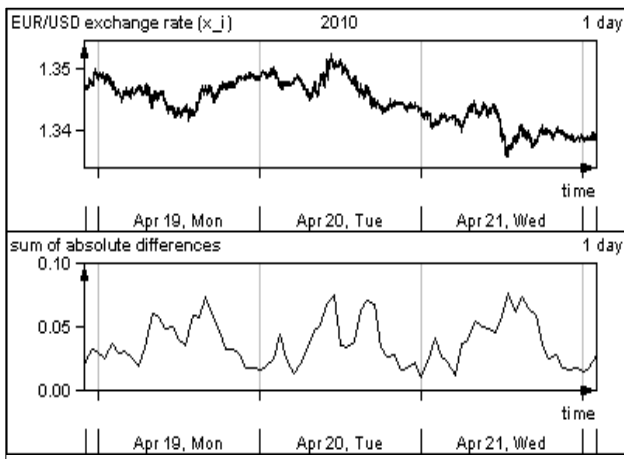


Figure 1. Initial data values x_i (graph I) and resulting SAD values y_j (graph II)

As seen in Figure 1, the intraday exchange rate volatility follows an evident regular pattern. This could be partly explained by continuous trading of foreign currency due to overlapping working hours of many financial centers with different trading volumes across the world. This is illustrated in Table 1 (Forex market hours, 2013) with the exchange rate data of April 2013.

Table 1

Financial Market Regions and Daily Trading Volumes

Region	Major countries	Major financial centers	Average daily trading volume, April 2013 (US billions)
Asian-Pacific	Japan, China, Australia, New Zealand, Russia	Tokyo, Hong Kong, Sydney, Wellington	966 (19%)
Europe	France, Germany, Switzerland, United Kingdom	London, Frankfurt	2 780 (55%)
America	Canada, Mexico, South America	New York, Chicago	1 159 (23%)
Others	---	---	151 (3%)

As seen in the Table 1, the larger part of trading volume takes place in Europe. The above mentioned facts about trading centers are represented by SAD values in Figure 2.

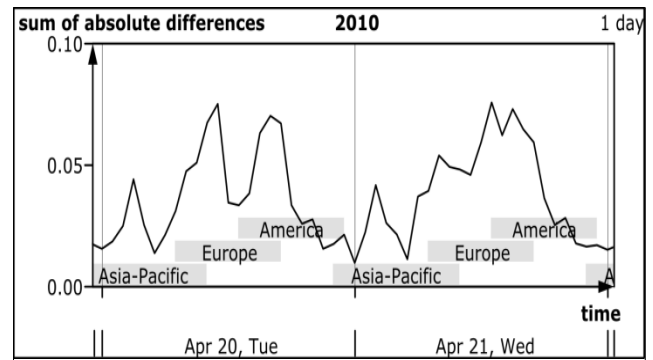


Figure 2. SAD values y_j and time spans of working hours of financial markets in regions

From the above graph it can be seen that European and American financial regions have the strongest influence on foreign exchange rate fluctuations. On the other hand, the intraday exchange rate seasonal volatility pattern changes slightly every day. For this reason the Holt-Winters model was applied.

The Holt-Winters Model and Forecasting Errors

The Holt-Winters model (Holt, 1957; Prajakta, 2004, Yar *et al.*, 1990) was developed to predict trends and seasonality from exponentially weighted averages. The additive version of the model can be described by the following formulas:

$$\begin{aligned}
 L_j &= \alpha(y_j - S_{j-s}) + (1 - \alpha)(L_{j-1} + b_{j-1}) \\
 b_j &= \beta(L_j - L_{j-1}) + (1 - \beta)b_{j-1} \\
 S_j &= \gamma(y_j - L_j) + (1 - \gamma)S_{j-s} \\
 F_{j+1} &= L_j + b_j + S_{j+1-s} \\
 j &= s + 1, s + 2, \dots
 \end{aligned} \quad (2)$$

where $\alpha, \beta, \gamma \in [0, 1]$ are the smoothing parameters, L_j is the smoothed level at time j , b_j is the change in the trend at moment j , S_j is the seasonal smooth at moment j , s is the number of periods in the season, and F_{j+1} is one step ahead

forecasted value. The initial values are calculated by the following formulas:

$$\begin{aligned} L_s &= \frac{1}{s} \\ b_s &= \frac{1}{s} \left[\frac{y_{s+1}-y_1}{s} + \frac{y_{s+2}-y_2}{s} + \dots + \frac{y_{s+s}-y_s}{s} \right] \\ S_j &= y_j - L_s, \quad j = 1, 2, \dots, s. \end{aligned} \quad (3)$$

The multiplicative version of the model is as follows:

$$\begin{aligned} L_j &= \alpha \left(\frac{y_j}{S_{j-s}} \right) + (1 - \alpha)(L_{j-1} + b_{j-1}) \\ b_j &= \beta(L_j - L_{j-1}) + (1 - \beta)b_{j-1} \\ S_j &= \gamma \left(\frac{y_j}{L_j} \right) + (1 - \gamma)S_{j-s} \\ F_{j+1} &= (L_j + b_j)S_{j+1-s} \\ j &= s + 1, s + 2, \dots \end{aligned} \quad (4)$$

The initial values of the model are computed using the same formulas as in the additive version of the model, except for seasonal variables:

$$S_j = \frac{y_j}{L_s}, \quad j = 1, 2, \dots, s \quad (5)$$

The data of the first two seasons is needed to calculate the initial value of b .

Since the time series of SAD values does not have a clear trend, the simplified version (without variable b) of the Holt-Winters model will be applied. Next the results of both models will be compared to validate the applicability of the simplified model. The simplified additive model is described as follows:

$$\begin{aligned} L_j &= \alpha(y_j - S_{j-s}) + (1 - \alpha)L_{j-1} \\ S_j &= \gamma(y_j - L_j) + (1 - \gamma)S_{j-s} \\ F_{j+1} &= L_j + S_{j+1-s} \\ j &= s + 1, s + 2, \dots \end{aligned} \quad (6)$$

The initial values L_s and S_j of the simplified model are calculated according to the above mentioned formulas (3). The reduced multiplicative model is redefined as follows:

$$\begin{aligned} L_j &= \alpha \left(\frac{y_j}{S_{j-s}} \right) + (1 - \alpha)L_{j-1} \\ S_j &= \gamma \left(\frac{y_j}{L_j} \right) + (1 - \gamma)S_{j-s} \\ F_{j+1} &= L_j S_{j+1-s} \\ j &= s + 1, s + 2, \dots \end{aligned} \quad (7)$$

The initial values of L_s and S_j are calculated in the same way as in the original multiplicative model.

The numerical approach will be used for selecting parameters of the optimal model. The forecasting precision of each model under investigation will be evaluated by using data from y_{i_1} to y_{i_2} . The mean absolute error (MAE) and root mean squared error (RMSE) calculated by the below formulas (Johnson *et al.*, 1990; Fat *et al.*, 2011)

$$\begin{aligned} MAE &= \frac{1}{n} \sum_{j=k_1}^{k_2} |F_j - y_j| = \frac{1}{n} \sum_{j=k_1}^{k_2} |e_j| \\ RMSE &= \sqrt{\frac{1}{n} \sum_{j=k_1}^{k_2} (F_j - y_j)^2} = \sqrt{\frac{1}{n} \sum_{j=k_1}^{k_2} e_j^2}, \quad n = i_2 - i_1 + 1 \end{aligned}$$

will be used to select the optimal parameters of the model. Let us consider an ordered sequence of errors for the discrete values from the interval $[y_{i_1}, y_{i_2}]$:

$$e_{j_1} \leq e_{j_2} \leq \dots \leq e_{j_n}, \quad e_{j_q} = F_{j_q} - y_{j_q} \quad (8)$$

The absolutely maximum values from the sequence of errors will be eliminated one by one until remaining terms are distributed by the normal law. The remaining sequence $e_{j_{min}}, e_{j_{min+1}}, \dots, e_{j_{max}}$ will be used to calculate the mean absolute normal error (MANE) according to the formula

$$MANE = \frac{1}{max - min + 1} \sum_{q=min}^{max} |e_{j_q}| \quad (9)$$

Otherwise, it is possible to evaluate the deviation from the normal distribution by applying the measure of deviation *RMSNE* according to the below formula:

$$\begin{aligned} RMSNE &= \sqrt{\frac{1}{max - min + 1} \sum_{q=min}^{max} (E_q - e_{j_q})^2}, \end{aligned} \quad (10)$$

where

$$E_q = \Phi^{-1}(P(j_q)) \cdot \sqrt{D(e_{j_q})} + E(e_{j_q}),$$

$q = min, min + 1, \dots, max,$

$P(j_q) = (q - min + 1)/(max - min + 1)$, $P(j_q)$ is percentile, Φ^{-1} is the inverse of normal cumulative distribution function, $E(e_{j_q})$, $D(e_{j_q})$ are the mean and variance of the selected subsequence. The portion of rejected errors (RE) which were caused by some extremely influential events, is calculated by the following formula:

$$RE = \frac{(min - 1) + (n - max)}{n} \quad (11)$$

The ‘expected maximum error’ (EME), under the absence of unpredictable events, is calculated as:

$$EME = \max(|e_{j_{min}}|, |e_{j_{max}}|) \quad (12)$$

Methodology

Foreign exchange rate dynamics is influenced by various factors. Some of them cause gradual changes, thus the Holt-Winters model is applicable only theoretically, while others are spontaneous and unprecedented. We assume that the extreme change of the exchange rate is the consequence of an unpredictable event (in the context of a certain model configuration) if its forecasted absolute error is larger than the expected maximum error. Thus, theoretically it a certain amount of the maximal absolute errors can be ignored under the assumption that these are the results of the events which cannot be properly modeled. Therefore the MANE statistics will be used to evaluate and interpret the performance of the model.

Two criteria – MAE and RMSE – will be investigated for the selection of the optimal model parameters. The model which minimizes MAE value can produce some extreme predicting errors as long as the mean of absolute errors remains close to minimal. On the other hand, a model with the same MAE value can produce more stable

predictions, thus the statistics ignores the dispersion of absolute forecasted errors. From the definition of RMSE, it can be seen that if can be minimized with respect to e , then the extreme errors can be minimized too.

$$RMSE = \sqrt{D(e) + (E(e))^2} \tag{13}$$

If two models with different sets of parameters have the same MAE values, this does not imply the equality of RMSE values. Therefore, the RMSE criterion gives more stable results in comparison to MAE. Which of these above two criteria should be used to get the optimal model?

For the selection of the optimal model, the series of the historical data are divided into six equal intervals $m_k, k = \overline{1,6}$.

The algorithm for the selection of the model parameters is developed in two stages. In the first stage, from a set of some combinations of parameters $(0, 0, 0), (0, 0, 0.02), \dots, (0, 0.02, 0.0), (0, 0.02, 0.02), \dots,$

$(1, 1, 0.98), (1, 1, 1)$ the triple, say $(\alpha_A, \beta_A, \gamma_A)$, which minimize statistics MAE and RMSE were selected. In the second stage, the domain of parameters $[\alpha_A - 0.02, \alpha_A + 0.02] \times [\beta_A - 0.02, \beta_A + 0.02] \times [\gamma_A - 0.02, \gamma_A + 0.02]$ was defined. The resulting domain of parameters must be adjusted so that all the parameter values fall into the interval $[0, 1]$. To find the optimal triple of the parameters from the domain, the parameters were selected by the step equal to 0.001.

Experiment Results

The interval of data from 10:00PM of 31 January, 2009, to 10:00PM of 24 December, 2012, was divided into six equal intervals m_1, m_2, \dots, m_6 which consisted of 3024 values each. The length of the interval m_i corresponds to 126 trading days. The beginning of i^{th} interval in the data interval is determined by the expression $d_i = 3024 \cdot (i - 1)$. The experiment setup is illustrated in Figure 3.

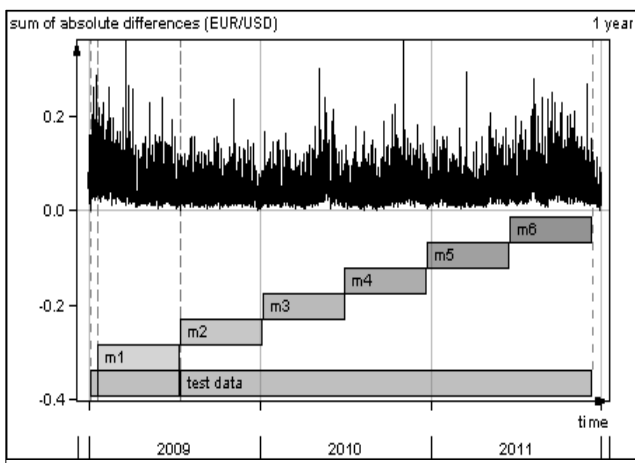


Figure 3. SAD values y_j and the selection of data intervals m_1, m_2, \dots, m_6 .

Figure 4 shows some irregularly distributed extreme values which can be the consequence of unpredictable events.

Experiments were carried out with the additive and multiplicative versions of the original and simplified Holt-Winters models.

Table 2

Optimal Parameters of the Original Additive Model for MAE Statistics

Intervals	MANE	RE	EME	α	β	γ
m_1	0,01260	1,46 %	0,05429	0,205	0,003	0,158
m_2	0,00899	1,16 %	0,03927	0,285	0,003	0,099
m_3	0,00984	2,48 %	0,04206	0,339	0,000	0,077
m_4	0,00976	1,46 %	0,04353	0,265	0,002	0,109
m_5	0,00945	2,74 %	0,03726	0,404	0,000	0,114
m_6	0,01210	2,05 %	0,05278	0,269	0,000	0,149
Mean	0,01046	1,89 %	0,04487	0,295	0,001	0,118

Table 2 shows the results of the optimal model selection which was performed using the Holt-Winters additive model minimizing MAE statistics. Analogous results were computed for other versions of the model and statistics. A summary of all the results is provided in Table 3.

Table 3

A Summary of Model Selection

Model	Criteria	$E(MANE)$	$E(\alpha)$	$E(\beta)$	$E(\gamma)$
Additive	MAE	0,01046	0,295	0,001	0,118
	RMSE	0,01057	0,303	0,001	0,110
Multiplicative	MAE	0,00983	0,298	0,002	0,111
	RMSE	0,00988	0,311	0,001	0,110
Additive (reduced)	MAE	0,01052	0,299		0,119
	RMSE	0,01054	0,298		0,109
Multiplicative (reduced)	MAE	0,00988	0,284		0,123
	RMSE	0,01002	0,310		0,111

Table 3 shows that the multiplicative models give slightly more accurate predicting results than the additive models. A rather low value of parameter β indicates that there is no trend in the time series.

Table 4

Summary of Rejected Errors

Model	Criteria	$E(MANE)$	$E(RE)$	$E(EME)$
Additive	MAE	0,01046	1,89 %	0,04487
	RMSE	0,01057	1,73 %	0,04619
Multiplicative	MAE	0,00983	2,90 %	0,04084
	RMSE	0,00988	2,89 %	0,04055
Additive (reduced)	MAE	0,01052	1,74 %	0,04611
	RMSE	0,01054	1,73 %	0,04597
Multiplicative (reduced)	MAE	0,00988	2,88 %	0,04064
	RMSE	0,01002	2,45 %	0,04165

The $E(RE)$ values in Table 4 show that the additive model has a statistical advantage over the multiplicative model while the values of (EME) show the opposite situation. Therefore, it cannot be stated for granted which model is better.

For each interval m_i , changing values d_i from 0 till the end of the data interval, all the statistics and their mean and standard deviation were computed with optimal parameters. The resulting statistics were used to plot the graphs to compare the performance of the model.

Table 5

Testing Results of the Optimal Original Additive Model for MAE Statistics

Model selection data interval	(1) $E(MANE)$	(2) $Std(MANE)$	(3) $E(ER)$	(4) $Std(ER)$
m_1	0,01021	0,0008267	1,79 %	0,34 %
m_2	0,01019	0,0008515	1,82 %	0,43 %
m_3	0,01027	0,0009417	2,00 %	0,72 %
m_4	0,01017	0,0008412	1,79 %	0,39 %
m_5	0,01017	0,0008735	2,11 %	0,64 %
m_6	0,01017	0,0008817	1,91 %	0,62 %
Mean	0,01020	0,0008694	1,90 %	0,52 %

The results presented in Table 5 were obtained with the original additive model. A summary of analogue results of the remaining models is given in Table 6.

Table 6

Summary of Model Testing Results

Model	Criteria	Mean of (1)	Mean of (2)	Mean of (3)	Mean of (4)
Additive	MAE	0,01020	0,0007238	1,90 %	0,52 %
	RMSE	0,01019	0,0008590	1,89 %	0,46 %
Multiplicative	MAE	0,00959	0,0009582	2,90 %	1,19 %
	RMSE	0,00960	0,000921	2,83 %	0,99 %
Additive (simplified)	MAE	0,01017	0,0008431	1,86 %	0,43 %
	RMSE	0,01018	0,0008521	1,87 %	0,44 %
	RMSE	0,00959	0,0008901	2,76 %	0,76 %

It can be seen from the results in Table 6 that the multiplicative models give slightly lower predicting errors. It follows that the simplified models are more applicable than the original models. The results of model testing with RMSE statistics are presented below.

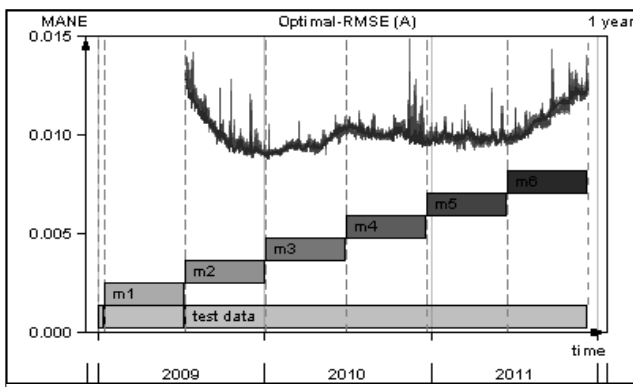


Figure 4. MANE values of the original additive model

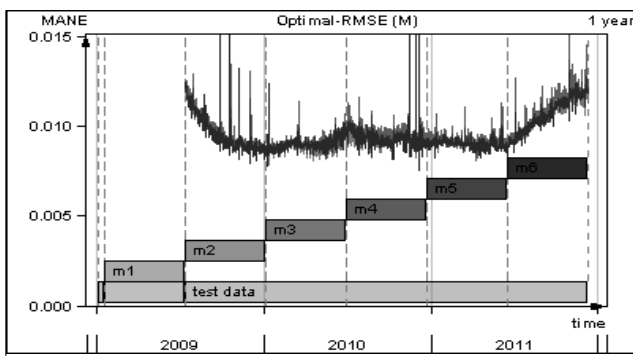


Figure 5. MANE values of the original multiplicative model

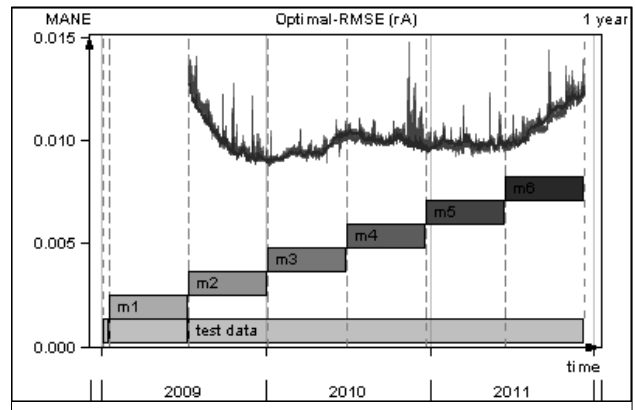


Figure 6. MANE values of the reduced additive model

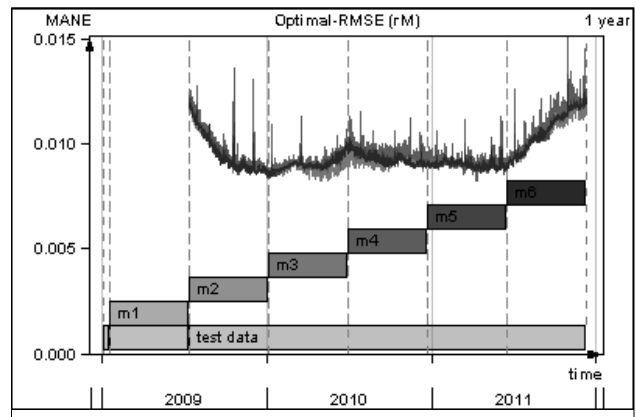


Figure 7. MANE values of the reduced multiplicative model

A careful study of Figures 4, 5, 6 and 7 allows determining the most reliable Holt-Winters model for modeling currency volatility. Figures 4 and 6 show that volatility prediction errors are not influenced a lot by using model parameters obtained from different data intervals, whereas Figures 5 and 7 show the opposite.

Figures 4 and 5 indicate that the additive model is less sensitive to the initial conditions in comparison to the multiplicative model, since the former model gives less extreme predicted values. Thus it is suggested to apply the original additive model in the investigation of exchange rate volatility. Figures 6 and 7 show that the simplified additive Holt-Winters model is more accurate than the reduced multiplicative one. The comparison of calculations in Figures 4 and 5 shows that the reduced additive model is slightly better than the original additive model.

The models can be used as tools in determining which time of the day is the best for executing currency exchange operations. Moreover, Figures 4, 5, 6 and 7 provide some insights on the accuracy of each model. If the model predicts a high volatility of EUR/USD exchange rate, it is suggested to invest in foreign exchange market for a short time period, which means that a participant of the market can gain profit by speculating with a high risk. Otherwise, money must be invested for a long term to gain possible profit since the exchange rate is less influenced by speculators.

Conclusion

A new time series was constructed for the investigation of the exchange rate volatility under a sum of absolute increments of the original time series during each hour. The additive and multiplicative versions of the original and simplified Holt-Winters models were applied to investigating exchange rate volatility. The estimated predicting errors (mean absolute error and root mean squared error) were used for the selection of the optimal values of parameters of each model. Four models were compared by its predicting ability by calculating mean absolute errors excluding the extreme values. The testing results revealed that the simplified models work better than the other versions of the original model. To determine the concrete version of the model for other currency pair, the models must be tested. Additionally, the models must be

investigated on the different locations of fixed length data of time series. The tested results showed that among four versions of the Holt-Winters model which is appropriate for predicting the exchange rate volatility of the EUR/USD currency pair and is a simplified additive model that minimizes the root mean square error.

In the future, the comparison between actual data and data obtained by means of different forecasting techniques, including four versions of the Holt-Winters model, will be carried out. Furthermore, a time series that allows discussing the sign of variation will be used.

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