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Balancing of turbomachine rotors by increasing the eccentricity identification accuracy

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1. Introduction

Various studies show that more than 40% of accidents are caused by excessive vibration of turbomachinery parts. Modern CAD systems, such as Solidworks and ANSYS, have proven themselves in solving some of turbomachinery design problems, but in the finished machine balancing problems, they can only serve as a tool in the hands of researchers.

Most modern turbomachinery rotors and powerful electrical devices are balanced in view of their flexibility during use [1], since for them methods of balancing of rigid rotors in the two extreme planes of correction in low-speed balancing machines are not effective [2, 3].

Such rotors are balanced on operating speeds in at least three correction planes in an effort to detect and compensate for imbalances that are normally distributed along the length of the rotor. It is required that deformation of the entire length of the rotor, or in places where they can focus largest imbalances should be initially measured [4]. Most often, in these places the rotor deflections are measured and which it is necessary to calculate eccentricities and the corresponding values of the imbalances, and then balancing loads [5].

Identification of the eccentricities of the measured deflection is an inverse problem. Here, by corollary (the measured deflections) it is necessary to find the cause of the rotor eccentricity. Complexities inherent to inverse problem of identifying the eccentricities arise from the incorrect setting of inverse problems [6].

Unfortunately, in the literature little attention is paid to the methods of overcoming the problems encountered in identifying the eccentricities of real turbomachinery rotors. One of the abovementioned problems is bad conditionality of systems of linear equations. As a result, their solution may be unstable, and the identified values of the parameters – inaccurate. Without the use of special methods for increasing the stability and reducing the scattering yield of desired values of eccentricities, identification methods can be ineffective.

In this paper, the authors offer effective methods for identification of eccentricities in real machine rotors with acceptable accuracy by obtaining stable solutions of systems of corresponding equations.

2. Solution of the inverse problem of identification of the turbopump rotor eccentricities

The test type turbopump unit TNA-150 (Fig. 1) had an increased vibration caused by rotor imbalance and it was necessary to understand the causes of increased vibration, to reduce the vibration, rotor deformation, stress and load on its bearings to the level of 300 N (according to the engineering specifications).

Since balancing the entire rotor on low speed machines in two planes of correction did not lead to the desired results, it was decided to balance the rotor to operational speed in three planes of correction, where the largest weight is loaded, namely in the planes of the two compressor disks 2 and 3 and the drive turbine 1 (Fig . 2).

The aim was to identify the results of measurement in the three sections of the rotor deflection magnitude and location of eccentricities (imbalance) of each of the compensating masses for further installation of balancing loads.

Fig. 1 Turbopump assembly

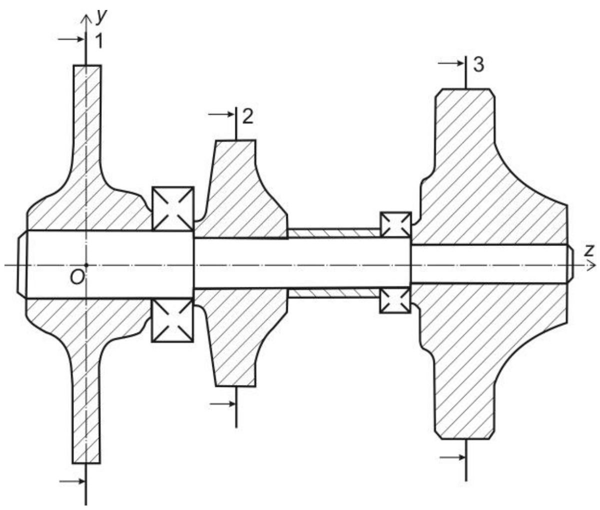

Fig. 2 Three-mass model of the turbopump assembly

The integro-differential dependencies resulting from the theory of bending allowed to write the equations of motion of the rotor, with the result that each of the three rotor sections in the projections on two mutually perpendicular planes were recorded by equations relating the unknown distribution of stiffness EJ, mass m, and projections e_y and

e_x, and eccentricities e with deflections y of the rotor shaft:

$$\alpha_0 K''_{zz}\left(Z,\omega_j\right)+2\alpha_1 K'\left(Z,\omega_j\right)+ \\ +\alpha_2 K\left(Z,\omega_j\right)-e_y\omega_j^2=\omega_j^2 y, \qquad (1)$$

where $\alpha_i=\alpha_i\left(Z\right)=\dfrac{1}{m}\dfrac{d^{(i)}EJ}{dZ^i}$; $i = 0, 1, 2$;

$K\left(Z,\omega\right)=y''/\left[1+\left(y'\right)^2\right]^{3/2}$ - the curvature of the elastic line of the rotor, Z - the coordinate of the rotor section, measured along the axis of rotation from point O (Fig. 2). Coefficients $\alpha_0, \alpha_1, \alpha_2, e_x, e_y$ are the unknown values.

To identify the stiffness, mass and inertial characteristics of the rotor, deflections were measured at four different angular frequencies: $n_1=14100$ rpm, $n_2=15000$ rpm, $n_3=15600$ rpm, $n_4=16000$ rpm. Using the obtained values of the projections of the rotor shaft deflection y_j, $j=\overline{1,4}$ measured at frequencies of rotation ω_j, $j=\overline{1,4}$, and four first derivatives $y_j, y_j', y_j'', y_j''', y_j^{IV}$, $j=\overline{1,4}$, constituted by two systems of linear equations of the type (1) for each of the calculated cross sections 1,2,3 which are identified by eccentricities, stiffness and mass.

For the 1st section (OY axis) we have:

$$\begin{bmatrix} -6\times10^{-8} & 1.33\times10^{-5} & 1.33\times10^{-4} & -218\times10^{4} \\ -9\times10^{-8} & 2.00\times10^{-5} & 1.61\times10^{-4} & -246\times10^{4} \\ -6\times10^{-8} & 1.50\times10^{-5} & 1.38\times10^{-4} & -267\times10^{4} \\ -9\times10^{-8} & 2.00\times10^{-5} & 1.61\times10^{-4} & -280\times10^{4} \end{bmatrix}\times\begin{bmatrix}\alpha_0\\ \alpha_1\\ \alpha_2\\ e_y\end{bmatrix}= \\ =\begin{bmatrix}-8285\\ -10363\\ -11209\\ -12352\end{bmatrix}$$

Similar matrix equations were formulated for discrete linear inverse problems for other cross-sections and planes.

The number of matrix, composed of equation systems $\boldsymbol{AX}=\boldsymbol{Y}$ appeared very high (Table 1).

Table 1

Conditionality of the matrix of type (1) linear equation system

Section number	axis	Value of condition number
1	OY	$cond\left(\boldsymbol{A}\right)=1.4\times10^{15}$
	OX	$cond\left(\boldsymbol{A}\right)=5.0\times10^{14}$
2	OY	$cond\left(\boldsymbol{A}\right)=1.7\times10^{14}$
	OX	$cond\left(\boldsymbol{A}\right)=6.4\times10^{15}$
3	OY	$cond\left(\boldsymbol{A}\right)=2.1\times10^{14}$
	OX	$cond\left(\boldsymbol{A}\right)=3.5\times10^{3}$

Apparently, the resulting solutions of systems of equations cannot be considered reliable. However, this conditionality may be called "imaginary". Indeed, for the analysis of the matrix A, it becomes clear that increased conditionality is caused not only by the proximity of the system to degenerate, but also a huge difference in the order of the coefficients, i.e. the difference between the values and norms of the matrix period. Applying the scaling of coefficients, we look for the following unknowns:

$\alpha_0'=\alpha_0\times10^{-11}$ cm^4/s^2; $\alpha_1'=\alpha_1\times10^{-9}$ cm^3/s^2;

$\alpha_2'=\alpha_2\times10^{-8}$ cm^2/s^2; $e_y'=e_y\times10^{2}$ cm and

$\boldsymbol{B}'=\boldsymbol{B}\times10^{-3}$.

Then, for the 1st section (axis OY) we have $cond\left(\boldsymbol{A}\right)=217$. Similarly, by scaling the coefficients of the system of units of linear equations, it was possible to reduce the conditionality of the matrix composed for section 1 (Ox axis) from 4.968×10^{14} up to 332, for section 2 (Oy axis) from 1.715×10^{14} to 25, for section 2 (Ox axis) from 6.397×10^{15} to 103, for section 3 (Oy axis) from 2.074×10^{14} to 176, for section 3 (Ox axis) from 7.453×10^{15} to 3453.

These matrices have acceptable conditionality and so the corresponding equations were solved using the statistical method with sustainability developed through additional measurements as well as with the use of linear filtering method of least squares estimator [7].

Another method of identifying unknown $\alpha_0, \alpha_1, \alpha_2, e_x, e_y$ in each of the three sections is also proposed. The analysis of the systems of equations formulated for OX and OY axes in section 1 shows that out of 8 equations only 5 unknown values could be found, because $\alpha_0, \alpha_1, \alpha_2$ are common unknown values for both systems of equations. This fact allows to simplify calculation of eccentricities, imbalances and location angles by solving one linear system, composed of two linear systems with standard linear transformations. For example, adding the corresponding matrices of the left and right side of the two linear systems for section 1 and forming the 5th equation by adding equations, we obtained a matrix system of equations $cond\left(\boldsymbol{A}\right)=3.1\times10^{14}$.

After scaling, we have a system of equations with $cond\left(\boldsymbol{A}\right)=724$:

$$\begin{bmatrix} -5.3 & -9.18 & 0.36 & -2.18 & -2.18 \\ -1.0 & 6.82 & 0.88 & -2.47 & -2.47 \\ -2.0 & 10.35 & 1.09 & -2.67 & -2.67 \\ -5.9 & 13.72 & 1.24 & -2.81 & -2.81 \\ -8.3 & -2.50 & 0.65 & -2.18 & -2.81 \end{bmatrix}\times\begin{bmatrix}\alpha_0'\\ \alpha_1'\\ \alpha_2'\\ e_x'\\ e_y'\end{bmatrix}=\begin{bmatrix}-0.7631\\ -0.9129\\ -0.9874\\ -1.1229\\ -1.1698\end{bmatrix}.$$

The next step of conditionality reduction was to apply scaling by searching vector $\boldsymbol{K}=\left[k_i\right]_{1\times n}$, at which $\min\limits_{\boldsymbol{K}} cond\left(\boldsymbol{A}'\right)$ is reached, where $\boldsymbol{A}'$ - matrix of equivalent system of linear equations (SLE), which includes the lines $\boldsymbol{A}'\left(j,:\right)=\boldsymbol{A}\left(j,:\right)k_j, j=\overline{1,n}$, i.e. the task is to find

$\min_{\boldsymbol{K}}\left\{f_1(\boldsymbol{K}), f_2(\boldsymbol{K}),..., f_k(\boldsymbol{K})\right\}$, $k \geq 2$ where $f(\boldsymbol{K}) = cond(\boldsymbol{A}')$. Equivalent SLE, optimized according to the criteria of conditioning minimum, looks the following way: $\boldsymbol{A}'\boldsymbol{X} = \boldsymbol{Y} \times diag\{k_1, k_2, ..., k_n\}$. To validate this, we used optimization to find vector $\boldsymbol{K} = [1\ 2.94\ 2.64\ 1.19\ 1.19]$ and obtained equivalent SLE which is as follows:

$$\begin{bmatrix} -5.30 & -9.18 & 0.36 & -2.18 & -2.18 \\ -2.94 & 20.07 & 2.59 & -7.26 & -7.26 \\ -5.28 & 27.34 & 2.89 & -7.05 & -7.04 \\ -7.17 & 16.39 & 1.48 & -3.35 & -3.35 \\ -9.91 & -2.98 & 0.77 & -2.61 & -3.35 \end{bmatrix} \times \begin{bmatrix} \alpha_0' \\ \alpha_1' \\ \alpha_2' \\ e_x' \\ e_y' \end{bmatrix} = \begin{bmatrix} -0.7631 \\ -2.6873 \\ -2.6084 \\ -1.3415 \\ -1.3964 \end{bmatrix}.$$

As a result, the conditionality was reduced to $cond(\boldsymbol{A}) = 564$, i.e. by 28%. Similarly, two SLEs are formed from 5 equations for sections 2 and 3 respectively. With this approach only 3 out of 6 systems of equations are solved.

Let us estimate the relative error for elements of the vector of absolute terms. The elements of the vector are $\omega_i^2 \times y_i$, $i = \overline{1,4}$. Applying the knowledge of the theory of errors, we find that the relative error of the product is $\delta(\omega^2 \times y) = \delta\omega^2 + \delta y$, and $\delta(\omega^2) = 2\delta\omega$. Given that the measurement error of rotational speed is 100 rpm = 10.47 rad/s ($\delta\omega = 0.0071$ rad/s), and error of deflection measurement is 1 μm ($\delta y = 0.026$), the relative error of the first element should be 4%.

It follows that for the solution of this problem without the use of regularization techniques, possible error in determining the unknown could be hundreds of percent. To increase the accuracy of calculations and solutions, to ensure the specified accuracy, a statistical method for increasing the stability of mathematical models was used [8]. It is possible to solve the system (1) with an accuracy of 5% (Table 2).

Table 2

The results of inverse problem solution

Section Number	e_x, m	e_y, m	α_0, m^3/s^2	α_1, m^2/s^2
1	-5×10^{-6}	-5.8×10^{-6}	185.65	–270.3
2:	-9×10^{-6}	1.7×10^{-6}	710.65	–247.18
3:	-6.2×10^{-6}	30×10^{-6}	280.83	–680.00

The identified values α_0 and α_1 allow to determine the values of stiffness more accurately than the static tests. For this purpose, each of the 3 sections mentioned found values of the mass m_i and stiffness EJ, i = 1, 2, 3 rotor shaft in accordance with the formulas:

$$m(Z) = M \times exp\left(\int_0^Z \frac{\alpha_1}{\alpha_0} dZ\right); \tag{2}$$

$$EJ(Z) = m(Z) \times \alpha_0(Z), \tag{3}$$

where M - rotor mass.

Further, we used the formulas:

$$D_i = M_i\sqrt{e_{xi}^2 + e_{yi}^2};\ \varphi_i = arctg\left(e_{yi}^2 / e_{xi}^2\right);\ i = 1, 2, 3,$$

to determine the magnitude of the imbalances of the rotor angle and compiled them with the OX axis of the selected coordinate system. The results are presented in Table 3. Finally, using the identified data of eccentricities, we compensate them.

Table 3

The results of solving the problem of identifying THA rotor imbalances

Identified values		Section 1	Section 2	Section 3
Stiffness	EJ_i, H m^2	414.7	1594	23998
Reduced mass	$m_i \times 10^{-3}$ kg/m	2.2	2.0	8.3
Imbalance	D_i, gr cm	23.7	2.48	30.6
Angle with Ox axis	φ_i, degrees	95°	170°	102°3′

According to the identified values of stiffness and weight, critical rotor frequency has been calculated, which is shown in the adopted dynamic model. For this purpose, the values of influence factors are calculated, using the known EJ values for the rotor sections and Mohr's integral. Then critical frequency of the rotor ω_1 = 1732 1/s. and ω_2 = 2625 1/s, corresponding to n_1 = 16500 rpm and n_2 = 25080 rpm is found. The difference between the first critical speed calculated from the identified masses and stiffness, and critical rotor speed measured when running TNA is 400 rpm, i.e. 2.49% of 16100 rpm.

For comparison of the critical difference between the actual rotor speed and the resulting solutions, a determinant secular equation is composed, based on static factors influence of 3400 rpm. That is, 21% of 16100 rpm. Improving the accuracy of calculations 8.4 times has been made possible thanks to the solution of inverse problems with the use of sustainable methods of making.

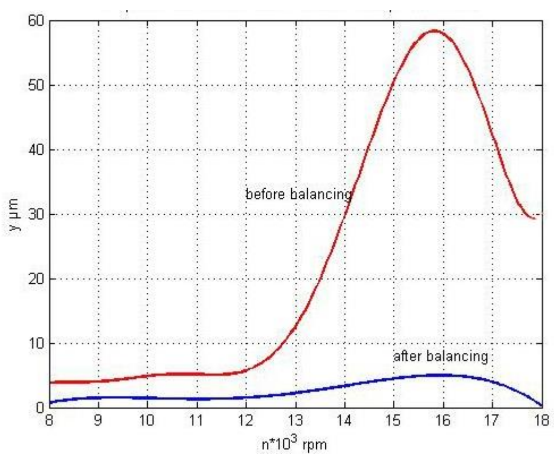

Fig. 3 Dependence of rotor deflection on rotation speed

After balancing of the rotor by setting a special corrective mass storage, a controlled launch was performed on passage from 0 to 18,000 rpm with oscilloscope readings of strain gauges and vibration sensors. The resulting deflections depending on the rotational speed in a section before and after rotor balancing are shown in Fig. 3.

As a result of balancing, the maximum deflection of the rotor shaft in the range 8000-18000 rpm. was reduced by about 6 times, the amplitude of vibration supports - by 4 times the static tension in the material of the shaft - by 3.5 times, and dynamic - by 3 times (Fig. 3).

3. Solution of the inverse problem of identification of aircraft engine compressor rotor eccentricities

The rotor of the disc-drum type compressor of gas turbine engine (GTE) AI-20 contains ten individual discs bearing rotor blades, tail rotor shaft and seal of front and rear bearing assemblies on their crowns (Fig. 4).

One way to identify eccentricities is solving a matrix equation on the basis of experimental data:

$$\boldsymbol{Y} = \boldsymbol{A}\left(\boldsymbol{Y}+\boldsymbol{e}\right)\omega^2, \tag{4}$$

where $\boldsymbol{Y} = [y_i]_{1\times n}$; $\boldsymbol{e} = [e_i]_{1\times n}$; $\boldsymbol{A} = [a_{ik}]_1^n$.

Here the coordinates of the vector $\boldsymbol{Y}$ have a deflection of the rotor shaft in the landing places of the discs, vector $\boldsymbol{e}$ - the eccentricities of the discs, and $\boldsymbol{A}$ -elements of the matrix are the product of the static coefficient of influence on the masses of the corresponding discs [9].

Assuming that $\widehat{\boldsymbol{A}} = \boldsymbol{A}\omega^2\left(1-\boldsymbol{A}\omega^2\right)^{-1}$, we arrive at the solution of discrete linear inverse problem of the following type:

$$\boldsymbol{Y} = \widehat{\boldsymbol{A}} \times \boldsymbol{e}\,. \tag{5}$$

Due to the fact that conditionality $cond\left(\widehat{\mathbf{A}}\right)$ is usually large and vector elements are measured with errors, the task of identifying the type of the eccentricities of the rotor (4) cannot be solved in practice, since its solutions will be false. Thus, the actual challenge in the way of solving this inverse problem is to overcome the instability of its solutions, caused by poor conditioning of the matrix $\widehat{\boldsymbol{A}}$. The problem will be incorrect and its solution will be unstable because small errors in $\boldsymbol{Y}$ will be highly increased in the solution $\boldsymbol{X}$. The research [8] shows that stability of solutions can be reached by applying multiple measurements, which in fact is using the method of least squares.

By increasing the number of measurements, the measurement error can be reduced. But in practice the way of infinite increase of measurement accuracy is not possible, because sooner or later the lack of information (for example, not knowing the exact value of corrections etc.), rather than scattering the arithmetic average, becomes the determining factor. Accumulating experimental data thus decreasing the standard deviation of the arithmetic average can only make sense as long as it is not negligible compared to the standard deviation analogue which takes into account the lack of information. Multiple measurement accuracy, therefore, is limited due to systematic error caused by the lack of information.

So, despite the fact that the Least Squares Estimator (LSE) is an unbiased estimator, it is unsustainable, and the method of least squares is ineffective for systems of linear algebraic equations with large numbers of conditionality. The cause of instability is the huge variance of the LSE. As mentioned in [7], likelihood function should only be used as a preliminary tool while solving the inverse problem. Instead, it is reasonable to rely on a certain communicative statistics that takes into account the systematic deviations of the compared random sequences.

To solve the inverse problem of determining the eccentricity of the rotor it is proposed to apply to LSEs linear filtering. The basic idea of filtering as a method of regularization is to consciously leave some bias in the estimate obtained, while significantly reducing its scattering. Consequently, it is necessary to find such an estimate, which is still acceptable at offset and the variance - significantly less than that of the LSE. With the purpose of filtering it is proposed to apply data compression and produce a truncated assessment. For this purpose it is suggested to use multivariate analysis of the data compression method - the method of principal component analysis and (PCA), known in statistics [7, 10].

Suppose the following equation is solved instead of (5):

$$\boldsymbol{AX} = \boldsymbol{Y} + \boldsymbol{\Delta Y}\,, \tag{6}$$

where $\boldsymbol{Y}$ - true value; $\boldsymbol{\Delta Y}$ - vector of “noise” values, with regularly distributed components $\Delta y_i \sim N\left(0, \sigma_i\right)$.

Then there is the multivariate normal variable $\boldsymbol{\Delta Y}$ with zero mean $\left\langle \boldsymbol{\Delta Y} \right\rangle = 0$ and covariance matrix $\boldsymbol{\Sigma} = cov\left(\boldsymbol{\Delta Y}\right)$.

As it is known, one of the most important roles in the analysis of the formation of the stability of solutions for linear inverse problems belongs to Fisher matrix $\boldsymbol{I}$, which is equal to the inverse of the covariance matrix of the LSE $\boldsymbol{\Omega} = \boldsymbol{I}^{-1}$. Fisher matrix for the LSE model (6) can be found from the formula $\boldsymbol{I} = \boldsymbol{A}^T \Sigma^{-1} \boldsymbol{A}$, where the covariance matrix of the "noise" is obtained through:

$$\boldsymbol{\Sigma} = \left(\boldsymbol{Y} - \bar{\boldsymbol{Y}}\right)^T \left(\boldsymbol{Y} - \bar{\boldsymbol{Y}}\right)$$

or by the formula:

$$\boldsymbol{I} = \left(\left(\boldsymbol{X} - \widehat{\boldsymbol{X}}\right)^T \left(\boldsymbol{X} - \widehat{\boldsymbol{X}}\right)\right)^{-1},$$

where $\widehat{\boldsymbol{X}}$ is LSE.

Spectral representation of the Fisher information matrix has the form:

$$\boldsymbol{I} = \boldsymbol{VDV}^T,\ \boldsymbol{D} = diag\left(\lambda_1, \lambda_2, ..., \lambda_n\right),\ \lambda_1 \ge \lambda_2 \ge ...\lambda_n > 0 \tag{5}$$

where $\left(\lambda_1, \lambda_2, ..., \lambda_n\right)$ - eigenvalues of the Fisher matrix, $\boldsymbol{V}$ - orthogonal matrix whose columns define the directions of the principal axes of the ellipsoidal region of admissible estimates of the problem set incorrectly (5) [7]. At the same time, the LSE converts according to the system of eigenvectors of the Fisher matrix:

$$\widehat{\boldsymbol{X}} = \boldsymbol{V}\ \widehat{\boldsymbol{p}}\,, \tag{7}$$

where $\widehat{p}_1, \widehat{p}_2, ..., \widehat{p}_n$ - principal LSE components. These are the components $\widehat{\boldsymbol{X}}$ in the coordinate system that is rotated relative to the initial system so that the coordinate axes were parallel to the main axes of LSE scattering ellipse.

As it is known, the trace of the covariance LSE matrix is equal to the sum of its eigenvalues:

$$tr\left(\boldsymbol{\Omega}\right) = \sum_{i=1}^{n} \left\langle \left(\widehat{x}_i - x_i\right)^2 \right\rangle = \sum_{i=1}^{n} \lambda_i^{-1} .$$

This shows that the total deviation from the true LSE object is defined by the range of matrix $\boldsymbol{I}$. The largest contribution to the total deviation is made by the smallest eigenvalues, i.e. the "tail" of the Fisher matrix. So, the essence of filtering is a compromise choice of such a large number of principal components $\nu \le n$ that provide sufficient accuracy of assessment with an acceptable variance. By increasing, ν it is possible to reach a more accurate representation of $\boldsymbol{X}$ the average through $\widehat{\boldsymbol{X}}_\nu$, but at the same time more and more terms from the "tail" of the Fisher matrix spectrum are taken into account, and it quickly deteriorates the quality of assessment.

Truncated estimate of the LSE is calculated as follows: $\boldsymbol{X}_{tr} = \boldsymbol{V}_{\nu_{min}} \widehat{\boldsymbol{p}}$. Taking into account that:

$$\widehat{\boldsymbol{p}} = \boldsymbol{V}^T \widehat{\boldsymbol{X}} , \tag{8}$$

we get:

$$\boldsymbol{X}_{tr} = \boldsymbol{V}_{\nu_{min}} \boldsymbol{V}^T \widehat{\boldsymbol{X}} . \tag{9}$$

Truncated estimation method was used to solve the inverse problem of identifying unknown eccentricities of aircraft gas turbine engine AI-20 compressor rotor (Fig. 4). A five-mass mathematical model of compressor rotor shown in Fig. 5 was set up to search eccentricities.

$$\widehat{\boldsymbol{A}} = \begin{bmatrix} 0.228 & 0.203 & 0.195 & 0.155 & 0.1299 \\ 0.184 & 0.246 & 0.284 & 0.246 & 0.2457 \\ 0.143 & 0.230 & 0.303 & 0.295 & 0.3472 \\ 0.111 & 0.195 & 0.290 & 0.336 & 0.4440 \\ 0.067 & 0.139 & 0.234 & 0.316 & 0.5200 \end{bmatrix}, c^{-2} .$$

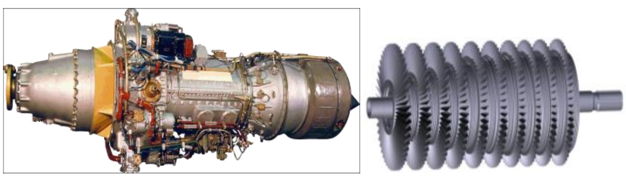

Fig. 4 Airplane engine AI-20 and compressor of engine

Critical rotor speed on rigid supports are 14000, 28900, 65300, 130600 and 419300 rpm. The number of matrix condition $cond(\mathbf{A}) \approx 573$. This means that the accuracy of measurement of the deflections of the rotor 10^{-5} m, which corresponds to a relative error of 6-10%, the error in determining eccentricities for normal inverse isolation system (3) can reach 5730%, that is, the resulting solution will be totally unreliable. In this situation using the LSE with 50 measurements can slightly improve the accuracy (upper estimate will decreases approximately by 7 times), which is also unacceptable.

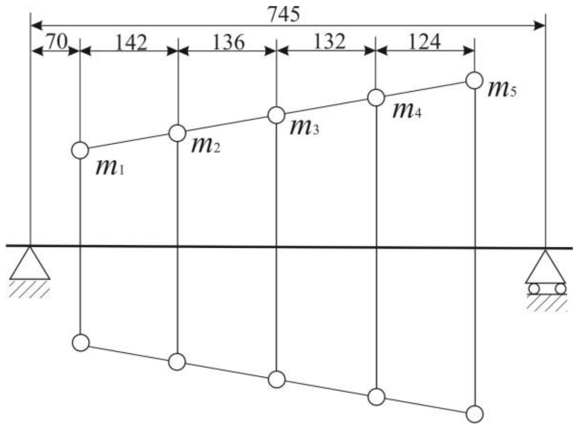

Fig. 5 Five-mass model of compressor rotor

The following numerical experiment was carried out with the help of principal component analysis (PCA) and MATLAB program to test the effectiveness of the proposed linear filtering method. On the basis of the specified sections of the exact values of eccentricities $\boldsymbol{e} = [77.4, 89.9, 105.0, 79.0, 59.5]^T \times 10^{-6}$ m the exact values of the rotor deflections $\boldsymbol{Y}$ were determined by solving the direct problem, in which the rotor matrix $\boldsymbol{A}$ is assumed to be given without errors. These values $\boldsymbol{Y}$ = [76.35, 100.23, 107.52, 109.53, 98.16]^T 10^{-6} m. have been taken for the expectation of deflections in the given sections. Further, the standard deviation $\sigma = \Delta/3$, where $\Delta = 10^{-5}$ m - measurement accuracy, is set using a computer random number generator to obtain different implementations of deflections prepared as random variables distributed by the normal distribution law with the above mentioned parameters. In this experiment, 50 deflection realizations generated in each of the examined sections were provided for. For each $\boldsymbol{Y}$ realization the corresponding $\boldsymbol{e}$ implementation was found and their expectation values $\widehat{\boldsymbol{e}}$, which coincide with the LSE, were calculated.

By carrying out the spectral decomposition of the Fisher matrix according to (7), a diagonal matrix $\boldsymbol{D}$ with the eigenvalues on the main diagonal (sample variance principal component analysis) and a matrix of eigenvectors $\boldsymbol{V}$ were obtained. Since the total sample variance was 82116 , the dispersion of the main component was 78.2% of the total variance, and the three main components reached 99.2% of the total variance, it was sufficient to choose three eigenvectors of covariance matrix ($\nu = 3$) for filtering of the estimation.

Filtered LSE, calculated according to formula (9) is $\boldsymbol{e}_{tr}$ = [84.64, 92.14, 97.31, 76.96, 62.88]$^T \times 10^{-6}$ m. Relative error of truncated estimates, calculated as:

$$\Delta\, \boldsymbol{e} = \left(\left\|\boldsymbol{e}_{tr}\right\| - \left\|\boldsymbol{e}\right\|\right) / \left\|\boldsymbol{e}\right\| , \tag{10}$$

reached $\Delta \boldsymbol{e}$ = 0.18%, while LSE made $\widehat{\boldsymbol{e}}$ = [188.3, 238.3, 419.8, 58.8, 74.6]$^T \times 10^{-6}$ m.

The relative error of the LSE was $\Delta\, \boldsymbol{e}$ = 182%, that is, the accuracy of the solution using a truncated assessment compared with a conventional LSE increased again by 1167 times. The results demonstrate a sufficiently high accuracy and efficiency of the described method for producing regular statistical solutions of linear inverse problems using LSE linear filtering method with the help of PCA method.

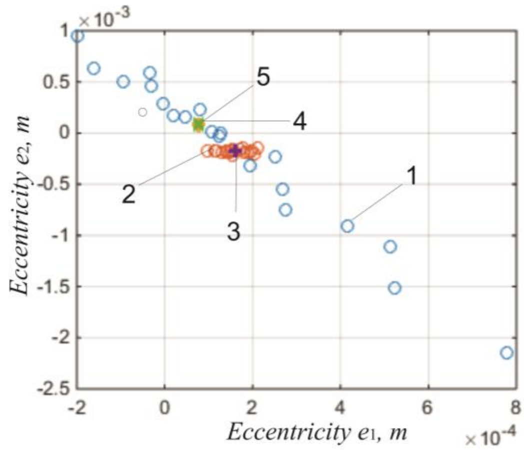

Fig. 6 Demonstration of the effectiveness of the filtration method of the LSE in solving the problem (5): *1* - normal inverse interchange; *2* - solutions that are compressed by LSE method; *3* - LSE; *4* - truncated LSE estimate; *5* - true eccentricities e_1, e_2

4. Conclusions

1. The current research presents the results of utilizing the methods for improving stability of linear discrete inverse problem solutions to identify the eccentricities according to measured deflections for TNA -150 turbopump unit. As a result of balancing, maximum rotor shaft deflections in the range of 2000-18000 rpm decreased approximately by 6 times; the amplitudes of vibrations in supports (bearings) – by 4 times; static stress in the material of the shaft – by 3.5 times; and dynamic stress – by 3 times.

2. Application of LSE linear filtering method using PCA has been offered to ensure the stability of the solutions of inverse problems for identification of eccentricities with measured deflections and compliance. The bottom line is that filtering should have such effect on the LSE, which could substantially reduce the ellipsoid of LSE scattering by compressing the information contained in the matrix of scattering, due to "truncating" the "tail" of the Fisher matrix spectrum.

3. The study validates high efficiency of using truncated estimates to solve the inverse problem of identifying unknown eccentricities in the rotor of aircraft engine AI-20 compressor using empirically determined compliance values and rotor deflections.

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BALANCING OF TURBOMACHINE ROTORS BY INCREASING THE ECCENTRICITY IDENTIFICATION ACCURACY

S u m m a r y

The article presents the results of research aimed at improving the identification of eccentricities by ensuring the stability of solutions of linear discrete inverse problems. The results of the application of scaling factors of systems with a view to reducing their dependence are suggested. Identified eccentricities imbalances and angles of their location allowed to balance the rotor turbopump unit TNA-150 and reduce the maximum deflection of the rotor shaft in the range of 2000-18000 rpm 6 times, the amplitude of vibration supports - 4 times, the static tension in the material of the shaft - 3.5 times, and dynamic tension- 3 times. The use of linear filtering LSEs based on the PCA method, has significantly reduced the LSE scattering and accurately identify the eccentricities of the compressor rotor of aircraft engine AI-20.

Keywords: eccentricity, imbalance, inverse problem, rotor, least squares estimator, principal components analysis.

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