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PRODUCTION EFFICIENCY OPTIMIZATION BY MODELLING MANUAL ASSEMBLY PROCESSES

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LIST OF ABBREVIATIONS

ALC – almost learning curve
AM – agile manufacturing
CLC – classical learning curve
CTD – cumulative trauma disorders
DFA– design for assembly
DFM– design for manufacture
DFMA – design for manufacture and assembly
INC / INW – invariant method for Crawford/Wright's model
JIT – just in time production
KPI – key performance indicators
LC – learning curve
MPP – monitoring of the production processes
OEE – overall equipment efficiency
PIC / PIW – point and interval method for Crawford/Wright's model
PMTS – predetermined motion and time studies
RMI – repetitive motion injuries
SMED – single minute exchange of die
SMI – strictly monotonically increasing
TIC / TIW – two intervals method for Crawford/Wright's model
TPC / TPW – two point method for Crawford/Wright's model
TPS – Toyota production system
VAVE – value analysis / value engineering

INTRODUCTION

The manual assembly of production items by humans continues, although this manufacturing technology is being widely replaced by robotic and automated equipment; however there are still many production fields where mechanical human work is inevitable due to a variety of reasons. These reasons include but are not limited to labor cost, production volumes, cost of equipment, task complexity, dangerous working environment and etc. (Konz & Johnson 2000). In spite of different manufacturing fields, the majority of manual assembly operations are standard (Boothroyd & Dewhurst 2002) and differ in quantities and combinations used to complete the final assembly product. The main technological parameter defining the performance, productivity and, finally, the cost of certain assembly task is the operating time to complete the task. Improvements (cost or productivity) can be made by operating time reduction i.e. by rearranging and changing quantities and combinations of the manual assembly operations.

However, a manual operating time is defined not only by assembly operations. Due to the fact that human work is involved, the assembly time is highly affected by cognitive human factors, i.e. learning phenomenon. This phenomenon defines operating time decrement as the human operator is becoming familiar with the task. The time improvement is defined by the learning curve. Learning curve models have been known for several decades (Wright 1936, Yelle 1979, Anzanello, 2011). Initially, the learning curves were based on a study of the processing time decrement as manufacturing continues (Bevis et. al., 1970) to forecast time (or cost) decrement achieved by large production volumes. Nevertheless, in typical mass production, the order quantities are huge and, therefore, start-up (learning) phase is soon completed and it does not make a significant impact to the total assembly time.

It is very important to emphasize that long time trend in the manufacturing industry clearly shows the fall of mass production (Womack et. al., 1990, Holweg & Pil, 2004; Holweg, 2006) and the spread of mass customization (Silveira et. al., 2001; Piller, 2004). Therefore, manufacturing companies are forced to reduce order quantities, increase product variety and shorten production lead times. In the manual assembly, when the order quantities are small, intermittent or even occasional, there is no possibility of completing the learning phase, so the production is always at the start-up (learning) phase i.e. at the beginning of the learning curve. As a result, the processing time is not stable, fluctuating also much higher than calculated standard production time. This is one of the major reasons why the interest of the learning effect has increased and re-emerged among production researchers recently (Fogliato & Anzanello 2011). In the last few years, many authors have addressed a variety of issues regarding the learning-forgetting effects connected to decreasing production quantities and mass customization: ramp-ups in production (Glock et. al., 2012), investment in learning curves (Seta et. al., 2012) and etc. To sum up, this topic is therefore very relevant and important in manufacturing engineering.

In addition to this, there is a global competition between manufacturing companies' demands for increased production capacity and lower production costs. Thus, assembly operators receive an increased work rate i.e. they are required to produce more during the same period of time or/and are forced to work overtime to complete production orders. The work by Gooyers and Stevenson (2012) reported that an increase in work rates demanded increased muscular effort, which leads to an elevated risk of musculoskeletal injury. These injuries can be prevented by job rotation (Helander, 2006), although, job rotation might result in recursive learning. Job rotation was also pointed out as the learning curve research direction by Fogliato and Anzanello (2011), however there is still very little research on this topic.

In this research, the manual assembly process as a manufacturing technology with small production lots is studied and the major technological process parameter (assembly time) is addressed. Since the operating time in such a manufacturing technology is highly affected by learning, the analysis, modelling and development of the learning curve is the main topic of the dissertation. Moreover, issues connected to learning time reduction by technological process modelling and ergonomic factors of the manual assembly are addressed as well. The outcome of the study is to propose the design directions for manufacturing technology (manual assembly) improvement. To test the adequacy of the proposed models, a company performing manual operations was selected. This company manually assembles an enormous variety of different products (more than four thousand) for the automotive industry and production lot sizes are small, fluctuating and changing rapidly for each product.

Goal of the work

Optimize (increase) the manual assembly process efficiency by modelling process parameters.

Tasks of the dissertation

In order to accomplish the stated goal of this work, the following tasks were established:

1. Define the appropriate research methodology in the context of manufacturing engineering.
2. Develop and apply new mathematical learning models that adequately approximate the operating time development (reduction) of the manufacturing process.
3. Create methodology to estimate parameters of the learning curve from the limited production data.
4. Define, state and solve manual assembly process efficiency optimization problem by employing appropriate LC models proposed in this dissertation.
5. Perform production data monitoring to test, evaluate and prove the adequacy of proposed models.

Scientific novelty

1. New mathematical learning models created.
2. New methodology of the learning curve parameter estimation proposed.

3. The majority of the other authors address the calculation and estimation of learning time, however in this research a method to reduce the learning time is proposed.

Importance of the work

Due to the fact that manufacturing order quantities are decreasing, the variety of products are increasing and the product lifecycle is getting shorter, there is a necessity to reduce the total production time in order to react to changing customer demand. In this research, the problem of direct total production time is addressed and solved by learning time reduction.

Key statements for defense

1. In this research, the new learning curve models that satisfy the general properties of the learning curves and approximate learning processes more accurately than traditional models are created and proved.
2. The parameters of the learning curve can be estimated by using deterministic (non-statistical) methods.
3. In this research, it is proved that efficiency of the complex manual assembly increases when the process is split into a certain (optimal) number of simpler processes.

Layout of the dissertation

The dissertation consists of an introduction chapter, 4 chapters and a conclusion chapter. The introduction to the dissertation represents the main idea of the work and motivation, the main goal and tasks of the work, the novelty and importance of the dissertation, and the key statements for defense. At the end of the dissertation a list of references, as well as a list of publications and conference contributions are presented as well.

The page count of the dissertation is 118. There are 179 formulas, 102 figures and 12 tables in the text. The list of references consists of 134 entries.

Approbation of the research results

The results of the dissertation have been presented at 9 international conferences (2 of them organized abroad) and published in 8 articles referred in international scientific databases (2 articles published in journals referred by ISI Web of Science).

1. REVIEW OF THE MANUAL ASSEMBLY PROCESS MODELLING

1.1. General trends in manufacturing systems

The most significant change in the manufacturing systems' development occurred at the beginning of last century (Womack, 1990). Prior to this change, manufacturing was performed only by highly skilled craftsmen. Such a production system had no standardization, work instructions or any modelling, optimization or improvement. Therefore, when Henry Ford introduced the assembly line it was in fact a revolution to the manufacturing that had been in existence. Ford actually created modern manufacturing principles that are being used up until now (see Fig. 1.1). Additionally, Ford's ideas of rationalization were welcomed by prominent Japanese researchers Ohno (1988) and Singo (1989). Also, the interest in Ford's ideas has re-emerged recently, when his book '*Today and Tomorrow*' was reprinted in several editions. The inventor of the modern assembly line, actually, introduced it following the principles of interchangeability and standardization. These principles are necessary to run an assembly line in any manufacturing plant (Thomopoulos, 2014). Ford created an assembly line for car production, but soon these principles shifted to other manufacturing fields.

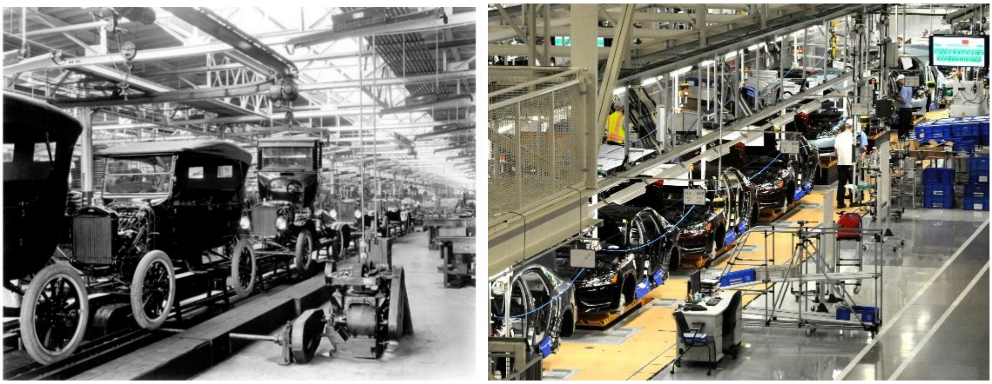

Fig. 1.1. Ford's assembly line (Weber, 2013) and the modern assembly line (Young, 2015)

Beside the technical achievements, such as assembly line and interchangeability, Ford actually introduced mass production as well. Mass production usually employs a production to stock policy. It helps to level and synchronize the production system. In addition, mass production tends to increase production order quantities. This helps to eliminate effect of the start-up inefficiencies, reduce the total manufacturing cost and improve production. However, mass production has two major drawbacks:

- It uses large order quantities, generates many defects, creates large inventories of unused production and ties-up capital.
- It is unable to react to changing needs of the customers and to propose a variety of different products.

On the other hand, the needs of the customer are constantly growing and the lifetime of the products are sharply declining: the market demands new and different products in a short period of time in small quantities (Silveira et. al., 2001; Piller, 2004). These conditions contribute to the fall of mass production and the demand for new, authentic manufacturing systems to satisfy market requirements and solve the shortcomings of mass production. Currently, there are two significant manufacturing systems to satisfy these demands:

- Agile manufacturing (AM).
- LEAN manufacturing.

AM focuses on the flexibility and ability to react to the changing customer needs (Gunasekaran, 1999). Even though AM is supposed to bring quick cost-effective responses to fluctuating product demand and help with rapid production launches for unplanned products, this does not work in a simple way and an additional calculation is needed to justify this approach (Elkins et. al., 2004). Although AM is addressing the flexibility and the customer demand, it is lacking in technical measures to reduce cost and improve the quality of the product, therefore a hybrid system of LEAN-Agile manufacturing is considered as well, with such an approach proposed recently by Elmoselhy (2013).

LEAN manufacturing is considered to be one of the last major changes in the manufacturing systems' development (Womack et. al., 1990) and still many authors address the issues connected to LEAN manufacturing. Some of them report benefits of LEAN implementation in general manufacturing (Khanchanapong et. al., 2014), in specific case studies (Yang et. al., 2015), accounting (Fullerton et. al., 2014) and environmental friendly production (Pampanelli et. al., 2014). LEAN production affects the whole range of the manufacturing sequence: from the component supply chain to the market of a final product. Although the major impact is to the manufacturing system: production planning and control, and manufacturing operations.

LEAN developed from the Toyota production system (TPS), therefore most principles and tools in the LEAN manufacturing and TPS are the same. To say in other words, LEAN is the international name of TPS, meaning wide application possibilities outside the Toyota motor company. There are a several concepts and definitions of LEAN manufacturing:

- 'LEAN is the production system that performs only the value-adding operations and considers other work as waste' (Ohno, 1988)
- 'LEAN is the system that uses zero inventories, produces zero defects and can offer endless product variety' (Holweg, Pil, 2004)
- 'LEAN production is lean, because it uses less of everything compared with mass production – half of the human effort in the factory, half the manufacturing space, half the investment in tools, half the engineering hours to develop a new product in half of the time' (Womack et. al., 1990)

Summary of these definitions suggests the main idea of LEAN manufacturing: it focuses on cost savings by ultimate elimination of waste, this enables companies to reduce production cycle times and propose endless product variety that leads to huge cost, human labor and the other savings when compared to convenient manufacturing systems. Therefore many companies adopted LEAN into their manufacturing systems (Holweg, Pil, 2004; Holweg, 2006). In addition, LEAN production spread from the automotive industry to other fields, such as electronics (Doolen, Hacker 2005), construction (Crowley, 1998) and the other manufacturing industries. Moreover, Toyota finally became the largest car producer in the world, with the highest profits, demoting the US car producer General Motors to second place (Holweg, 2006). These facts show the superiority of LEAN production and explain why LEAN is actually the system that current manufacturing is focusing on.

1.2. Assembly process modelling

Regarding the process modelling, the assembly line is still a very common production technology in today's manufacturing industry. A lot of manual assembly lines were replaced by robotic assembly lines; however there are still many production fields where mechanical human work still remains as the main assembly technology.

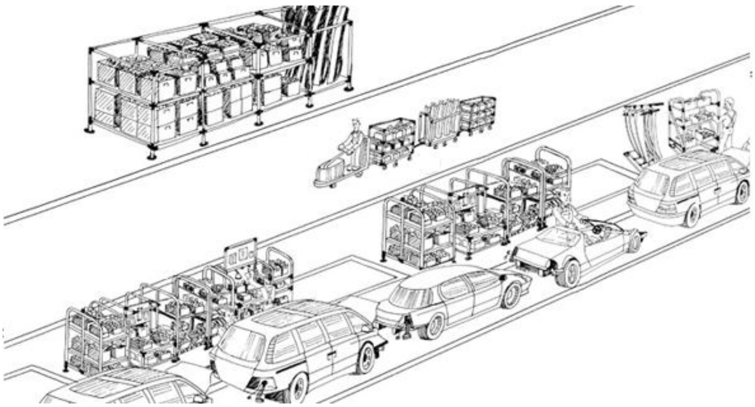

Fig. 1.2. Mixed model assembly line (IPA Magazine, 2015)

Such examples are: final assembly in the automotive industry, automotive wiring harness industry and the other production fields where robotic assembly is too costly to be competitive. To sum up, nowadays there are plenty of different methods of how assembly is performed (Freiboth et. al., 1997, Thomopoulos, 2014):

- single model assembly
- batch assembly
- mixed model assembly
- one station assembly
- cellular assembly
- robotic assembly

The classical assembly line was created by Ford and it is dedicated to single product produced in larger quantities. Such an assembly line is fixed, so the product change demands technical rearrangements of the whole assembly line. Classical assembly line was developed to form the batch assembly. In the batch assembly there is more than one product, however each product is processed as a single model line and the planned inventory define the time interval for each product run. Prior to the start of the new model assembly line it needs to be set-up and adjusted. There is no possibility to run production of different models simultaneously.

Due to the fact that the first two assembly systems are lacking flexibility, a new approach to the assembly organization was introduced. Such an approach enables organizations to assemble different products at the same assembly line at the same time and is called the mixed model assembly (see Fig. 1.2). The major problems regarding mixed model assembly are line balancing and sequencing (Thomopoulos, 2014). In the case of make-to-order production, these problems become more significant and there are many recent reports on this particular problem (Tiacci, 2015; Kucukkoc, Zhang 2015; Hazir & Dolgui 2015). Mixed model assembly is the most flexible, because it can react to the individual needs of every customer, however it is also the most complex to implement technically. Since the customer order defines the planning and sequencing, every order becomes unique, with its own quantity and bill of material. If the customer demand is stable, the assembly line balancing and sequencing becomes quite simple, but when demand fluctuates and arrives at random and long term intervals it becomes hardly possible to balance such an assembly system.

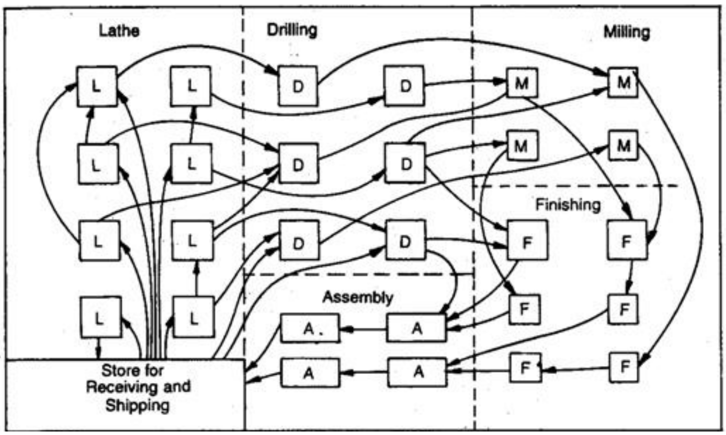

Fig. 1.3. Cellular manufacturing layout (Black, 2007)

Due to the fact that the market is forcing the manufacturing industry to provide small batches of customized products, many manufacturing companies shift from assembly lines to assembly cells (Molleman et. al., 2002; Johnson, 2005; Black, 2007) or U-shape assembly lines (Miltenburg, 2001).

One station usually handles one assembly task (or one product) by one (or two) assembly operators. When order quantities are very small or singular, one station assembly is superior comparing to other assembly systems. However, such an assembly system requires more time for setup and adjustment and has lower efficiency rates, since one operator has to handle a full range of assembly operations. In addition, one station assembly demands high skilled operators, especially for high complexity product assembly. Since this type of assembly has the largest flexibility, the definition of the assembly system depends on the tasks assigned for certain assembly stations (cells). Therefore, the certain network of several working cells constitutes a cellular assembly system (see Fig. 1.3) which can be regarded as a mixed model assembly system. The research (Johnson, 2005) declared about a 50 % increase in production output and a 50 % decrease on average flow time per batch from the same manufacturing area when compared with the mixed model assembly line.

Robotic assembly is often considered as the alternative to the manual assembly (Aguirre et. al., 1997). Robots can perform the same task for a long period of time without losing quality, when human operators are fallible to a variety of ergonomic issues, fatigue and etc. This is the reason why robots are replacing human work in the manufacturing industry to an increasing extent. The major drawback of the robot implementation is the extremely high investment cost and in low volume production these investments would be uncompetitive. Therefore, economical calculation models determine the choice (Aguirre & Raucent, 1994).

If the robotic alternative is refused, the major question arises when management has to make the decision and choose a particular assembly process for a certain product or product group. The choice is usually based on such data:

- planned production volume
- current manufacturing situation
- previous experience
- calculations and assumptions

Regarding the assembly process flow, it is very important to decide the appropriate job division, number of work stations and etc. Analytical tools addressing these issues are presented in (Zulch, 1997). To calculate the data for decision making is quite a complex problem, because uncertainty always exists. Such a calculation example can be found in (Abdel-Malek & Resare, 2000). The research proposed a methodology to integrate product structure, operational constraints and available budget to make a decision of the appropriate assembly cell design, production planning and purchase of equipment. More tools to address decision making are available in (Abdullah et. al., 2003). The authors of the research emphasized the need for methods for the selection of the assembly type (assembly line, cellular manufacturing, mixed model assembly, etc.) that would help to render the final decision. Such methodology can also be found in (Su, 2007). The author developed a case-based assembly sequence planning tool, which can be used as a versatile tool to any specific case of assembly. This tool enables a comparison between different assembly scenarios to make a robust decision. Nevertheless, the proposed tools perform well in the case of larger production quantities and more stable production environment. In the case of instability, these tools fail to provide robust results and it is this reason that it is quite popular among production researchers to propose specific (non-standard) assembly process models. Some specific alternatives to the assembly line were proposed by Engström and Jonsson (1996). The authors proposed a parallelization of the assembly processes for productivity and quality improvements. Large scale modelling of operations and assembly processes of high complexity wiring harness components are presented by

Estrada F(1997), who addressed the assembly process of the automotive wiring harness to be suitable for the just in time (JIT) production, however, it addressed the single product and with stable production demand.

Process modelling for the optimization is also addressed for specific case studies. Weigert et. al. (2011) addressed the general assembly situation and proposed model handling multiple parallel assembly processes for optimization, production scheduling, supply of materials and etc. Another report (Panhalkar et. al., 2014) addressed the automotive assembly case and proposed the optimal assembly sequence that minimizes assembly time. However, none of these reports included learning effects into the assembly time calculation.

1.3. Modelling of mechanical assembly operations

The total time to complete the sequence of the manufacturing and assembly operations defines the cost of the final product. There are two ways for cost improvement:

1) Increase production quantities (mass production)
2) Change, simplify or redesign manufacturing and assembly operations.

There are two common ways in production engineering to achieve these cost reductions: design for manufacture and assembly (DFMA) and Value analysis / Value engineering (VAVE). Design for assembly is usually sub-divided into design for assembly (DFA) and design for manufacture (DFM). In this research the manual assembly is the main topic, therefore only DFA will be considered.

Design for assembly (DFA)

Initial objective of this technique was to create simple models of the manufacturing processes, so that they can be used without specific manufacturing knowledge (Dewhurst & Boothroyd, 1988). Today, DFA is dedicated to assist designers' in simplifying the product structure, reducing assembly costs, and measuring the improvements of a certain assembly (Boothroyd & Dewhurst 2002). Also, the technique has been simplified and further improved recently (Moultrie & Maier 2014).

From the technical side, DFA provides guidelines for the part handling, insertion and fastening.

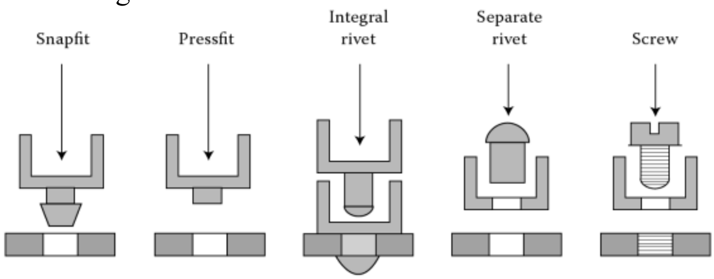

Fig. 1.4. Different fastening methods (Boothroyd & Dewhurst 2002)

There are many component connection methods, but each of them differs in complexity and time (Fig. 1.4). The simplest fastening method is snap-fit and the most complex one is the bolted connection. Such an approach, by evaluating each assembly operation, is the core of the DFA. The major aspects of component handling, insertion and fastening are:

- Clear distinction between symmetrical and asymmetrical components.
- If the parts need asymmetrical, make this asymmetry clearly visible.
- Avoid components that tangle, stick together, are very small or flexible.
- Avoid resistance in the part insertion.
- Standardize the components to use the benefit of increased order quantity.
- Avoid necessity of component holding down to maintain their position.
- Part should be finally located after being released, avoid any adjustments or positioning after component is inserted.
- Use the most simple fastening methods (Fig. 1.4)

The example of DFA application to the particular assembly is presented in Fig. 1.5.

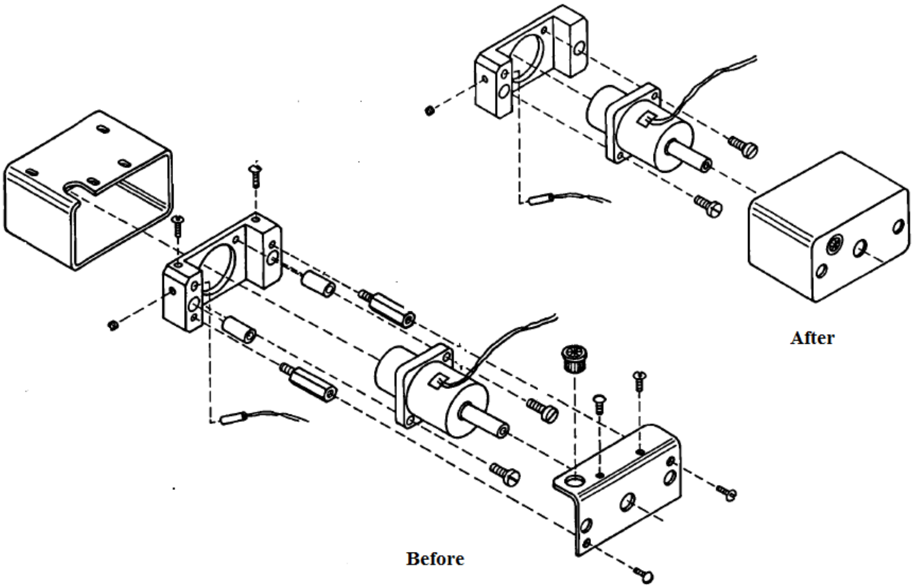

Fig. 1.5. Application of DFA (courtesy, Boothroyd & Dewhurst 2002)

The original design has the assembly time of 160 sec. After the DFA improvements the assembly time was reduced to 46 sec. In addition, number of different components was also reduced and the product extremely simplified. The assembly time improvements were achieved by applying the DFA technique. DFA possess high application versatility: it can be applied to any product where mechanical assembly is performed, including extremely high complexity and time consuming manual assemblies of wiring harness components (Ong & Boothroyd, 1991).

Complexity modelling

According to DFA operation sheets, more operations and more components would require more direct standard time to complete the final assembly. In addition to this, an increased number of different components leads to increased complexity and therefore additional indirect time is required to complete the assembly i.e. more time is needed for thinking and decision making (learning phenomenon). Many researchers report about the relationship between the variety of assembly parameters and complexity. Researches are focused on two main topics: complexity modelling and complexity reduction by grouping and simplifying.

Full scale modelling of the manual assembly operations of wheel support assembly were presented in the research by Wang, et. al. (2005a & 2005b). Such modelling includes all ingoing components, their dimensions, mating and assembly information. The results of this particular study suggest that mathematical modelling enables the selection of the best variants for the actual assembly.

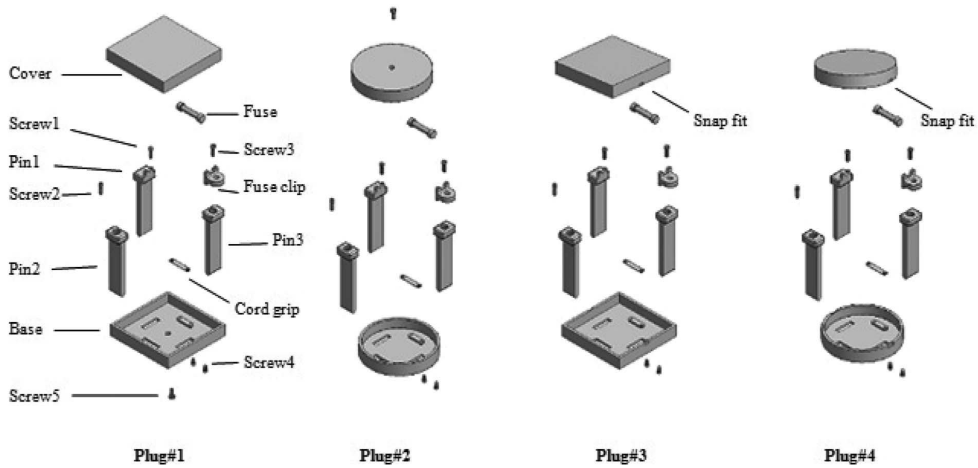

Fig. 1.6. Different assembly assortments (courtesy Samy & ElMaraghy, 2010)

Samy and ElMaraghy (2010) proposed a mathematical model for measuring a products' assembly complexity in terms of difficulties during the handling and insertion processes. Four different assemblies (see Fig. 1.6) were studied and the complexity results presented in Table 1.1.

Table 1.1 Complexities of different assemblies (Samy, ElMaraghy, 2010)

Product	Product complexity	Total assembly time from DFA analysis (s)
Plug1	5,74	38,66
Plug2	5,70	37,02
Plug3	4,72	31,16
Plug4	4,70	29,52

As it can be seen from Table 1.1, complexity increases with the increment of the assembly time. Methodology can be used as a supplementary technique for DFA for complexity reduction at the early design stages. Similar results were found by Mathieson et. al. (2013), however the authors of the research used statistical methods.

Interesting results are reported by Singer et. al. (2014). The research focuses on planning improvement of low volume complex products and addresses the gap between the product documentation stage and the manual assembly production process. The idea is to supplement the bill of materials with additional information: specific low volume assembly instructions and standard assembly times to have the assembly model with quite a short time interval. This documentation can be further used for production planning and control.

Another group of researchers report complexity reductions by component grouping (Madan et. al., 1995, Kannan & Jayabalan, 2001), reorienting (Pan & Smith, 2006) and kitting (Hanson & Medbo, 2012). There studies show, that grouping of constituting components simplifies the assembly task, thus significantly reducing the thinking and decision making time of the operator.

Value analysis / Value engineering (VAVE)

The DFA method is usually dedicated to the design stage of the product, however it can be used for an existing process as well. Another technique to improve an existing process is VAVE. Since the production cost can be reduced by eliminating waste, this technique addresses the identification of the value adding operations from waste and other operations which do not add any value. The main idea of VAVE is to work smart and not hard (Konz & Johnson 2000). Also, VAVE is referred as the LEAN tool to reduce processing cost (Singo, 1989, Womack et. al., 1990). There are six steps of VAVE:

- Select a problem
- Gather and collect the information
- Define a products functions and point out the value adding operations.
- Create solutions
- Evaluate solutions
- Recommend solutions for a particular problem

VAVE application leads to a huge amount of savings of money and manufacturing hours. Compared with DFA, VAVE is a more general direction whereas DFA is a technical tool for manufacturing time reduction. Moreover, VAVE enables the elimination of not only the non-value adding operations, but also non-value adding processes. This is very important to consider while designing the assembly lines or assembly cells.

1.4. Effects of vibrations and repetitive motions on human work

LEAN manufacturing system omits efficiency as the main criteria; however the majority of manufacturing companies, especially in the automotive industry, perceive efficiency as the major competitiveness factor. In manual assembly based companies, efficiency is mostly based on worker operational performance, i.e. time of the manual assembly operations. Manual assembly time of the operators is affected by a variety of factors, such as motivation, working skills, working environment, quality of tools, work organization and ergonomic aspects. In addition, it is reported that a good ergonomic situation leads to increased productivity and profit (Oxenburg, 1993). Many ergonomic factors can be quite easily provided to make the work place safe and comfortable, but with regard to the manual and semi-automatic assembly, the most dangerous factors for the operator's health remain the vibrations and repetitive motions, due to the fact that they directly affect the operator's body and its parts (Griffin, 1996). Coupled to this is the global competition in the manufacturing industries demands for increased production capacity and lower production costs, thus assembly operators receive an increased work rate i.e. they are required to produce more during the same period of time or/and are forced to work overtime to complete production orders. Gooyers and Stevenson (2012) reported that increased work rates requires increased muscular effort, which leads to the elevated risk of musculoskeletal injury. Companies competing for each customer order often omit the most important ergonomic factors and are then forced to pay compensation costs to employees for damaged health and work related traumas (Bonzani et. al., 1997). On the other hand, many authors claim (Neese et. al., 1993; Griffin, 1996; Murphy et. al.,) that traumas caused by vibrations and repetitive motions can be avoided if they are measured and correctly distributed between assembly operators.

In the manual assembly process, due to the fact that parts and components of assembly need to be manipulated and a variety of hand tools should be applied, two major risk factors exist: segmental vibrations and repetitive motion injuries (RMI) or cumulative trauma disorders (CDT) (Helander, 2006). In most cases, these two risk factors both affect the human body simultaneously and both lead to the same injuries and professional diseases. In manual assembly, segmental vibration occurs for the operators who manipulate power tools by hand, such as drills, saws, heaters, hammers and similar equipment that vibrate and transmit the vibration through the hand (Griffin, 1996). The most common injury in such cases is white finger disease. This disease occurs due to prolonged usage of hand tools vibrating at a 20-100 Hz frequency. Such tools include the major of power tools (drills, impact wrenches and etc.). If such a vibration continues, this leads to permanent damage of nerves and blood vessels in the hand (Helander, 2006). A prolonged segmental vibration also leads to other injuries, such as hand-arm vibration syndrome and carpal tunnel syndrome (Cederlund et. al., 1999). In regard to the manual assembly performance, this means temporal or even permanent impairment of hand functions.

RMI and CTD are being widely recognized in manufacturing ergonomics for the last 30 years (Konz & Johnson 2000), (Bonzani et. al., 1997), (Putz-Anderson, 1988) (Zetterberg &, Ofverholm, 1999) and they occur as a result from these operational activities (Helander, 2006):

- repetitive hand movements with high force;
- flexion and extension of hand; high force pinch grip.

The particular operations causing these factors include: grinding, working with a press, assembly of small components (wrapping, wiring etc.), belt conveyor assembly, packing operations and etc. To sum up, these operations encompass most of the activities performed in manual assembly, especially in high volume production and if repetitive motions continue for longer time intervals they lead to CTD. The major RMI instances are carpal tunnel syndrome, cubital tunnel syndrome and tenosynovitis. More specific, RMI and CTD can be found in scientific literature (Konz & Johnson 2000), (Helander, 2006), (Putz-Anderson, 1988). The symptoms of these RMI include numbness, inflammation of tendons, swelling and similar injuries that prevent the operator from working. More injuries can be found in Table 1.2.

Table 1.2. Variety of different CTD and hazard operations (Helander, 2006)

Type of manual operation	Injury	Causing factors
Buffing/Grinding	Tenosynovitis, carpal tunnel, thoracic outlet	Repetitive motions of the wrist, ulnar deviation with force, repetitive forearm pronation, vibrations.
Punch press operations	Tendonitis of wrist	Repetitive forceful wrist extension/flexion
Overhead assembly	Tenosynovitis, tendonitis, De Quervain's, thoracic outlet	Repetitive ulnar deviation, prolonged hyperextension of arms, hands sustained above shoulders
Belt conveyor assembly	Tendinitis, carpal tunnel, thoracic outlet	Arms extended, abducted or flexed. Repetitive motions of the wrist
Small parts assembly (wiring, bandage wrap)	Tendonitis of wrist, thoracic outlet, epicondylitis.	Prolonged restricted posture, forceful ulnar deviation and thumb pressure, repetitive motion of the wrist, forceful wrist extension and pronation
Packing	Tendinitis, carpal tunnel, De Quervain's	Prolonged load on shoulders, repetitive wrist motions, overexertion, forceful ulnar deviation

The significance of repetitive motions and vibration effects to human work is reported by many researchers. The major parameter defining the risk of vibrations and CTD is the time of which exposure to the risk factors last (Wells, 2007). Therefore, there are plenty of calculation methods available to measure an optimal exposure time to the risk factors (Tanaka & McGlothlin, 1993), (Kristensen et. al., 1997), (Dong, 2006), (Merritt & Gopalakrishnan, 1994); however, most of them suffer from uncertainty and accuracy problems, but some recent research address this issue (Ainsa et. al., 2011), (Moschioni et. al., 2011). It is not enough to solely know that vibration effects human work, it is also important to measure the direct impact of risk factors to the assembly time of the operator. However, there is a lack of such models in scientific literature.

1.5. Learning curve models

The performance of the task improves as the task is repeated. Mathematically the learning is defined by a certain function; learning curves (in this dissertation classical learning curve – CLC) which show a time (or cost) decrement as the argument (number of units) increases. This learning phenomenon was firstly reported by Wright (Wright, 1936) after studying the assembly of airplanes. Since then, CLC has become an important industrial engineering topic and it has been used for predicting future costs, analyzing and controlling the performance and efficiency of certain individuals, groups, organizations and etc. The usage of CLC has spread from manufacturing to other fields, such as healthcare institutions, military, education, training and the other sectors, however manufacturing, especially manual assembly based industry, is at the top of the interest. Initially, CLC was used to predict and forecast operating time and production cost decrement as production continues (Bevis, et. al. 1970). Since manufacturing is shifting from mass production with high production volume and low diversity to LEAN production and Mass Customization with small production quantity and an almost endless product variety, the manual assembly based production systems are encountering serious issues, mainly caused by the never ending learning phase; quantities are just too small to complete it and the time for learning constitutes a major part of the total task processing time. As reported in articles (Anzanello, Fogliatto, 2011), this is the reason why CLC problems are re-emerging as an important issue among production researchers.

There are many various LC models in scientific literature and applications. Wright (Wright, 1936) proposed a cumulative average learning curve (CLC) based on power function:

$$y_W(x) = \beta x^{-\alpha_W}, \quad (1.1)$$

where y_W is the average time of all units produced up to the x-th unit, the parameter α_W is a slope coefficient, β is the amount of direct labor time required to produce the first unit. Crawford's model is as follows (Crawford, 1944; Yelle, 1979):

$$y_c(x) = \beta x^{-\alpha_c}, \quad (1.2)$$

where x is the unit number, y_c is the amount of direct labor time required to produce the x-th unit, the parameter α_c ($\alpha_c > 0$) is a slope coefficient, β is the amount of direct labor time required to produce the first unit. Crawford's model is usually regarded as the unit model LC. Some authors also use the Plateau model (Baloff, 1971); (Teplitz, 1991); (Li, Rajagopalan 1998):

$$y_p(x) = \beta x^{-\alpha_p} + \gamma, \quad (1.3)$$

where x, y_p, α_p, and β are the same as in (1.2), γ ($\gamma > 0$) is the constant that describes when the steady-state is reached after the learning is concluded or when machinery limitations block workers' improvement. Baloff (1971) studied this plateauing phenomenon and found it to be extensively present in machine intensive manufacturing, however even in fully manual assembly this phenomenon also exists. Comparison of different power function based LC models is represented in Fig. 1.7.

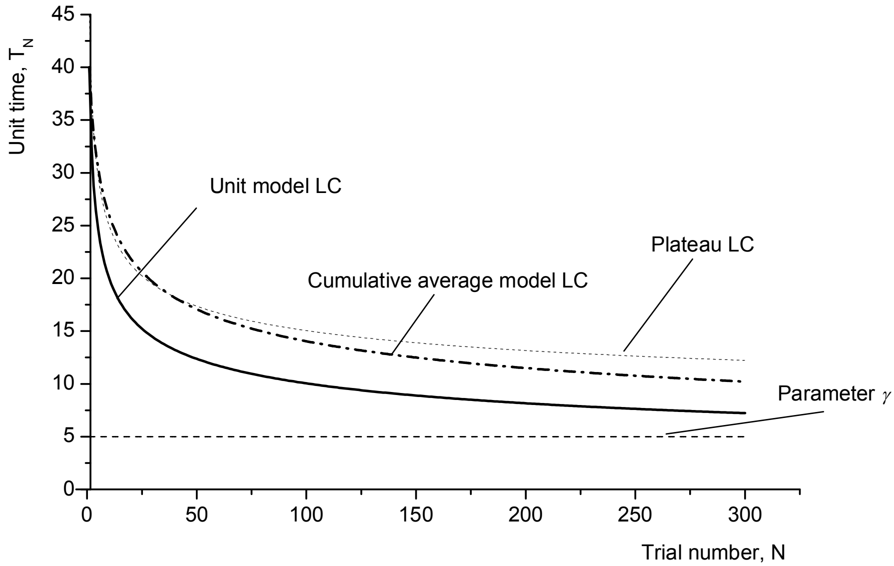

Fig. 1.7. Comparison of different LC models

Although all these three LC models are based on power functions, their application is different. Unit and cumulative average models are based on the same production data in Fig. 1.7, however their curves and parameters are different. Therefore these two models are completely different when it comes to application. The Plateau model has a positive asymptote to be reached after the learning phase is fully completed. However, this can be achieved while trial numbers increase to infinity, which is not really possible in a real manufacturing situation. Power models are the most popular and have very broad application possibilities. Superiority of power function as the best fit for a learning curve was been proved by intensive study and data (Newell, Rosenbloom, 1981), however many drawbacks of this model exists, therefore CLC research is open for improvements. Improvements were created by introducing new LC models. One of the main problems regarding power functions was the inability to consider previous experience. This was solved by creating the Stanford-B model (Teplitz, 1991; Badiru, 1992):

$$y_B(x) = \beta(x+B)^{-\alpha_B}, \tag{1.4}$$

where x, and β are the same as in (1.2), the parameter α_B ($\alpha_B > 0$) is a slope coefficient for the Stanford-B model, parameter B determines previous experience of the assembly operator, which actually shifts assembly time downwards.

When machinery is presented in the manufacturing process, it affects both the assembly time and the performance of the human operator. CLC has no possibility to consider these effects, therefore the learning model was further improved with parameters that evaluates effects of equipment and was called the DeJong's model (Badiru, 1992):

$$y_D(x) = \beta\left(M + (1-M)x^{-\alpha_D}\right), \qquad (1.5)$$

where parameter α_D ($\alpha_D > 0$) is a slope coefficient for DeJong's model, $M\left(0 \le M \le 1\right)$ is an incompressibility factor that defines the work ratio between human operators and machines.

The S-Curve model combines previously presented models (Stanford-B and DeJong's). The main idea of this model is to address the gradual start-up and avoid steep decrement of the assembly time (Badiru, 1992):

$$y_S(x) = \beta\left(M + (1-M)(x+B)^{-\alpha_S}\right), \qquad (1.6)$$

where α_S ($\alpha_S > 0$) is a slope coefficient for the S-Curve model. All other coefficients are the same as for Stanford-B and DeJong's models. The graphical comparison of these three models is presented in Fig. 1.8.

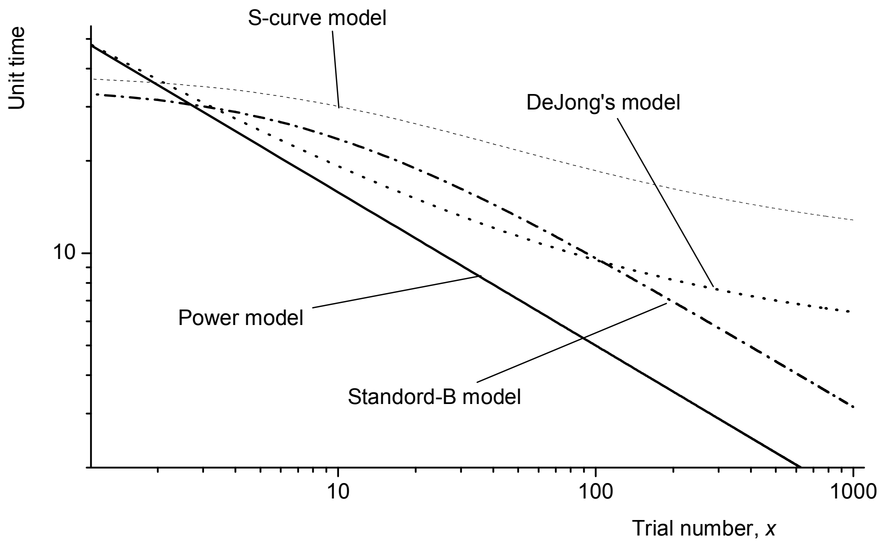

Fig. 1.8. Stanford-B and DeJong's and S-Curve LC models in LOG-LOG scale

LOG-LOG scale enables the observation of obvious differences between the proposed models (Fig. 1.8). The most simple power model results in a straight line according to the logarithmic scale. Other models produce approximate production data in a better way than a power model; however a lot of issues might occur due to the determination of parameters. However, all these models are power functions with different modifications.

Other authors (Li, Rajagopalan, 1998) reported the limitations of the traditional CLC models and proposed an analytical model to calculate the impact of knowledge depreciation and the plateauing phenomena to the CLC and total processing time. Smunt (1999) continues to unravel the shortcomings of the conventional CLC, thus eliminating misunderstandings of the CLC application and proposes mid-unit CLC model as the solution for the stated problems. The research (Waterworth, 2000) reports estimation errors of 30% due to misunderstandings and misapplications of the traditional CLC and proposes a theory for the correct application. On the other hand, a universal calculation algorithm proposed in work by Janiak and Rudek (2008), avoids major drawbacks of CLC fitting to particular production data, because it is open to any CLC model. Shortcomings and drawbacks of traditional CLC were being solved by using a dual phase learning assumption. This was initially reported by Dar-El, et. al., (1995.). The idea is based on cognitive and motor improvements with different CLC's combined into the one model. The proposed model was further improved by Jaber and Glock (2013). Mathematically, such a model is a combination of cognitive and motor learning functions:

$$y_{CM}(x) = \chi \beta x^{-\alpha_C} + (1-\chi)\beta x^{-\alpha_M}, \tag{1.7}$$

where α_C and α_M are slope coefficients for cognitive and motor learning curves, χ is a ratio between cognitive and motor work in the assembly task. The cognitive component in the equation (1.7) improves faster, as the operator learns the operations, whereas the motor components improve slower and might have some additional restrictions. The proposed model was applied to production data (Jaber, Glock, 2013) and provided a better fit to compare with other models.

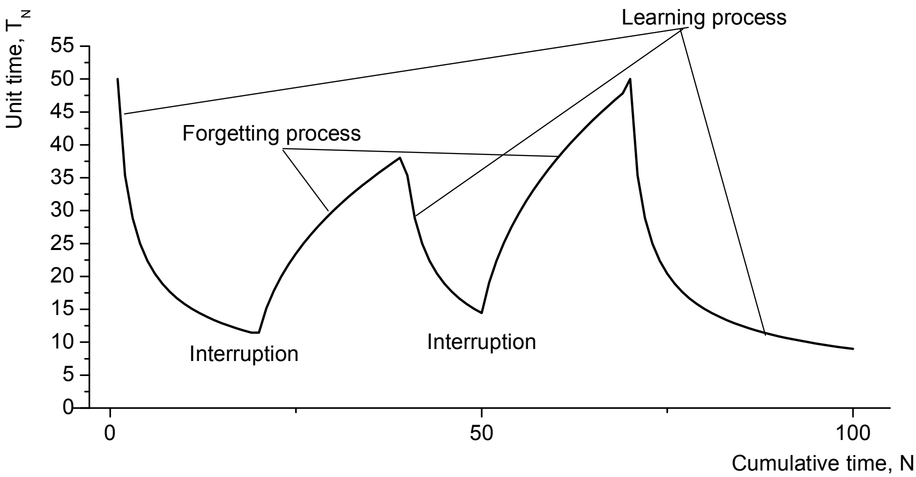

Fig. 1.9. Interruption, forgetting and recursive learning

Many authors emphasized traditional CLC limitations arising with production stops due to reworks and re-adjustments. Such stoppages lead to the forgetting phenomenon. A typical situation of interruption, forgetting and recursive learning is depicted in Fig. 1.9. The work by Globerson, et. al. (1989) reported significant impact of breaks to forgetting. Other researches (Jaber, Bonney, 1996), (Jaber, Bonney, 1997) proposed the analytical method to predict this impact and the performance after the forgetting phase. Learning-forgetting models were also improved by using dual phase learning curves, such improvements can be found in work by Jaber and Kher (2002). The comparison of three different potential learning/forgetting models could be found in Jaber and Sikstrom (2004).

In addition, quality learning curves were developed for the imperfect production processes generating defects requiring reworks by Jaber and Guiffrida (2004), as well as production interruptions (Jabe, Guiffrida, 2008).

Nevertheless, most of the previously discussed researchers used power model for its modifications. Other authors proposed different models to define learning. To begin with, an exponential function (Heathcote et. al., 2000) was proposed to define learning. The exponential function was tested on the same data as the power function and the results provided by the exponential learning curve approximated data more accurately. The exponential learning curve has the following form:

$$y_e(x) = \beta_e e^{-\alpha_e x}, \tag{1.8}$$

where y_e is unit time of the x-th unit, the parameters α_e and β_e is slope coefficient and assembly time of the first unit for the exponential learning curve. The principal difference of exponential and power functions can be visible in the Fig. 1.10.

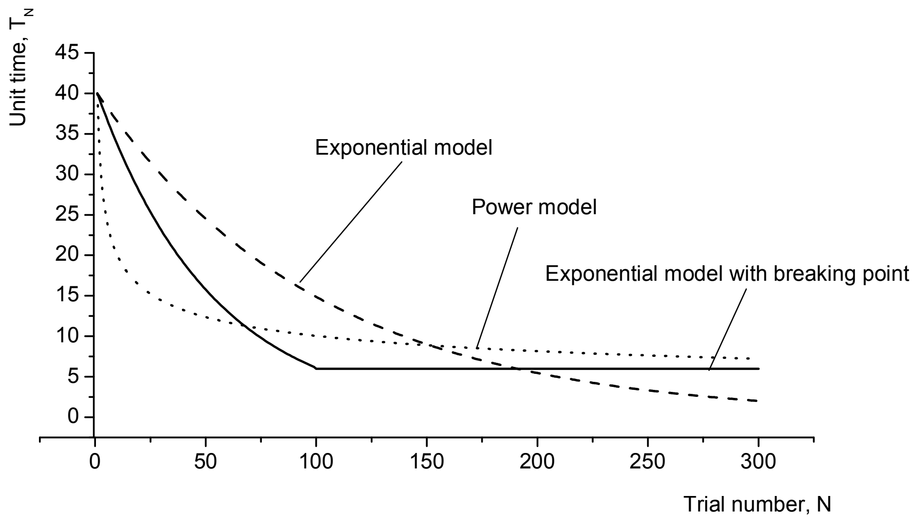

Fig. 1.10. Exponential vs. power learning curves

Exponential learning curves do not have as steep a decrement of time in the beginning of the production cycle as the power functions do. This might be a preferable property while applying it against raw production data. However, the exponential function does not have a horizontal asymptote, therefore it requires some improvements. Monfared and Jenab (2011) proposed a learning model to be applied in demand-based manufacturing, where the traditional power model might be not applicable. The proposed model consists of a double segment curve with breakpoints depending on orders manufactured. The LC used in the research was exponential, and after the breaking point processing time was assumed to be stable. The proposed method was tested on particular manufacturing data and provided better results than power functions. The proposed model is depicted in Fig. 1.10. The model itself was based on time:

$$y(t)=\begin{cases}\dfrac{\delta}{\lambda^{\delta}}e^{-\left(\frac{t}{\lambda}\right)^{\delta-1}t^{\delta-1}},t<t_0\\ \dfrac{\delta}{\lambda^{\delta}}e^{-\left(\frac{t_0}{\lambda}\right)^{\delta-1}t_0^{\delta-1}},t\geq t_0\end{cases}, \tag{1.9}$$

where t represents time, t_0 denotes breakpoint time, λ is a scale parameter, and δ is a cognitive factor. It is obvious from the equation (1.9) that increased complexity results in higher operating times. Also, there is a certain breaking point after which the steady-state (standard) assembly time is reached.

A separate family of LC curves are parametric hyperbolic functions. These functions are developed by analyzing organizational learning (Uzumeri, Nembhard 1998). This model was further improved and developed to the experiential learning model (Nembhard, Uzumeri, 2000). There are two major types of hyperbolic LC: 1) based on two parameters; 2) based on two or three parameters. The equation of the two parameter hyperbolic function is:

$$y(x)=k\left(\frac{x}{x+r}\right), \tag{1.10}$$

where k is a maximum performance level, x is the number of produced units, and r is a learning rate. The 3-parameter hyperbolic LC is supplemented with an additional variable defining prior experience:

$$y(x)=k\left(\frac{x+p}{x+p+r}\right), \tag{1.11}$$

where k, x, r is the same as per equation (1.10) and p is a previous experience of the operator. Also, $y,k,p,x\geq 0$ and $p+r>0$. For hyperbolic LC, three major cases can exist:

- $r>0$, curve shows time increment until maximum performance level k; ordinary learning situation
- $r\to 0$, no improvement is achieved.
- $r<0$, working performance declines as a result of fatigue or forgetting.

Even the hyperbolic models are quite simple to use and appears to be more robust than other LC models, however its application is limited to specific case studies and application is not as wide as of power models.

The studies (Anzanello, Fogliatto, 2011, Badiru, 1992) identified the need for multivariate models, although Badiru (1992) declared limitations of such a model application. Although the Wright based univariate CLC models dominate in the most literature, an advanced multivariate learning model was presented recently in the work by Ramsey, et.al. (2009). The main concept of the multivariate LC model is to combine different variables into one model (Anzanello, Fogliatto, 2011):

$$y_M(x) = K\prod_{i=1}^{n} c_i x_i^{-a_i} , \qquad (1.12)$$

where K is the time to perform the first assembly task, c_i is the coefficient depending on variable i, n is the number of independent variables. Other parameters are defined by i. Time (or cost) relationship with two variables is depicted in Fig. 1.11.

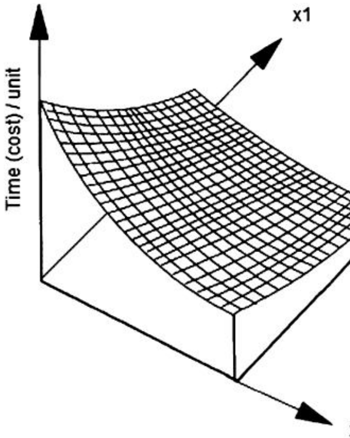

Fig. 1.11. Bivariate learning surface (adopted from Badiru, 1992)

The computerization and appearance of sophisticated devices in the manufacturing environment enables a lot of different data connected to manufacturing operations to be collect: operator experience, stoppages, training time and etc. This enables the analysis and implementation of multivariate LC models. However, these models are quite complex and suffer from uncertainties received with the production data. This is the reason why multivariate models are not as widely applied in comparison with univariate models.

There are more specific LC models available in scientific literature (Tepliz, 1991; Heizer, Render 2006; Dar-El, 2000), however these models are mostly based on specific production problems and do not have a significant contribution to general LC application, in addition the majority of them are derived from power, hyperbolic and exponential LC models.

1.6. Task complexity implications on LC

LC parameters are highly impacted by task complexity. The complexity affects both motor and cognitive skills of the assembly operator. Certain task attributes define task complexity, which has a finite impact on the improvement of cognitive skills (learning). These attributes were addressed in research (Richardson, et. al. 2004) that affects the operator learning abilities. In this research group, different products were studied to find the influence of each task attribute. Analysis showed that changes of components or operations affect the thinking and decision making time of the assembly operator. The main task attributes reported by Richardson, et. al. (2004):

- Selections
- Symmetrical planes
- Fastening points
- Fastenings
- Components
- Novel assemblies
- Component groups
- Assembly steps

Regarding this list, the most important task attributes were emphasized: components, novel assemblies, fastenings and component groups as having the most significant impact to assembly difficulty. The number of different components increase thinking time as do fastenings. Novel assemblies has a significant impact, since the operators have to learn them anew. On the other hand, grouping of components into component groups reduces the difficulty sharply. These assembly task attributes can not only be used for a task evaluation, but also for new assembly task complexity prediction at the early design stage (Richardson, et. al. 2006).

Beside the general knowledge that task complexity affects operator performance, another issue is the direct relationship between task complexity attributes and assembly time expressed by the LC. Such an approach can be found in research (Pananiswami, Bishop, 1991), which proposed a method to relate slope coefficient of the LC with task complexity and some behavioral aspects of the assembly operators.

The impact of product structure to manual assembly performance via LC was reported in Prabhu et. al. 1995. In this report, the complex assembly was considered and different assembly structures were evaluated. Widely applied technique PMTS (predetermined motion and time studies) declared the same results for all product structures, however the real assembly data indicated large differences between assemblies. Consequently, the learning curve parameters impact was seen on both: assembly of the first unit β and learning slope α. The major outcome of this study is that product assembly structure has a significant impact on manual assembly time and this issue might be overlooked in manufacturing. On the other hand, the research concluded that some types of product structures result in less complexity, however the relationship between task attributes and LC parameters was not proposed.

Statistical relationships can be found in research by Nembhard (2000) and Nembhard and Osothsilp (2002). The authors used 3-parameter hyperbolic LC (1.11) and performed statistical analysis to evaluate the impact of task complexity on parameters. Both reports concluded the statistically significant impact of task complexity on LC parameters, such as learning rate and forgetting rate. In addition, approximated values were proposed. However, an analytical relationship was not proposed.

Some recent reports proposed such an analytical relationship between complexity and LC parameters. Jenab and Liu (2010) presented a model for the relative complexity calculation. Methodology was addressed to many manufacturing issues, such as: assembly time estimation for cost calculation and scheduling, product life cycle cost estimation for future products, resource allocation, effective balance of manufacturing and assembly. Further research (Monfared, Jenab 2011) extends the relative complexity application, declares complexity impact on learning, and presents an expression to calculate task complexity of a particular operation. The complexity calculation is presented in the following form:

$$CP(t_0) = 1 - e^{-\left(\frac{t_0}{\lambda}\right)^{\beta}}, \qquad (1.13)$$

where $CP(t_0)$ is a complexity as a function of breakpoint time t_0, and the other parameters are the same as per (1.9) equation. The graph of this complexity function is depicted in Fig. 1.12.

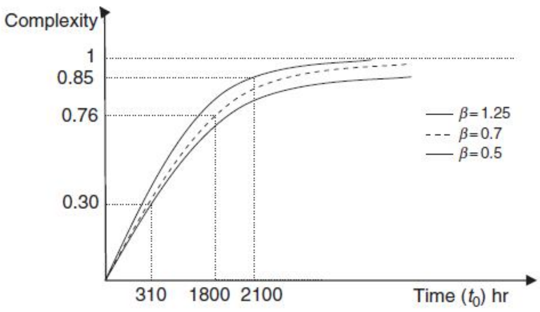

Fig. 1.12. Complexity as a function of breakpoint t_0 (from Monfared, Jenab 2011)

This research shows the connection between task complexity and operating time. However, the complexity of the operations is calculated according to statistical analysis and regression. There is still no link between task attributes and complexity, only the formula relating complexity and certain exponential LC parameters. This complexity model lacks versatility options, because excessive statistical analysis has to be performed to gain the values of necessary parameters. In addition, there are no considerations of uncertainty situations.

1.7. LC application in production

Beside general LC research and review, there are plenty of particular LC applications to certain manufacturing problems, such as (Anzanello, Fogliatto, 2011): production planning and scheduling, product development, job rotation, cost estimation and etc. Many researchers addressing the issue of production planning and control report benefit of the LC application, especially on low volume production and large product variety. McCrery and Krajewski (1999) showed that LC models could be used as a solution for flexibility and efficiency improvement, especially for order-based manufacturing with increased product variety.

Several authors address planning improvement using learning curves. The paper by Smunt and Watts (2003) addresses the processing time estimation from limited shop floor data and concludes that estimated learning curves could be used for better allocation of labor resources, thus creating a smoother workflow at the factory through planning improvement. On the other hand, the same paper points out the lack of possibility to gain such detailed data to be used for curve fitting, because companies rarely collect and share such data with researchers. In spite of this, even limited data could be applicable and useful for learning curve application.

A case study by Gunawan (2010) used various univariate LC models to fit the data from a sheet metal company. The author proclaimed power law LC as the best fit for the particular production field for workforce planning. Moreover, the author reported the benefit of LC application to labor resource allocation.

Authors Gabel and Riedmiller (2012; Anzanello and Fogliatto (2007; Anzanello and Fogliatto (2010) proposed some analytical and deterministic planning methods with implemented LC. The study (Gabel, Riedmiller, 2012) reported that empirical evaluation showed an effective solution to the job-shop scheduling problems. In other works (Anzanello, Fogliatto, 2007; Anzanello, Fogliatto, 2010), a case study was performed at a shoe manufacturing company. The results show that satisfactory workload balance and optimal schedules were achieved after implementing learning curve models into process planning.

Other works (Cohen et. al. 2006); (Neidigh, Harrison, 2010) address the production optimization with learning models. The research (Cohen et. al. 2006) focused on optimal work allocation and concluded after empirical calculations that savings of LC-based work allocation grows (compared with traditional line balancing), as order quantity reduces and the number of operations increases. The study by Neidigh and Harrison (2010) concentrated on an optimal work schedule and optimal order size and proposed an optimal deterministic planning method to satisfy the demand accurately and minimize production costs. The empirical results from the company with heavy non-linear learning effects confirmed the approach to be adequate and realistic compared with other methods.

Some authors address similar planning issues similar to the ones in this research. The inefficiencies of traditional balanced assembly lines while coping with unequal operator speed due to learning are reported in Montano et. al. (2007). The authors study the impact of variability to the general assembly line performance and their findings show that the introduction of new operators cause major inefficiency of the traditional assembly balancing. Also, the paper provides an analytical approach to improve planning in the case of variability of operators.

The paper by Glock et. al. (2012) deals with the ramp-up period caused by the learning impact during the growth of demand and proposes a planning model in stabilizing the production process and inventory levels, in addition it points out the synchronization as the major source of performance improvement. The same paper suggests that synchronization could be achieved with assigning extra operators to the assembly process. On the other hand, even skilled operators need a learning phase to achieve steady-state performance. Therefore, in highly manual assembly; operator shifting might not be the preferable solution.

LC studies are still widely considered in production engineering and still new applications occur. Here, the very recent research reports LC to be an effective solution for production time on optimal lot size (Kumar, Goswami 2015).

Regarding the product development, the report by Seta et. al. (2012) also emphasized the greatest impact of learning effect for products with a high complexity and low order quantity. In other words, in the case of LC implementation, the largest impact will be seen on the more complex products.

Job rotation is also a production issue where LC can be effectively applied. However, job rotation is needed due to the reduction of work related traumas and injuries. On the other hand, the changing of operators in the assembly lines due to absenteeism or planning cause bottlenecks (Cohen, 2012). Typical situation is depicted in Fig. 1.13.

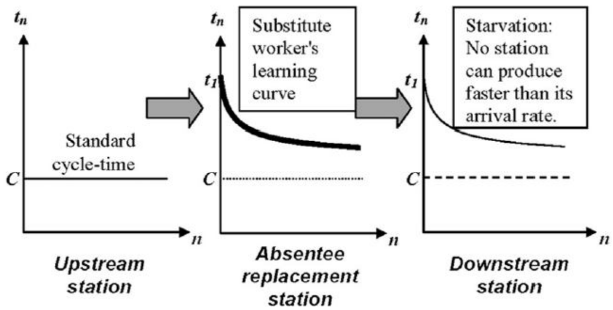

Fig. 1.13. Bottleneck due to operator change (from Cohen, 2012)

This issue is not widely addressed among scientific researchers, though some reports exist (Ortega, 2001); (Allwood, Lee, 2004).

Cost estimation by utilizing LC models are also considered. Smunt (1999) reported that the unit LC model and cumulative average LC model could be used both for cost estimation and suggested mid-unit approach. Variety of different cost models based on LC can be found in work by Lee (1997).

Parameter estimation

Regarding the LC application, two major problems exist: correct model selection and parameter estimation. Both problems are solved by employing statistics. However, uncertainties can result in inaccurate LC application and a lot of errors. Parameter estimation is an even more important problem. However, most parameter estimation methods in the LC application are traditionally statistical with full production data (Newell, Rosenbloom, 1981; Heathcote et. al. 2000; Heizer, Render, 2006; Globerson, Gold, 1997). Even though some authors proposed some advanced statistical methods, such as parameter estimation from a single random sample (Goldberg, Touw, 2003), parameter estimation from 'messy' manufacturing data (Smunt, Watts, 2003), they still violate conditions of statistical method application. Therefore, deterministic parameter estimation methods appear to be relevant for more accurate LC application. Some authors already apply deterministic methods (Kreinovich, 1995). Also, Wakker (2008) provided the deterministic method for the empirical researchers who use the power family curves to fit the data. Note that LC curves belong to this family. On the other hand, there is still a lack of such approaches in LC application.

1.8. Section conclusions and formulation of problems

Recent trends in manufacturing clearly show an irreversible process of the reduction of order quantities and increment of product variety. In the automated assembly, a lot of different technical measures exit to cope with this situation. However, in the fully manual assembly, existing technical measures (DFA, PMTS, VAVE) appear to not be effective enough to manage assembly processes at the beginning of the LC. Therefore, LC research has re-emerged recently to address issues occurring in the new manufacturing environment.

Although many authors report the benefit of LC application, the majority of existing LC models are dedicated to large volume production and might be inapplicable for low volume and large variety production. In addition, many proposed models are specific case studies and they lack versatility. In addition to this, there are very few reports with analytical tools connecting ergonomic factors and complexity with direct assembly time. Finally, most research addresses the learning time measurement and not the reduction of it.

The scientific literature review completed emphasizes the major problems for this dissertation:

- Define the appropriate research methodology in the context of manufacturing engineering.
- Develop and apply new mathematical learning models that adequately approximate the operating time development (reduction) of the manufacturing process.
- Create methodology to estimate parameters of the learning curve from the limited production data.
- Define, state and solve the manual assembly process efficiency optimization problem by employing appropriate LC models proposed in this dissertation.
- Perform production data monitoring to test, evaluate and prove the adequacy of the proposed models.

2. RESEARCH METHODOLOGY

2.1. Introduction

In this section the research methodology is determined. As the goal of the dissertation is to optimize production efficiency by manual operations modelling, a lot of pre-work has to be done in order to fulfil the main tasks of the dissertation. This pre-work includes the following tasks:

- Present and define a manual assembly product and its assembly process
- Propose the mathematical model for this product
- State a optimization problem to increase efficiency
- Propose the methodology of LC model selection
- Propose the methodology for production data monitoring
- Present the methodology to evaluate adequacy of selected models

All these tasks will be completed in this section in order to provide sufficient background for the next steps.

2.2. Manual wiring harness assembly

Wiring harness

One of the most time consuming assemblies is the manual assembly of the wiring harness components (Aguirre, Raucent, 1994). This is because wiring harness products might contain a variety of typical assembly operations (insertion, fastening, positioning, screwing, etc.) plus they demand some special manual operations such as wiring, wrapping, piercing and etc. Therefore, manual assembly of the wiring harness is very representative with regard to manual assembly. It is the main reason why in this research, as a typical manual assembly process, wiring harness production will be studied. The wiring harness performs an electrical circuit function in automobiles, construction equipment, buses, trucks, ships, aircraft, household appliances, and etc. to connect various electrical and electronic devices, sensors and other equipment. Even the main function is electrical; the production of the wiring harness is a pure mechanical assembly process: manual, semi automatic, automatic, depending on production volume (Aguirre, Raucent, 1994). There are three stages of wiring harness manufacturing:

- Design of the wiring harness
- Assembly of the wiring harness
- Installation of the wiring harness

The first and the last stages are usually performed by the final user of the wiring harnesses. The second stage is usually delegated to supplying companies due to outsourcing. Therefore, the supplier receives the technical documentation of the wiring harness, performs assembly and delivers the required quantity of wiring harness products to the customer. Thus, wiring harness suppliers are usually demand based or order based companies, they do not create their own product, cannot change the stated components or layout of the wiring harness and have to deliver the products only according to the technical specifications.

According to the wiring harness definition (Boothroyd, Dewhurst 2002), the main wiring harness components are terminated cables and wires (circuits), housings and connectors. All this information is usually presented in 2D drawings (see Fig. 2.1 and Fig. 2.2)

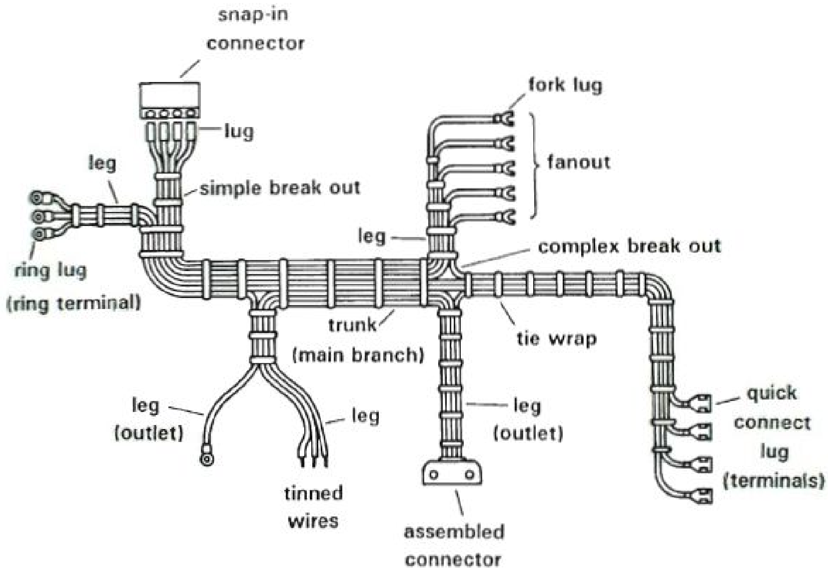

Fig. 2.1. Wiring harness components and terminology (Boothroyd, Dewhurst 2002)

The harness is composed of the main trunk, with main circuits and branches, break-outs and legs. Any branch of the main trunk needs to reach a certain electrical device (Fig. 2.1). The end of the each circuit could be crimped with terminal, soldered, tined or just cut. Several cables could be assembled into splice by crimping or ultrasound. To fix the branches, cable ties or tape wrap could be used. When cable harnesses serve in a very aggressive environment (for instance engine harness) heatproof corrugated tubes are used and etc. Corrugated tubes are usually connected to housings and connectors by adapters, branching points are secured by manifolds. Where there is no aggressive environment presence, cables could be only wrapped by PVC hose of without any tubing. Depending on the environment and the field, where the harness will have to serve, other additional components, such as non standard parts, shrink sleeves, other mechanical assemblies, could be encountered. A schematic view of the wiring harness with protective elements is depicted in Fig. 2.2. To sum up, the main wiring harness components:

- Terminated circuits
- Housing and connectors
- Wrapping material (tubes, hoses, tapes, etc.)
- Cable ties
- Adapters and manifolds

- Soldering material
- Additional components (screws, fuses, relays, etc.)

As it was stated at the beginning, all of these components are fully defined in the technical documentation of the wiring harness.

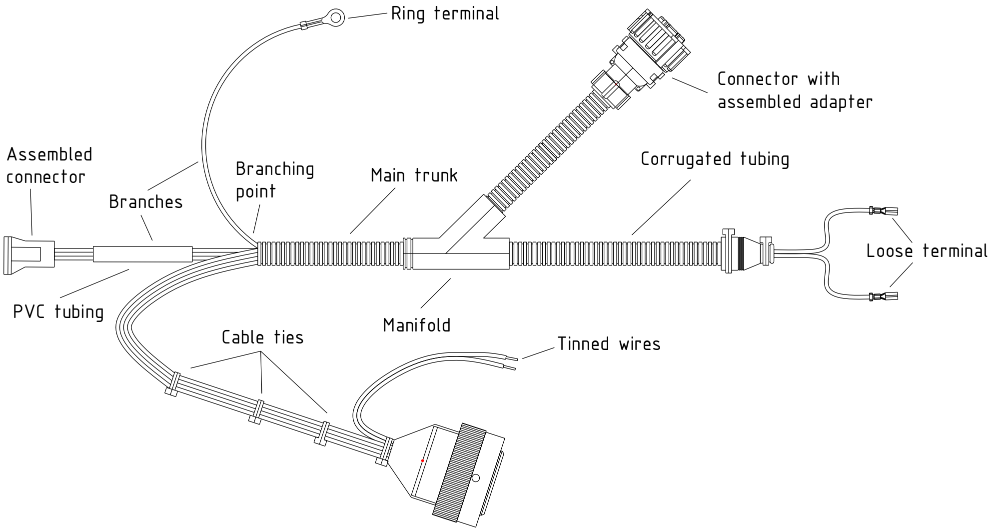

Fig. 2.2. Wiring harness with protective hoses

The whole production range of the wiring harness consists of the following manufacturing steps (Ong, Boothroyd 1991):

- Wire preparation, when wires are being cut and terminals mounted.
- Installation, which encompass laying and positioning cables and wires on the assembly board according to the wiring harness layout;
- Securing, when wires and cables are wrapped and bundled together, by ties, tapes, tubing etc.;
- Attachment, which involves plugging terminals into the housings and connectors, assembling other additional components, performing mechanical assemblies, sticking labels and markings etc.

Since the first step is usually automatic, only the next three steps require manual assembly of the product when the operator performs assembly operations on the assembly jig. The final assembly is performed at the working cell where all necessary production resources are provided during set-up: raw materials, semi-products, tooling, assembly jig and etc. Only after the previous order is fully completed is the manufacturing cell dismantled and rearranged for the new product.

During the assembly, an operator performs a series of small operations on each step, thus installing all necessary constituting components until the final product is fully assembled. At the beginning of the assembly, lots of time is wasted due to the start-up phase. The operator is forced to check drawings, standards and perform other additional start-up functions. The wiring harnesses are similar, however, there are four major types of wiring harness depending on the function: power cables, the common harness, the engine harness and the electrical center harness. Assembly departments (or operators) are usually dedicated to a certain harness type. Moreover, each harness has its own specific layout and circuit scheme. At the beginning of the assembly, the operator needs to check the documentation before each assembly step: before plugging a terminal, wrapping tape, etc. So, the overall performance of the assembly process is continuously improving until the steady-state performance is reached, i.e. the operator does not need to think before installing a certain component. Obviously, small production quantities prevent the operator from reaching this steady-state performance.

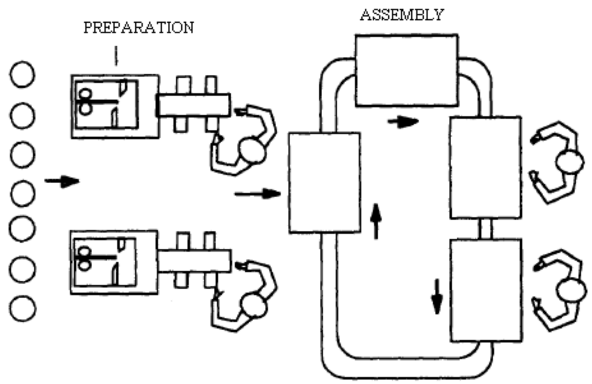

Fig. 2.3. Division of the components preparation and assembly (Aguirre et. al. 1997)

As the first step of the wiring preparation is mostly an automatic process, the next three steps are assembly of the wiring harness performed by an operator on the assembly jig. Therefore, the whole process flow is divided into components preparation and the final assembly (see Fig. 2.3).

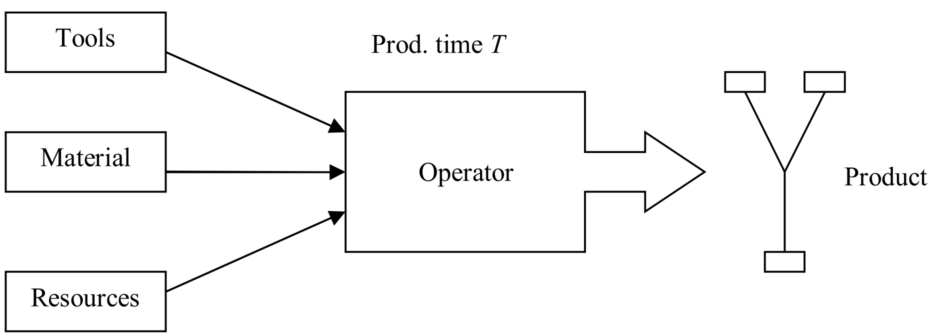

Fig. 2.4. Single cell wiring harness assembly

Please note, that in this research it is assumed that all prepared components, circuits with crimped terminals, needed tubing are introduced in the final assembly of a particular wiring harness, so the study is focused on the manual assembly of all these components into the final product. When manual assembly of the wiring harness is selected, the main question arises regarding job division and assembly process layout. Three options are available:

- Cell assembly (Fig. 2.4)
- Line assembly (Fig. 2.5)
- Parallel assembly (Fig. 2.6)

The choice of assembly process is quite difficult, although the production volume actually defines this decision. But when production volumes are not stable, this criterion can hardly be used.

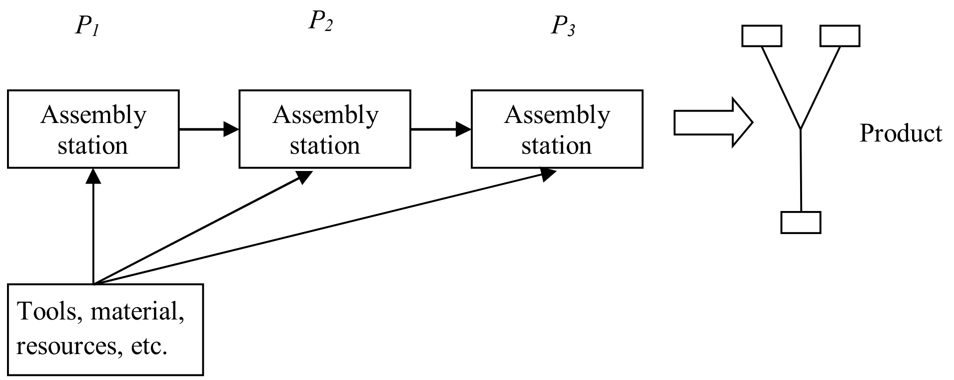

Fig. 2.5 Sequential wiring harness assembly

Therefore, some other criteria needs to be considered. Cell assembly has the largest flexibility and lowest technical investment costs. Line or parallel assembly involves more work stations, so the investment cost increases.

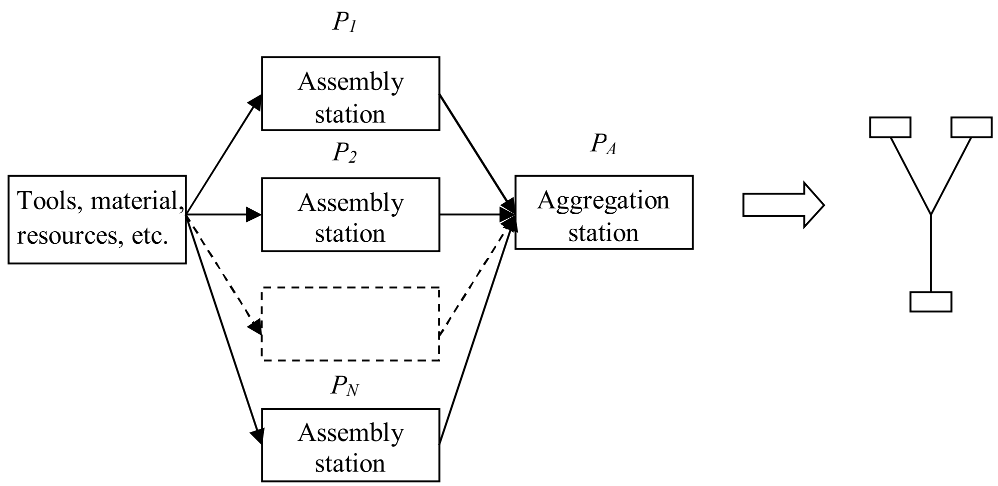

Fig. 2.6 Parallel wiring harness assembly

However, flexibility is reduced, because one specific assembly line is more or less dedicated to a certain product and rearrangement takes additional time. On the other hand, job division and more workstations enable companies to split complex products and reduce complexity in such a way. Complexity reduction leads to increased productivity. Therefore, this might be a solution for the extremely complex products assembled in low volume. Usually, such products are being assembled in a single assembly cell, to avoid high investment costs and maintain flexibility. However there is a lack of analytical tools to facilitate decision making in this case.

Wiring harness assembly time

During the assembly of the wiring harness, all components must be installed and assembled into the final product. As it was stated before, all operations are being performed on an assembly jig (Fig. 2.7)

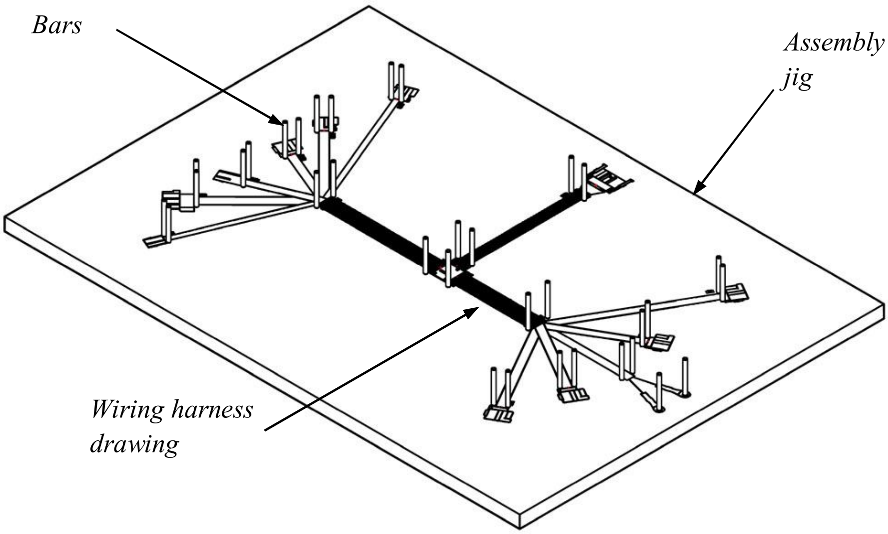

Fig. 2.7. Wiring harness assembly jig

On the assembly jig, the operator performs three assembly steps (installation, securing, and attachment).

There is two ways of assembly time measurement: the total assembly time could be measured and also time of each operation could be measured. The total time measurement is quite inaccurate and can only be measured after the actual production has started. Therefore a time measurement of particular operations is more preferable.

To determine assembly time of the wiring harness by separate operation time measurement, it needs to be emphasized that each component of the wiring harness has a specific mounting operation or several operations and each operation has a certain unique processing time i.e. to pick and place component, pull the hose, assemble terminal into housing, wrap the cable tie and etc. In addition to this, the assembly time of a specific component does not or almost does not depend on the particular wiring harness. Even though the different wiring harnesses have a different number of different components, the basic operation time of specific mounting procedures will be the same, for all. This basic operation time represents necessary – steady state (or standard) operation time, which is reached after the operator has finished the learning phase. So there is the universal standard operation list, which could be used for each wiring harness (Boothroyd, Dewhurst 2002), (Ong, Boothroyd 1991).

In the standard DFA sheets and standard operation lists, most assembly operations are defined. However, non-standard assembly operations could be incomparable and harder to define, any such assembly operation time could easily be determined during a time study. When a time study is completed, the necessary processing time is determined and, if needed, calculated. Any different manufacturing or assembly operation will have some common description. So one can easily determine three different groups of operations:

- Processes that depend on the number of operations only
- Processes that linearly depend on the number of operations used and the number of sub-operations or any quantitative measure (length, area, etc.)
- Processes that non-linearly depend on the number of operations used and any other parameters.

There are two groups of component assembly operations, simple and complex. Simple operation represents a single process and depends only on the number of components used i.e. the assembly of terminal, fixing of cable tie. The complex assembly operation depends on the number of components and the number of sub-operations or component length.

To begin with the simplest case, i.e. processes that depend on the number of operations used, a simple equation is defined:

$$T_{op} = t_{op} q , \tag{2.1}$$

where t_{op} – assigned processing time of a single operation, q – number of operations used in product. Another case is when the linear relationship is determined. During the time study of electrical wiring harness assembly (Ong, Boothroyd 1991), applied linear regression analysis showed linear equations not only between the number of operations used and assembly time, but even the operation time and the other parameters. After regression analysis (Ong, Boothroyd 1991), the linear relationship between the number of sub-operations/length and assembly time was determined:

$$T_{op}^{v} = t_{const} + t_{adds} s_{op} + t_{addl} l_{op} , \tag{2.2}$$

where t_{const} - constant operation time, t_{adds} - additional time for each sub-assembly, s_{op} - number of sub-assemblies, t_{addl} - additional time for each meter of component, l_{op} - length of component. Constant operation time has the meaning of a certain fixed time needed to perform an operation. Subassembly or length addresses the variable time fraction of an operation. For instance, to pull the hose, constant assembly time is needed to pick a hose, prepare for pulling and grasp bundle of wires. The variable assembly time, depending on the length of the hose, refers to the distance on which the hose is actually pulled (Fig. 2.10). The same situation occurs with circuit layout. The constant time is needed to pick and place the circuit and variable time depending on length is required as well. A similar situation is with the sub-operations: the number of wires in the splice (Fig. 2.8), the number of wires in the branch and etc.

Fig. 2.8. Different numbers wires in splices

Crimping of splice is a constant time, but each additional wire requires additional insertion operation. The linear regression is depicted in Fig. 2.9.

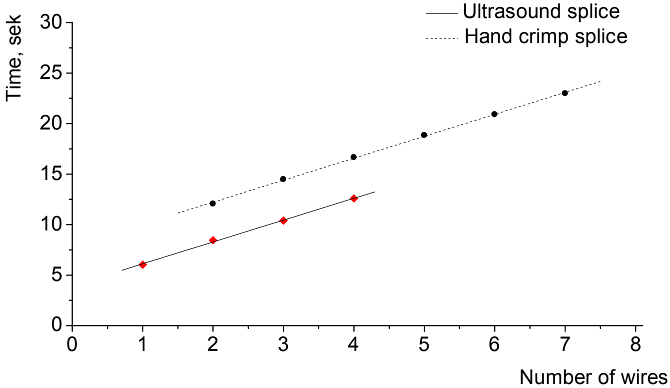

Fig. 2.9. Linear regression of wire quantity in the splice

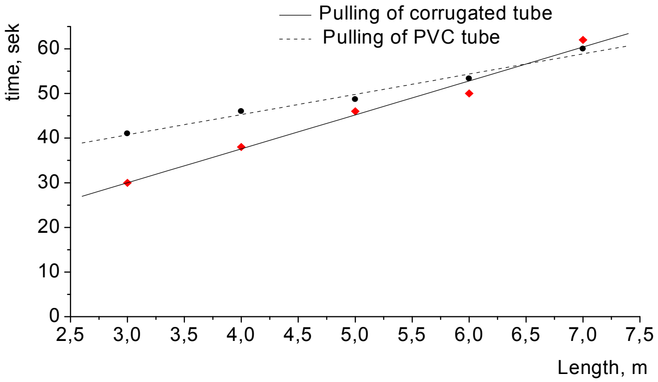

Fig. 2.10. Linear regression of assembly time vs. hose length

This linear regression can be applied to any operation that has additional sub-operations. In other words, the whole range assembly operations can be modelled in such a way. In addition, such an approach enables to simplify complex operations by dividing them in to a certain number of sub-operations. Therefore, equation (2.2) can be generalized to be applicable to any operation. The general equation is defined as follows:

$$T_{op} = t_1^{op} q_{op} + t_2^{op} q_{op} + t_1^{p} q_p + t_2^{p} q_p + \ldots + t_k^{op} q_{op} + t_l^{p} q_p \, , \tag{2.3}$$

where t_i^{op} and t_i^{p} time norms of accordingly different operation and quantitative measure elements, k and l – numbers of different elements, q_{op} and q_p are quantities of operations and sub operations used in a particular product. Linear equation has two constants, therefore equation (2.2) could be divided into two parts; the first defining the number of operations used and the second defining any other processing parameter:

$$T^{op} = t_1^{op} q_{op} + t_2^{op} q_{op} + \ldots + t_k^{op} q_{op} = q_{op} \left(t_1^{op} + t_2^{op} + \ldots + t_k^{op} \right), \tag{2.4}$$

$$T^{p} = t_1^{p} q_p + t_2^{p} q_p + \ldots + t_l^{p} q_p = q_p \left(t_1^{p} + t_2^{p} + \ldots + t_l^{p} \right). \tag{2.5}$$

In the case of a non-linear relationship, the operation model would have such an equation:

$$T_{op} = f\left(q_{op}, q_p \right), \tag{2.6}$$

where *f*() is any nonlinear function depending on two parameters. However, any nonlinear expressions cause simplification problems and results in high complexity of the model. It becomes impossible to group several operations and the operation time must be calculated separately for each operation. This would increase the model preparation time and not necessary make the results more accurate. So, if it is possible, it is better to apply linear expressions.

Mathematical model

By using equations (2.3)-(2.6), the whole range of the assembly operations can be defined; including setup operations. It is necessary to state that some of the operations, even in manual assembly, show a constant time (such as assembling with power tools, or usage of heating equipment for shrinking procedures).

Therefore, in order to fully define the process of the wiring harness assembly for modelling, necessary information should be collected. The following information is needed:

- Operation number determines the operation; a certain number identifies a certain operation. For instance, total number of operation m is selected.
- Sub-operation/operation processing time t_i is used to calculate total assembly time after.

- Factor determining the number of operators performing a certain operation w_i.
- Operation group g_i is used to classify the operations into groups (setup-up operation, machinery operation, assembly operation and etc.)
- Ergonomic factor e_i is determining how much time of the total operation time contains a hazard to operator health and safety.

Accordingly, a represented parameters data matrix is constructed:

$$D = \begin{pmatrix} 1 & t_1 & w_1 & g_1 & e_1 \\ 2 & t_2 & w_2 & g_2 & e_2 \\ 3 & t_3 & w_3 & g_3 & e_3 \\ \dots & \dots & \dots & \dots & \dots \\ m & t_m & w_m & g_m & e_m \end{pmatrix}. \tag{2.7}$$

There are two types of processing time of each operation t_i the first depending on operation quantity only and the second depending on quantitative measure. In general cases, both types are calculated according to the formula (simplified expressions (2.4) and (2.5):

$$t_i = \sum_{j=1}^{v} t_j^{el} , \tag{2.8}$$

where v is a number of constituting operation elements and t_j^{el} is the time norms of constituting operations.

The next step is the creation of the production matrix P. If number n of different products is being produced, each unit will have its own sets of manufacturing operations q:

$$P = \begin{pmatrix} q_{11} & q_{12} & \dots & q_{1n} \\ q_{21} & q_{22} & \dots & q_{2n} \\ \dots & \dots & \dots & \dots \\ q_{m1} & q_{m2} & \dots & q_{mn} \end{pmatrix}. \tag{2.9}$$

The processing time of a certain group g_c for a particular product z is calculated according to the expression:

$$T(g_c) = \sum_{i=1}^{m} D_{i,2} D_{i,3} D_{i,4} D_{i,5} P_{i,z}^{z}, \text{ for } i:\ g_i = g_c . \tag{2.10}$$

Using this formula, the total operation time of all groups is calculated. However, formula (2.10) is quite complex, therefore simplification is also available.

So in general, the vector of all operations with the same operation times as in (2.7) is defined:

$$\mathbf{D} = \begin{pmatrix} t_1 & t_2 & t_3 & \dots & t_m \end{pmatrix}, \tag{2.11}$$

where t_i is operation time of the particular operation, m is total number of all possible operations. During the time study, every operation was thoroughly evaluated, not only on single, but on several different products and different types of components and performed up to the several hundred times, to warrant that the pure steady state operational time is reached. Each different wiring harness product will have different numbers and quantities of operations. This information is easily collected from the product structure. So for every product operation, quantity vector is determined:

$$\mathbf{W} = \begin{pmatrix} q_1 & q_2 & q_3 & \dots & q_m \end{pmatrix}, \tag{2.12}$$

where q_i is the quantity of i operation. If a particular unit does not have the certain operation the quantity is obviously 0. When fully defined vectors **D** and **W** are given, the total assembly time is calculated as a scalar product:

$$T_c = \mathbf{DW}. \tag{2.13}$$

After (2.13) is applied, the total assembly time of a selected wiring harness product is determined. Since T_c is the sum of the steady state operation times, T_c as well represent the steady state (standard) assembly time of the wiring harness. Obviously, if the detailed wiring harness production time sheet is needed, (2.10) has to be applied.

2.3. Efficiency optimization problem

To define production performance, key performance indicators (KPI) are used. These indicators define major aspects of production performance, including quality, operational performance, sickness, time waste, efficiency loses, production breakages and etc. Efficiency is affected by a variety of factors, such as:

- Customer demand fluctuations
- Order size
- Production organization
- Production planning and control
- Technology level
- Personnel motivation and etc.

Some of the factors come from outside and, therefore, cannot be affected, but other factors can be controlled. Since the group of various factors, parameters, properties, decisions and etc. define the overall efficiency, so obviously there exist an optimal set of these parameters to reach the highest possible efficiency and in order to reach it, first of all the optimal set of parameters should be derived.

There is a lack of research that directly connects time spent for learning with key performance indicators (i.e. efficiency). Often, the learning time is inevitable, but also the learning time does not create value, but is rather a waste of operating time and should not be only calculated but also minimized for production improvement. Below, the methodology to incorporate the learning time into the general efficiency calculation technique and propose the possible optimization problem for the optimal parameter calculation, which could be the background for further production efficiency, research of an improvement via LC application is required.

Overall equipment efficiency (OEE) is a common LEAN tool to calculate the overall performance of the production system. Even though it is dedicated to the equipment efficiency calculation it could be used for any production unit i.e. assembly line, job-shop, work cell and etc. This tool plays a very important role in the production system, because it helps to identify the ratio, of how much time is spent in order to produce value, which will be sold to the customer. The OEE could be calculated according to the simple formula [12]:

$$OEE = A_{eff} P_{eff} Q_{eff} , \tag{2.14}$$

where A_{eff} is availability effectiveness, P_{eff} is performance effectiveness, Q_{eff} is quality effectiveness. Each component of OEE is calculated as follows:

$$A_{eff} = T_{OP} / T_{PL} , \tag{2.15}$$

where T_{OP} is plant operating time, T_{PL} is planned production time.

$$P_{eff} = T_{ICT} / T_{OP} , \tag{2.16}$$

where T_{ICT} is ideal (standard) cycle time of the operations.

$$Q_{eff} = \left(Q_{OP} - Q_D\right) / Q_{OP} , \tag{2.17}$$

where Q_D is the number of defective products produced during operating time, Q_{OP} is overall quantity produced during operating time. If we supplement (2.14) equation with the (2.15), (2.16) and (2.17) equation, the OEE equation becomes the form of:

$$OEE = \frac{T_{OP}}{T_{PL}} \times \frac{T_{ICT}}{T_{OP}} \times \frac{Q_{OP} - Q_D}{Q_{OP}} , \tag{2.18}$$

Now, the major problem arises of the introduction of the learning factor into the (2.18) equation. Let's suppose that the learning time is T_L. At the most simple case, learning time can be included into the performance component as the ideal cycle time:

$$OEE = \frac{T_{OP}}{T_{PL}} \times \frac{\left(T_C + T_L\right)}{T_{OP}} \times \frac{Q_{OP} - Q_D}{Q_{OP}} , \tag{2.19}$$

where T_C – ideal cycle (standard time) of the operation (without learning time).

One can use (2.19) if the learning time is assumed to be an inevitable part of the processing time. On the other hand, learning time appears to be waste and the equation (2.19) cannot be used for the efficiency optimization, so the formula needs to be improved. Obviously, learning reduces operator speed and there are major reasons causing the reduced of speed of the operation:

- Rough Running
- Under Nameplate Capacity
- Under Design Capacity
- Equipment Wear
- Operator Inefficiency

If the operator has to learn the task he/she loses speed, loses the efficiency and reduces performance, so the learning time should be included into the equation (2.18) in form of the objective function as follows:

$$F(T_L) = \frac{T_{OP}}{T_{PL}} \times \frac{T_{ICT}}{T_{ICT} + T_A + T_L} \times \frac{Q_{OP} - Q_D}{Q_{OP}}, \quad (2.20)$$

where T_L – integrated learning time, T_A – other time waste during total operating time. This time T_A appears due to other factors such as rough running equipment wear, operator inefficiency and cannot be calculated in the analytical form. Please note that the ideal cycle time is standard operation time, i. e. $T_{ICT} = T_C$. Also it is needed to be emphasized the operating time is:

$$T_{OP} = T_{ICT} + T_A + T_L. \quad (2.21)$$

Further, the optimization problem is stated. However:

$$\max_{T_L} F(T_L) = F\left(\min_{\alpha,\beta,T_{ICT}} T_L\right), \quad (2.22)$$

therefore, final optimization problem becomes to find:

$$\min T_L(\alpha, \beta, T_{ICT}), \quad (2.23)$$

subject to

$$\alpha > 0, \beta > 0, T_{ICT} > 0, \quad (2.24)$$

where α and β – parameters of the learning curve $y(x) = \beta x^{-\alpha}$. Please note that optimization problem (2.23) is general and additional constrains can be included regarding the learning time calculation. In addition, here it is assumed that T_L is an integrated learning time which is calculated according to complex expressions. These expressions will be defined in section 3.

2.4. LC model selection

LCs have already been considered for quite a long time, but their application is urgent, as far as many enterprises are striving to apply the LC to determine their production process time, but they face various problems; most important of which are errors due to the wrong LC application. In this research, a manufacturing system with unstable production quantities is addressed. So it is necessary to study thoroughly the LC fitting to wiring harness production, with regard to product variety, complexity and production volume. Traditional LC fitting and parameter estimation in most literature is based on statistical methods. Statistical methods are suitable when one has enough data to fit. In the production system with huge order fluctuations and a big variety of products, only limited data could be provided for the analysis, because it would be just too costly to gain full research data. On such a limited data, statistical fitting methods simply will not work. This issue must be considered while selecting a LC model for manual wiring harness production modelling.

At present, a lot of LC models are applied that are widely reviewed at the beginning of the dissertation. The summary of used models is presented in the Table 2.1.

Table 2.1. Variety of different LC models

Model	Mathematical expression	Number of parameters
Wright's	$y_W(x) = \beta x^{-\alpha_W}$	2
Crawford's	$y_c(x) = \beta x^{-\alpha_c}$	2
Plateau	$y_p(x) = \beta x^{-\alpha_p} + \gamma$	3
Stanford-B	$y_B(x) = \beta(x+B)^{-\alpha_B}$	3
Dejong's	$y_D(x) = \beta\left(M + (1-M)x^{-\alpha_D}\right)$	3
S-Curve	$y_S(x) = \beta\left(M + (1-M)(x+B)^{-\alpha_S}\right)$	4
Dual phase	$y_{CM}(x) = \chi\beta x^{-\alpha_C} + (1-\chi)\beta x^{-\alpha_M}$	5
2-parameter hyperbolic	$y(x) = k\left(\frac{x}{x+r}\right)$	2
3-parameter hyperbolic	$y(x) = k\left(\frac{x+p}{x+p+r}\right)$	3
Exponential	$y_e(x) = \beta_e e^{-\alpha_e x}$	2
Exponential with breaking point	$y(t) = \begin{cases} \frac{\delta}{\lambda^\delta} e^{-\left(\frac{t}{\lambda}\right)^{\delta-1} t^{\delta-1}}, t < t_0 \\ \frac{\delta}{\lambda^\delta} e^{-\left(\frac{t_0}{\lambda}\right)^{\delta-1} t_0^{\delta-1}}, t \geq t_0 \end{cases}$	3

When applying the LC model in practice, there are three basic things that defined the accuracy of the applied model:

- Adequacy to the specific character of production under consideration
- Rather exact method of LC parameters restoration

- Sufficient quantity of the production made according to which the producing time is predicted.

This work investigates the LC application to the manual assembly process of the automotive wiring harness. Raw production data has been collected. Observing the production process, certain tendencies of assembly time dynamics have been noticed:

1) assembly time mostly decreases for the first products;

2) assembly time after large quantities of repetitive cycles is stable and does not decrease.

In this work, the power LC model for learning phase modeling of the production process was used, because it meets the following requirements:

- it is simple (minimal number of parameters);
- it well approximates the whole length of the LC;
- it is able to define the stabilization point of the learning rate;
- monitoring of the producing process for the necessary data for restoring parameters of the model is rather cheap.

A lot of learning curve models have been proposed (Table 2.1), but only two Wright‘s (cumulative) and Crawford‘s (unit) models and their modifications are in widespread use. Also, power model equation is the simplest and more common to use for a wide variety of processes. Sophisticated models, such as hyperbolic and exponential, are rather used for specific manufacturing issues. Therefore, a power function based LC will be employed for further research.

2.5. Methodology for production data monitoring

Raw production data was collected in the selected wiring harness production company. During the research period, the company allowed researchers to monitor the production processes and collect production data. Three types of measurements were performed:

- Measuring of certain product assembly time and the assembly time of the whole batch; it is simple (minimal number of parameters);
- Measuring time of every and each of the several sequential assembly batches;
- Measuring time of each assembled unit.

Since the learning curve differs for different workers, therefore in this research the high skilled and motivated assembly operators with similar performance have to be employed to avoid variances.

All types of production monitoring are presented below, with the detailed parameters recovered.

A time measurement of the certain assembly cycle and whole batch assembly time measurement is quite simple, since a very small number of parameters have to be measured. These parameters are:

- Time T_0 to produce x_0 unit;
- Total time T_{12} required to produce units from x_1 up to x_2

Regarding the measuring complexity, it is also quite simple, because it is enough to visit the assembly station only two times to gain the necessary data. One time analyst can gain data of several assembly stations simultaneously. Therefore the price of such measurement is quite low. Graphical representation of such production data is presented in Fig. 2.11. This measurement can be made on the first assembly cycle, or any other cycle within the batch.

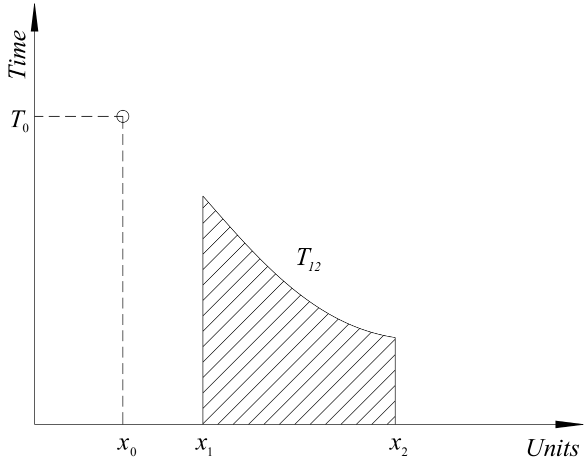

Fig. 2.11. One assembly point and assembly batch time measurement

Measuring time of all of the several sequential assembly batches demand for at least two measurements of production batches. Obviously, there can be more measurements (up to *n*), but two is the minimum:

- T_{12} required to assembly units from x_1 to x_2
- T_{34} required to assembly units from x_3 up to x_4
- $T_{i,i+1}$ required to assembly units from x_i up to x_{i+1} where $i = 1....n$

Graphical representation of such production data is presented in Fig. 2.12.

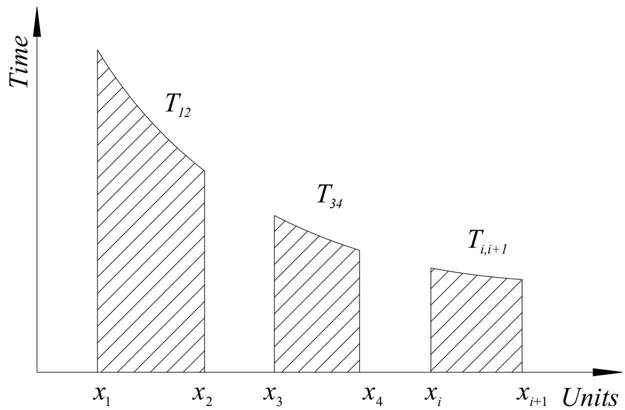

Fig. 2.12. Assembly batch time measurement

The quantity of measurement n is not limited. However, the shorter the intervals data is obtained the more accurate the data is delivered and the random error of measurements is reduced. The boundaries of the intervals can coincide with each other i.e. $x_2 = x_3$, but cannot intersect.

Measuring the time of each assembled unit (full monitoring of the manual assembly process) the LC can be obtained as one or several random samples. Note that such a way, by fixing each complete cycle, is most frequently used in the investigation in LC parameters restoration. However, it is also the most expensive, since it requires more expenditure for monitoring the production process: scanning equipment, its management and administration, as only one group of researchers can tackle only one experiment at a time.

If learning curves are obtained experimentally, such production data is available:

$$x_i^{(k)}, y_i^{(k)}, x_i^{(k)} < x_{i+1}^{(k)}, i = 1,2,...N, k = 1,2,...K, \qquad (2.25)$$

where N is the number of points measured in each different experiment k, K is the total number of experiments made. Then, if K is sufficiently large, all statistical parameters (mean values, standard deviation and etc.) and confidence intervals of LC parameters with high statistical significance can be calculated. However, this method is expensive (more data is needed, more monitoring equipment and control is also required). In this research, it is supposed that only the limited data available from unstable, fluctuating manufacturing environment is provided. The limited production data is considered as a single random sample (i.e. K = 1 in equation (2.25)). Graphical representation of such production data is presented in Fig. 2.13.

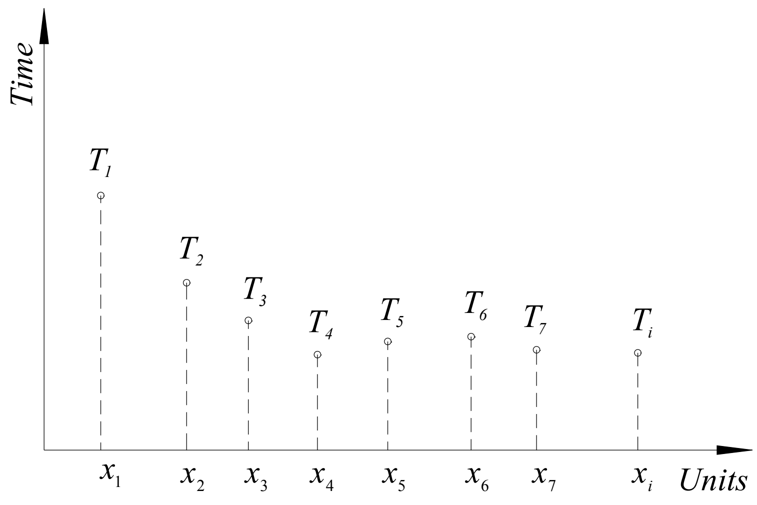

Fig. 2.13. Measuring time of each assembled unit

The choice of the production data monitoring method depended on two main criteria: cost and accuracy with the goal to find the balance between these two extremities. The first method is simple and low cost, but with the probability of random error the highest. By measuring only one certain assembly cycle this has to be done very carefully, otherwise the data fitting will be poor and estimated LC parameters will be inaccurate.

Assembly data collection by measuring time of the several assembly batches is even simpler. In addition, this data collection method is relatively cheap, measured average data is also less sensitive to random errors. Several experiments can be performed simultaneously. Additionally, automatic data collection equipment can be employed.

The last measurement possess the highest accuracy, but the cost of such a monitoring method is also the highest.

2.6. Methodology of adequacy evaluation

After the monitored production data is collected (Fig. 2.14), fitting methods will be applied to estimate the appropriate LC. However, errors will still remain. Therefore, the adequacy of the proposed methods have to be evaluated in order to decide about the model's robustness and compare them to each other.

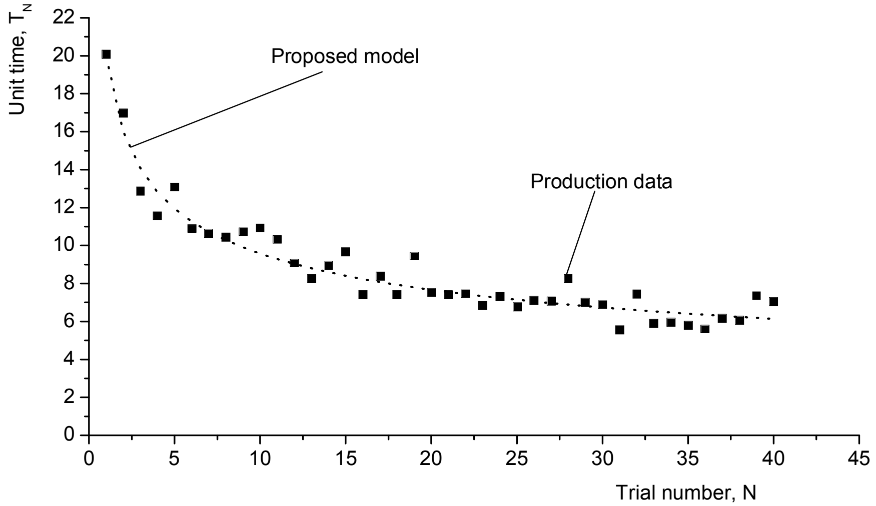

Fig. 2.14. Production data fitting

Assuming that the monitored production data is presented in such a form:

$$\mathbf{X} = (x^{(1)},...,x^{(N)}), \quad \mathbf{W} = (w^{(1)},...,w^{(N)}), \; x^{(i)} < x^{(i+1)}, \tag{2.26}$$

where $x^{(i)}$ is number of unit, $w^{(i)}$ is processing time of the x^{th} unit. Let $\mathbf{W}_{cal} = \left(f(x^{(1)},\alpha,\beta),..., f(x^{(N)},\alpha,\beta) \right)$ are recovered results by using any proposed model. The accuracy of approximation is measured by norm:

$$\delta_{abs} = \left\| \mathbf{W}_{cal} - \mathbf{W} \right\| \tag{2.27}$$

and relative norm (Pryce, 1984):

$$\delta_{rel} = \left\| \mathbf{W}_{cal} - \mathbf{W} \right\| / \left\| \mathbf{W}_{cal} \right\|, \qquad (2.28)$$

where $\left\| \mathbf{X} \right\| = \sqrt{\sum_{i=1}^{N} \left(x^{(i)} \right)^2}$ is Hilbert-Schmidt norm.

It is assumed in this dissertation that the adequacy of the model is proved when the model has relative error less or equal of 5%.

2.7. Conclusions and main results of the section

From the methodological part, the following conclusions and generalizations are derived:

1. Wiring harness component as a typical manual assembly product is defined and its main terminology and components presented.
2. Different wiring harness assembly technologies are introduced and discussed in detail. The most important problem arises due to the selection of appropriate number of assembly cells.
3. A mathematical model connecting manual wiring harness assembly operations and total assembly time has been created. This model enables modelling of an assembly process of wiring harness.
4. Efficiency optimization problem is stated according to OEE methodology. In this problem, the learning time is organized to be reduced, not only calculated.
5. The power function based LCs were selected for further research, since they satisfy the requirements of simplicity and applicability.
6. A methodology for the raw production data monitoring is presented. Three different monitoring techniques are introduced.
7. The Hilbert-Schmidt norm is selected as the main method to measure the accuracy of the proposed LC models

3. ANALYTICAL RESEARCH

3.1. Introduction

In this section analytical research regarding LC application is performed. According to the literature review, methodology and information from manufacturing and the main directions of the analytical research are established. They are briefly presented below. Firstly, based on the tendencies observed in most experiments done, a premise that in a assembly process there exists a Plateau phase, i.e. the assembly time is decreasing until the steady-state assembly time T_C is reached. Therefore, the problem is to derive an adequate, mathematically grounded LC model, which will enable the restoration LC completely and predict the assembly process.

The next direction is to create a versatile LC model based on the generalized power model applicable to manufacturing situations, when general LC model does not have sufficient accuracy. The proposed generalized model is based on the solutions of special (with perturbation parameter) differential equations. These solutions define the approximate learning curves (ALC). In this research, sufficient conditions for the perturbation parameter and other parameters of ALC are determined. Also, mathematical analysis of the ALC is performed to explore the versatility possibilities and establish the foundation of such an ALC modelling. Also, ALC modelling is employed to measure the direct impact of risk factors (vibrations and repetitive motions) to the assembly time of the operator that were impossible to achieve by using classical LC models.

Another important issue is the LC parameter estimation from limited production data. Small, fluctuating manufacturing quantities provide only limited data, however the impact of learning is the greatest on such manufacturing. On such limited data, statistical fitting methods simply will not work. Therefore, there is an obvious need and challenge to propose alternative (non-statistical, i.e. deterministic) LC fitting methods beyond the statistical methods to fit the limited production data with satisfactory accuracy.

Moreover, since the unstable manufacturing environment is studied, the last direction of this research is to propose a mathematical model of evaluation of the extent of how the fluctuating quantities, prototype production, poor planning, unplanned customer orders effect the processing time due to learning factors in the manual demand-based wiring harness industry.

Previously defined directions (new LC models and parameter estimation methods are essential to reach the main outcome) lead to the main goal of this research and the last part of the analytical research section is dedicated to the manual assembly process modelling (complex process splitting) for learning time reduction. In other words, it is dedicated to the solution of the optimization problem (2.23) with actual process parameter values and real products.

The main topics of the analytical research can be summarized as:

- Development of new models
- Deterministic parameter recovery methods

Issues are addressed in the following sub-sections.

3.2. Plateau LC with stabilization parameter

Let a full experiment is done and data

$$\{x_i, y_i\}, x_i < x_{i+1}, i = 1,2,...n \qquad (3.1)$$

is obtained. Only “cheep” data is used, i.e. $x_0 = x_q$, $y_0 = y_q$ and the operation time is $T_c = y(x_c,\alpha)$, if the time of later operations “almost” does not change ($y(x,\alpha) \approx const$, when $x \ge x_c$). Parameters x_c and α are unknown. The word “almost” is treated here as a decrease in absolute value of a derivative $\partial y(x,\alpha)/\partial x$ up to an adequately chosen value $\varepsilon > 0$. We proof below that the parameter $\alpha = \alpha_\varepsilon$ is a unique solution of the equation

$$\left|\frac{\partial y}{\partial x}\right|\Bigg|_{x=x_c(T_c,\alpha)} = \varepsilon, \qquad (3.2)$$

and $x_c(T_c,\alpha_\varepsilon) = x_0 (y_0/T_c)^{\frac{1}{\alpha_\varepsilon}}$ is an abscissa of the unique intersection point of LC $y(x,\alpha_\varepsilon) = y_1\left(\frac{x}{x_1}\right)^{-\alpha_\varepsilon}$ crossing the point (x_0, y_0) and the line $y = T_c$. Thus if α_ε and $x_c(T_c,\alpha_\varepsilon)$ could be found it is possible to completely restore LC:

$$Y(x,x_0,y_0,\alpha_\varepsilon) = \begin{cases} y_0\left(\dfrac{x}{x_0}\right)^{-\alpha_\varepsilon}, & \text{when } 0 < x < x_c(T_c,\alpha_\varepsilon) \\ T_c, & \text{when } x \ge x_c(T_c,\alpha_\varepsilon) \end{cases}. \qquad (3.3)$$

Proposition 1. If $x_0 \ge 1$, $y_0 > 0, \alpha > 0$, $0 < T_c < y_0$ then a bundle of curves (according of α)

$$y(x,\alpha) = y_0\left(\frac{x}{x_0}\right)^{-\alpha}. \qquad (3.4)$$

have only one common point (x_0, y_0), and each curve of the bundle crosses the line $y = T_c$ only at one point, the abscissa of which is

$$x_c(T_c,\alpha) = x_0\left(\frac{y_0}{T_c}\right)^{\frac{1}{\alpha}} > 0. \qquad (3.5)$$

Proof. The bundle of curves (3.4) has only one common point (x_0, y_0), because $y(x_0,\alpha) = y_0 (x_0/x_0)^{-\alpha} = y_0, \forall \alpha > 0$.
Besides, this point is unique, because when at least one more point appears, then $(x_{01}, y_{01}) \neq (x_0, y_0) \Rightarrow y_0 (x_0/x_0,\alpha)^{-\alpha_0} = y_{01} (x_{01}/x_{01})^{-\alpha_{10}} \Rightarrow y_0 = y_{01}$.

It is shown that (3.5) is a unique solution of the equation $y(x,\alpha) - T_s = 0$. Resulting in

$$y(x_c(T_c,\alpha),\alpha) - T_s = y_0 \left(\frac{x_0 (y_0/T_c)^{\frac{1}{\alpha}}}{x_0} \right)^{-\alpha} - T_c = T_c - T_c = 0.$$

There exists solution (3.5) and it is unique if $y(1,\alpha) \geq T_c$, since a derivative of $y(x,\alpha)$ $\frac{\partial y}{\partial x} = -\alpha \frac{y_0}{x} \left(\frac{x}{x_0} \right)^{-\alpha} < 0$, i.e. $y(x,\alpha)$ is strictly monotonously decreasing (SMD) and $\lim_{x \to +0} y(x,\alpha) = y_0$, $\lim_{x \to +\infty} y(x,\alpha) = 0$.

Proposition 2. If $x_0 \geq 1, y_0 > 0, \alpha > 0$, $0 < T_c < y_0$, then the equation (3.2) has a unique solution $\alpha(\varepsilon) > 0$ for any $\varepsilon > 0$, besides the function $\alpha(\varepsilon)$ is strictly monotonously increasing (SMI).

Proof. The absolute value of $\frac{\partial y}{\partial x}$ at points $x = x_c(T_c,\alpha)$, is equal to:

$$f_\alpha(\alpha) \equiv \left| \frac{\partial y}{\partial x} \right| \Bigg|_{x = x_c(\alpha)} = \alpha \frac{y_0}{x_0} \left(\frac{T_c}{y_0} \right)^{\frac{1+\alpha}{\alpha}} = \alpha \frac{T_c}{x_0} \left(\frac{T_c}{y_0} \right)^{\frac{1}{\alpha}} > 0 \,, \tag{3.6}$$

therefore the equation (3.2) becomes:

$$\alpha \frac{T_c}{x_0} \left(\frac{T_c}{y_0} \right)^{\frac{1}{\alpha}} = \varepsilon \quad \text{or} \quad \frac{T_c}{x_0 \varepsilon} = \frac{1}{\alpha} \exp\left\{ \frac{1}{\alpha} \ln(\frac{y_0}{T_c}) \right\} \tag{3.7}$$

and after multiplying both sides of the equation (3.7) by $c = \ln(y_0/T_c) > 0$ the following is arrived at

$$cd(\varepsilon) = (c/\alpha) \exp\{c/\alpha\}, \tag{3.8}$$

where $d(\varepsilon) = \frac{T_c}{x_0 \varepsilon}$. From (3.8) it follows that the function $W(cd(\varepsilon)) = c/\alpha$ is Lambert's function (Olver et. al. 2010), (Dence, 2013) and

$$\alpha(\varepsilon) = \frac{c}{W(cd(\varepsilon))} = \frac{\ln(y_0/T_c)}{W\left[(T_c/x_0 \varepsilon) \ln(y_0/T_c) \right]}. \tag{3.9}$$

From the properties of Lambert‘s function (Fukushima, 2013) and (3.9) it follows that the function $\alpha(\varepsilon)$ is single-valued, positive, SMI, and $\alpha(0)=0$, because $cd(\varepsilon)>0$ is SMD. This proves **Proposition 2**.

Proposition 3. If $x_0 \geq 1,\ y_0 > 0,\ \alpha > 0$, $0 < T_c < y_0$, then the function $x_c(T_c, \alpha(\varepsilon))$ is SMD and concave.

Proof. It follows from **Proposition 1** that $x_c(T_c, \alpha(\varepsilon)) > 0$, therefore derivatives of implicit function $x_c(T_c, \varepsilon)$ are of constant sign:

$$\frac{\partial x_c}{\partial \varepsilon} = x_c(T_c, \varepsilon)\frac{\alpha'(\varepsilon)}{\alpha^2(\varepsilon)}\ln\left(\frac{T_c}{y_0}\right) < 0 \text{ and}$$

$$\frac{\partial^2 x_c}{\partial \varepsilon^2} = \left(\alpha'_\varepsilon(\varepsilon)\right)^2 \frac{\partial^2 x_c}{\partial \alpha^2} + \frac{\partial x_c}{\partial \alpha}\alpha''_\varepsilon(\varepsilon) > 0\,.$$

Thus after finding α_ε (there is no analytical solution), we completely restore LC (3.3) and the learning rate stabilization point $x_c(T_c, \alpha_\varepsilon)$, using only two experimental data (x_0, y_0) and T_c.

3.3. Almost Learning Curve Model

In this sub-section the definition of the classical learning curve (CLC) and almost learning curve (ALC) as solutions of the differential equation are presented. Since the CLC is defined as $y(x,\alpha,\beta) = \beta x^{-\alpha}, \beta > 0, 0 < \alpha < 1, x \geq 1$, CLC is the solution of Cauchy problem

$$\begin{cases} L_\alpha(y) = 0 \\ y(1,\alpha) = \beta \end{cases}, \tag{3.10}$$

where $L_\alpha(y) = y' + \alpha x^{-1} y$. Let the solution

$$y(x,\alpha,\beta) = \beta x^{-\alpha} \tag{3.11}$$

of problem (3.10) be a definition of CLC. Considering the more general Cauchy problem

$$\begin{cases} L_\alpha(y) = 0 \\ y(x_1,\alpha) = y_1 \end{cases}, \tag{3.12}$$

whose solution is

$$y(x,\alpha) = x_1^{\alpha} y_1 x^{-\alpha}, \tag{3.13}$$

then $\beta = y(1,\alpha)$. The properties of the bundle (with respect to the α) of solutions (3.13) follows from direct differentiation:

$$y \in C^{(2)}[1,+\infty),\ \text{if } x_1 \ge 1, y_1 > 0,\ \alpha \in (0,1),$$

$$y(x,\alpha) > 0, y_x'(x,\alpha) \le 0, y_x''(x,\alpha) \ge 0, \tag{3.14}$$

$$y_a = \lim_{x\to+\infty} y_h(x,\alpha) \ge 0. \tag{3.15}$$

Analyzing the Cauchy problem

$$\begin{cases} L_\alpha(w) = \varepsilon x^{-r},\ r \ge 1, \\ y(x_1,\alpha) = y_1 \end{cases}, \tag{3.16}$$

then the function

$$w(x,\alpha,\varepsilon,r) = x_1^{s(\alpha,r)}\left[\frac{c(\alpha,r)-\varepsilon}{s(\alpha,r)}\right]x^{-\alpha} + \frac{\varepsilon}{s(\alpha,r)}x^{1-r} \tag{3.17}$$

is a solution of (3.16). Where $s(\alpha,r) = \alpha + (1-r)$, $c(\alpha,r) = x_1^{r-1}y_1 s(\alpha,r)$. Such solutions are called the ALC. They are almost LC models (do not confuse with almost learning) and new parameters are curve fitting parameters to achieve more accuracy. Stating sufficient conditions for parameters α, ε and r such that solution (3.17) satisfy (3.14)-(3.15). In addition to this, conditions when solution (3.17) have positive horizontal asymptote (i.e. plateauing phenomenon. Note that under conditions $x \in [1,+\infty)$, $\alpha \in (0,1)$, $r \in [1,+\infty)$ and $\varepsilon \in (-\infty,+\infty)$ the solution (3.17) exists and is unique, because $\frac{\alpha}{x}, \frac{\varepsilon}{x^r} \in C^{(0)}[1,+\infty)$ (Roberts, 2010).

Proposition 1. Under the condition

$$(\alpha,\varepsilon,r) \in D_1 = \begin{Bmatrix} 1 \le r < 2 \\ \alpha_0(r,\varepsilon) < \alpha < 1 \\ 0 \le \varepsilon \le c(\alpha,r) \end{Bmatrix} \tag{3.18}$$

(where $\alpha_0 = \varepsilon\left(x_1^{r-1}y_1\right)^{-1} + (r-1)$) the Cauchy problem (3.16) solution (3.17) is ALC.

Proof. Introducing the following notation

$$C_1 = \frac{c(\alpha,r)-\varepsilon}{s(\alpha,r)},\ C_2 = \frac{\varepsilon}{s(\alpha,r)},\ C_3 = \frac{\varepsilon(r-1)}{s(\alpha,r)}, \tag{3.19}$$

then from (3.17) we have

$$w(x,\alpha,\varepsilon,r) = x_1^s C_1 x^{-\alpha} + C_2 x^{1-r}, \tag{3.20}$$

$$w_x'(x,\alpha,\varepsilon,r) = -\alpha x_1^s C_1 x^{-(\alpha+1)} - C_3 x^{-r}, \tag{3.21}$$

$$w_x''(x,\alpha,\varepsilon,r) = \alpha(\alpha+1)x_1^s C_1 x^{-(\alpha+2)} + C_3 r x^{-(r+1)}. \quad (3.22)$$

From (3.20) - (3.22) follows that if

$$C_1 > 0,\ C_2 \geq 0,\ C_3 \geq 0, \quad (3.23)$$

for all $x \geq 1$, then functions $w > 0,\ w_x' \leq 0,\ w_x'' \geq 0$ for all $x \geq 1$.
If $(\alpha,\varepsilon,r) \in D_1$, then $\varepsilon \geq 0$; $c(\alpha,r) \geq 0$, $s(\alpha,r) \geq 0$ if $\alpha \geq r-1$; $\alpha_0 \geq r-1$ and $c(\alpha,r) - \varepsilon > 0$.
$C_1 > 0$, because C_1 is a strictly monotone increasing with respect to the α ($(C_1)'_\alpha = \varepsilon / s(\alpha,r)^2 > 0$, because $\varepsilon > 0$) and $C_1 = 0$ when $\alpha = \alpha_0$. From $s(\alpha,r) > 0$ and $(r-1) \geq 0$ follows that $C_2 \geq 0,\ C_3 \geq 0$.
Showing that $D_1 \neq \varnothing$. By integrating the function $c(\alpha,r)$ with respect to the α, results in

$$\operatorname{mes} D_1 = \int_{\alpha_0}^{1} c(\alpha,r)d\alpha > 0, \quad (3.24)$$

because $c(\alpha,r) > 0$ and always exist $\varepsilon > 0$, that $0 < \alpha_0 < 1$. The equation of horizontal asymptote for solution (3.20)

$$w_a = \lim_{x\to+\infty} \frac{\varepsilon}{s(\alpha,r)} x^{1-r} = \begin{cases} \varepsilon/\alpha > 0, & if\ r = 1, \\ 0, & if\ 1 < r < 2 \end{cases} \quad (3.25)$$

Proposition 2. Under the condition

$$(\alpha,\varepsilon,r) \in D_2(r) = \begin{Bmatrix} r \geq 2 \\ 0 < \alpha < \alpha_0(r,\varepsilon) \\ 0 \geq \varepsilon \geq c(\alpha,r) \end{Bmatrix} \quad (3.26)$$

Cauchy problem (3.16) solution (3.17) is ALC.

Proof. If $(\alpha,\varepsilon,r) \in D_2$, then $\varepsilon \leq 0$; $c(\alpha,r) \leq 0$, $s(\alpha,r) \leq 0$ if $\alpha \leq r-1$; $\alpha_0 \leq r-1$ and, $c(\alpha,r) - \varepsilon \leq 0$. $C_1 > 0$, because C_1 is a strictly monotone decreasing with respect to the α ($(C_1)'_\alpha = \varepsilon / s(\alpha,r)^2 < 0$, because $\varepsilon < 0$) and $C_1 = 0$ when $\alpha = \alpha_0$. From $s(\alpha,r) < 0$ and $(r-1) > 0$ it follows that $C_2 \geq 0,\ C_3 \geq 0$.
Showing that $D_2 \neq \varnothing$. By integrating the function $|c(\alpha,r)|$ with respect to the α, it results in

$$\operatorname{mes} D_2 = \int_{\alpha_0}^{1} |c(\alpha,r)| d\alpha > 0. \quad (3.27)$$

because $c(\alpha,r)<0$ and always exist $\varepsilon<0$, that $0<\alpha_0<1$. The equation of horizontal asymptote for solution (3.20)

$$w_a = \lim_{x\to+\infty} \frac{\varepsilon}{s(\alpha,r)} x^{1-r} = 0 \tag{3.28}$$

because $(1-r)<0$.

At the end of the section calculation, examples of domains $D_1(r)$ and $D_2(r)$ are presented.

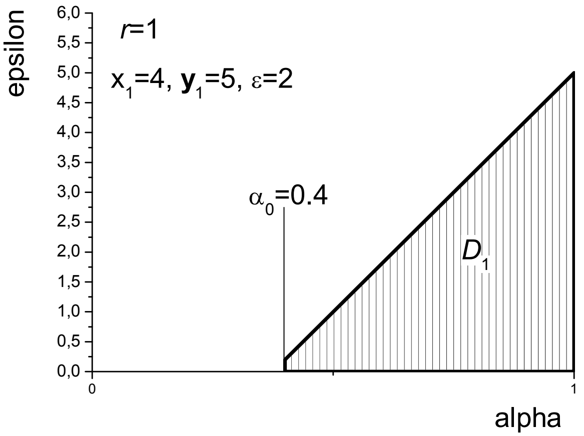

Fig. 3.1. Domain $D_1(r)$ when $1\le r<2$.

If $x_1=4$, $y_1=5$, $\varepsilon=2$ and $r=1$, then domain $D_1(r)$ is trapezoid with vertexes $(\alpha_0,0),(1,0),(1,c(1,r)),(\alpha_0,c(\alpha_0,r))$ Fig. 3.1.

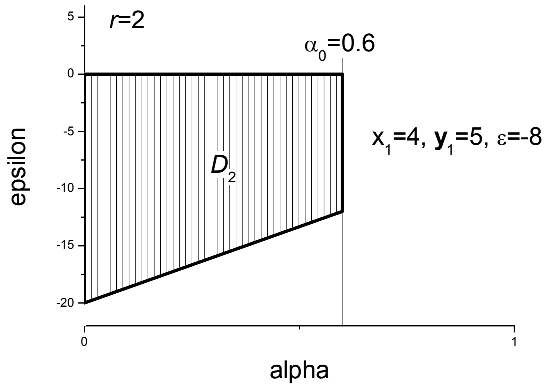

Fig. 3.2. Domain $D_2(r)$ when $r\ge 2$.

If $x_1 = 4$, $y_1 = 5$, $\varepsilon = -8$ and $r = 2$, then domain $D_2(r)$ is trapezoid with vertexes $(0, c(0,r)), (1, c(\alpha_0, r)), (1, c(\alpha_0, r)), (0,0)$ Fig. 3.2. Here $c(\alpha, r) = x_1^{r-1} y_1 \left(\alpha + 1 - r\right)$ and $\alpha_0 = \varepsilon\left(x_1^{r-1} y_1\right) + \left(r - 1\right)$.

To compare the developed ALC model (3.17) with a traditional CLC model, first of all an optimal CLC is obtained for the experimental data. The optimal parameters of CLC, i.e. coefficient α_y and the number n of the data point x_n which minimizes norm δ_y are:

$$\operatorname*{Arg\,min}_{\substack{0<\alpha<1 \\ x_i, 1\le i\le N}} \delta_y = \begin{bmatrix} \alpha_y \\ n \end{bmatrix}. \tag{3.29}$$

Then the norm δ_y can be calculated from the solution (3.13) of the Cauchy problem $y(x^{(n)}, \alpha_y) = y^{(n)}$. The optimal parameters of ALC, i.e. α_w, ε_w and r_w which minimizes norm δ_w are:

$$\operatorname*{Arg\,min}_{\substack{0<\alpha<1 \\ \varepsilon\in D_i(r)}} \delta_w = \begin{bmatrix} \alpha_w \\ \varepsilon_w \\ r_w \end{bmatrix}, \tag{3.30}$$

where $D_i(r), i = 1,2$ is domain of constraints for $\alpha_w, \varepsilon_w, r_w$. Then the norm δ_w can be calculated from the solution (3.17) of the Cauchy problem $w(x^{(n)}, \alpha_w, \varepsilon_w, r_w) = x^{(n)}$. The accuracy of the data approximation by models CLC and ALC is compared by $\Delta = \delta_y / \delta_w$.

3.4. Impact of vibrations and repetitive motions on LC model

Learning curve (CLC) shows a time decrement as the argument (number of units) increases. However, CLC cannot encompass any risk factors, due to the fact that it is a monotonously decreasing function. In order to include vibration exposure to the learning curve CLC, it must be treated as a solution of the differential equation. As it is already known that CLC is defined as $y(x, \alpha, \beta) = \beta x^{-\alpha}, \beta > 0, 0 < \alpha < 1, x \ge 1$. The CLC can now be considered as the solution of the Cauchy problem:

$$\begin{cases} L_\alpha[y] = 0 \\ y(1, \alpha) = \beta \end{cases} \tag{3.31}$$

where $L_\alpha[y] = y' + \alpha x^{-1} y$. Again, let the solution

$$y(x, \alpha, \beta) = \beta x^{-\alpha} \tag{3.32}$$

of the problem (3.31) be a definition of CLC. Considering the more general Cauchy problem

$$\begin{cases} L_\alpha[y] = 0 \\ y(x_1, \alpha) = y_1 \end{cases} \tag{3.33}$$

whose solution is

$$y(x, \alpha) = x_1^{\alpha} y_1 x^{-\alpha} \tag{3.34}$$

then $\beta = y(1, \alpha)$. The properties of the bundle (3.34) (with respect to α) of solutions (3.34) follows from direct differentiation:

$$y \in C^{(2)}[1, +\infty), \text{ if } x_1 \geq 1, y_1 > 0\,,\ \alpha \in (0,1) \tag{3.35}$$

$$y(x, \alpha) > 0, y_x'(x, \alpha) \leq 0, y_x''(x, \alpha) \geq 0 \tag{3.36}$$

$$y_a = \lim_{x \to +\infty} y_h(x, \alpha) = 0 \tag{3.37}$$

To connect assembly time and exposure to vibrations, a small additive fraction of time to define injury development to each repetitive cycle should be added. Thus, assembly time decreases as the learning phase is completed, but starts to climb up slightly as an injury starts to develop due to exposure to vibrations and repetitive motions.

Analyzing the Cauchy problem with small perturbation parameter (injury development)

$$\begin{cases} L_\alpha[w] = \varepsilon,\ \varepsilon > 0 \\ w(x_1) = w_1 \ \ x_1 \geq 1,\ w_1 > 0 \end{cases} \tag{3.38}$$

and its solution

$$w(x, x_1, w_1, \alpha, \varepsilon) = (1+\alpha)^{-1} x[\varepsilon + v(x, x_1, w_1, \alpha, \varepsilon)] \tag{3.39}$$

where $v(x, x_1, w_1, \alpha, \varepsilon) = x_1^{\alpha} S(x_1, w_1, \alpha, \varepsilon) x^{-(1+\alpha)}$, $S(x_1, w_1, \alpha, \varepsilon) = (1+\alpha)w_1 - \varepsilon x_1$.

Stating sufficient conditions for parameters α, ε, x_1 and y_1 such that solution (3.39) satisfy (3.35) and

$$w(x, x_1, w_1, \alpha, \varepsilon) > 0,\ w_x''(x, x_1, w_1, \alpha, \varepsilon)) \geq 0 \tag{3.40}$$

$$w(x, x_1, w_1, \alpha, \varepsilon) \text{ have one and only one minimum when } x \in (1, \infty) \tag{3.41}$$

$$\lim_{x \to +\infty} w(x, x_1, w_1, \alpha, \varepsilon) = +\infty \tag{3.42}$$

Note that under conditions $x \in [1, +\infty)$, $\alpha \in (0,1)$ and $\varepsilon \in (0, +\infty)$ the solution (3.39) exists and is unique, because $\dfrac{\alpha}{x}, \dfrac{\varepsilon}{x} \in C^{(0)}[1, +\infty)$.

Proposition. Under the conditions

$$x_1 \geq 1, w_1 > 0, \; 0 < \alpha < 1, \; 0 < \varepsilon, \; (1+\alpha) w_1 > \varepsilon x_1 \tag{3.43}$$

the solution (3.39) of the Cauchy problem (3.38) satisfies conditions (3.35), (3.40) – (3.42).

Proof. Under the conditions (3.43) number $S(x_1, w_1, \alpha, \varepsilon) > 0$ and function $w(x, x_1, w_1, \alpha, \varepsilon) > 0$, hence derivative

$$w''_x(x, x_1, w_1, \alpha, \varepsilon) = \alpha x^{-1} v(x, x_1, w_1, \alpha, \varepsilon) > 0 \tag{3.44}$$

i.e. solution (3.39) is a positive and convex function.

$$w'_x(x, x_1, w_1, \alpha, \varepsilon) = (1+\alpha)^{-1} [\varepsilon - \alpha v(x, x_1, w_1, \alpha, \varepsilon)] \tag{3.45}$$

is strictly monotone increasing and equation

$$w'_x(x, x_1, w_1, \alpha, \varepsilon) = 0 \text{ or } \alpha v(x, x_1, w_1, \alpha, \varepsilon) = \varepsilon \tag{3.46}$$

have a unique solution

$$x_{\min} = \operatorname{Arg}\min_{x \geq 1} w(x, x_1, w_1, \alpha, \varepsilon) = [\alpha \varepsilon^{-1} x_1^{\alpha} S(x_1, w_1, \alpha, \varepsilon)]^{\frac{1}{1+\alpha}} > 0, \tag{3.47}$$

hence minimum time for the $x_{\min}$ cycle is:

$$y_{\min} = \min_{x \geq 1} w(x, x_1, w_1, \alpha, \varepsilon) = w(x_{\min}, x_1, w_1, \alpha, \varepsilon) = \alpha^{-1} \varepsilon x_{\min}. \tag{3.48}$$

In this way it is proved that positive and convex solution (3.39) is a unimodal function and reaches its minimum at the point $x_{\min} \in (1, \infty)$.

Perturbation parameter is calculated according to the formula:

$$\varepsilon = t_r (t_c / t_r)^{-1} = t_r^2 t_c^{-1}, \tag{3.49}$$

where t_r is risk time, t_c is cumulative working time, when the operator starts to feel uncomfortable with the assembly task.

3.5. Methods of LC parameter estimation

In this article, several deterministic (interpolation or approximation) methods are presented. The choice of method depends on the type of the data (unit or cumulative average) obtained from monitoring of the production process i.e. C or W models are being applied. Each method proposed ensures the existence and uniqueness of estimated LC parameters. Several methods are proposed for the selection because the limited production data provides a variety of different data sets. The methods proposed:

- Two point (TPC) and (TPW), for C and W models;
- Invariant (INC) and (INW), for C and W models;
- Point and interval (PIC), for C model;
- Two intervals TIC), for C model;
- Point and interval (PIW), for W model;

- Two intervals (TIW), for W model.

Two point method (TPC, TPW)

Supposing that from monitoring of the production process (MPP) two different LC points are obtained: (x_1, y_1), (x_2, y_2) and $1 \le x_1 < x_2, 0 < y_1 > y_2$. Obviously there is no difference whether to use the W or C model. Then LC parameters α, β are uniquely defined as:

$$\alpha = -\frac{\ln(y_1) - \ln(y_2)}{\ln(x_1) - \ln(x_2)} > 0 \text{ and } \beta = y_1 x_1^{\alpha} = y_2 x_2^{\alpha} > 0. \qquad (3.50)$$

Showing that through two points (x_1, y_1) and (x_2, y_2), one and only one curve $y = \beta x^{-\alpha}$ goes with parameters (3.50). It follows from

$$\begin{cases} y_1 = \beta x_1^{-\alpha} \\ y_2 = \beta x_2^{-\alpha} \end{cases} \Rightarrow \begin{cases} \ln y_1 = \ln \beta - \alpha \ln x_1 \\ \ln y_2 = \ln \beta - \alpha \ln x_2 \end{cases} \Rightarrow \ln y_1 - \ln y_2 = -\alpha\left(\ln x_1 - \ln x_2\right), \qquad (3.51)$$

and

$$y_1 = \beta x_1^{-\alpha} \Rightarrow \beta = y_1 x_1^{\alpha}, \; y_2 = \beta x_1^{-\alpha} \Rightarrow \beta = y_2 x_2^{\alpha}. \qquad (3.52)$$

Suppose that two different curves exist: $y = \beta_1 x^{-\alpha_1}$ and $y = \beta_2 x^{-\alpha_2}$ going through two points (x_1, y_1) and (x_2, y_2), then $y_i = \beta_i x_i^{-\alpha_i}$, $i = 1,2$ hence $\beta_1 / \beta_2 = x_1^{\alpha_1 - \alpha_2}$ and $\beta_1 / \beta_2 = x_2^{\alpha_1 - \alpha_2}$, but this will only be when $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, because $x_1 \ne x_2$.

Point and interval method (PIC)

Suppose that from MPP are obtained: the marginal time T_0, to produce x_0 unit and the total time T_{12} required to produce units from x_1 up to x_2. Since the total time T_{12} has been measured, only C model can be applied. If $T_{12} / T_0 < x_2 - x_1$ and $1 \le x_0 < x_1 < x_2$, then LC parameters α, β are uniquely defined as a solution (α_T, β_T) of the two separate equations

$$x_0^{\alpha_T} \int_{x_1}^{x_2} x^{-\alpha_T} dx = \frac{T_{12}}{T_0} \qquad (3.53)$$

$$\beta_T = \frac{T_0}{x_0^{-\alpha_T}} = \frac{T_{12}}{\int_{x_1}^{x_2} x^{-\alpha_T} dx} \qquad (3.54)$$

besides $\alpha_T > 0, \beta_T > 0$.

The function

$$F(\alpha, x_1, x_2) = \int_{x_1}^{x_2} x^{-\alpha} dx = \begin{cases} (x_2 - x_1) \text{ if } \alpha = 0 \\ (\ln x_2 - \ln x_1) \text{ if } \alpha = 1 \\ \dfrac{(x_2^{1-\alpha} - x_1^{1-\alpha})}{(1-\alpha)} \text{ if } \alpha \neq 1 \\ 0 \text{ if } \alpha \to +\infty \end{cases} \quad (3.55)$$

is positive, continuous and strictly monotonically decreasing (SMD). $F(\alpha, x_1, x_2) > 0$ as an integral of the positive function. The continuity of (3.55) follows from that $\lim_{\alpha \to 1-0} \int_{x_1}^{x_2} x^{-\alpha} dx = \lim_{\alpha \to 1+0} \int_{x_1}^{x_2} x^{-\alpha} dx = (\ln x_2 - \ln x_1)$. The function (3.55) is SMD with respect to α because

$$\frac{\partial}{\partial \alpha} \int_{x_1}^{x_2} x^{-\alpha} dx = -\int_{x_1}^{x_2} x^{-\alpha} \ln(x) dx < 0. \quad (3.56)$$

If $T_{12}/T_0 < x_2 - x_1$, there exists a unique solution α_T>0 of the equation (3.53). It follows from that $\inf_{\alpha} x_0^{\alpha} F(\alpha, x_1, x_2) = 0, \sup_{\alpha} x_0^{\alpha} F(\alpha, x_1, x_2) = x_2 - x_1$. Whenever α_T is calculated, then

$$\beta_T = T_0 x_0^{\alpha_T} = T_{12} \Big/ \int_{x_1}^{x_2} x^{-\alpha_T} dx > 0 \quad (3.57)$$

Two intervals method (TIC)

Suppose that from MPP are obtained: the total time T_{12} required to produce units from x_1 to x_2 and the total time T_{34} required to produce units from x_3 up to x_4. Since the total time T_{12} and T_{34} are used, C model can be used. If $\frac{x_2 - x_1}{x_4 - x_3} < \frac{T_{12}}{T_{34}}$, and $1 \le x_1 < x_2 < x_3 < x_4$, then LC parameters α_T and β_T are uniquely defined as a solution of two separated equations

$$\frac{\int_{x_1}^{x_2} x^{-\alpha} dx}{\int_{x_3}^{x_4} x^{-\alpha} dx} = \frac{T_{12}}{T_{34}} \quad (3.58)$$

and

$$\beta_T = \frac{T_{12}}{\int_{x_1}^{x_2} x^{-\alpha_T} dx} = \frac{T_{34}}{\int_{x_3}^{x_4} x^{-\alpha_T} dx}, \tag{3.59}$$

besides $\alpha_T > 0,\ \beta_T > 0$.

Equation (3.58) is equivalent to equation $F(\alpha, x_1, x_2, x_3, x_4) = \frac{T_{12}}{T_{34}}$, here

$$F(\alpha, x_1, x_2, x_3, x_4) = \begin{cases} \frac{x_2 - x_1}{x_4 - x_3} \text{ if } \alpha = 0 \\ \frac{\ln x_2 - \ln x_1}{\ln x_4 - \ln x_3} \text{ if } \alpha = 1 \\ \frac{x_2^{1-\alpha} - x_1^{1-\alpha}}{x_4^{1-\alpha} - x_3^{1-\alpha}} \text{ if } \alpha \neq 1 \\ \infty \qquad \mathit{if}\ \alpha \to \infty \end{cases} \tag{3.60}$$

Function (3.60) is positive, continuous and strictly monotonically increasing (SMI). The continuity follows from

$$\lim_{\alpha \to 1-0} F(\alpha, x_1, x_2, x_3, x_4) = \lim_{\alpha \to 1+0} F(\alpha, x_1, x_2, x_3, x_4) = \frac{\ln x_2 - \ln x_1}{\ln x_4 - \ln x_3}. \tag{3.61}$$

Note that $F(\alpha, x_1, x_2, x_3, x_4) = \int_{x_1}^{x_2} \left(\int_{x3}^{x_4} \left(\frac{x}{t} \right)^{-\alpha} dx \right)^{-1} dt$. Showing that the function (3.60) is SMI. It follows from

$$\frac{\partial}{\partial \alpha} \left[\left(\int_{x3}^{x_4} \left(\frac{x}{t} \right)^{-\alpha} dx \right)^{-1} \right] = \int_{x_3}^{x_4} \left[\left(\frac{x}{t} \right)^{-\alpha} \ln \left(\frac{x}{t} \right) \right] dx \left[\int_{x_3}^{x_4} \left(\frac{x}{t} \right)^{-\alpha} dx \right]^{-2} > 0 \tag{3.62}$$

for all $t \in (x_1, x_2)$. The proposition $\lim_{\alpha \to +\infty} F(\alpha, x_1, x_2, x_3, x_4) = +\infty$ follows from that

$$\lim_{x \to +\infty} \frac{x_2^{1-\alpha} - x_1^{1-\alpha}}{x_4^{1-\alpha} - x_3^{1-\alpha}} = \lim_{x \to +\infty} \left(\frac{x_1}{x_3} \right)^{1-\alpha} \cdot \lim_{x \to +\infty} \frac{(x_2/x_1)^{1-\alpha} - 1}{(x_4/x_3)^{1-\alpha} - 1} = \lim_{x \to +\infty} \left(\frac{x_1}{x_3} \right)^{1-\alpha} = +\infty \tag{3.63}$$

because $1 \le x_1 < x_2 < x_3 < x_4$. Hence (3.60) is SMI and positive. Therefore equations (3.58) and (3.59) have unique solutions $\alpha_T > 0,\ \beta_T > 0$ if the condition $\frac{x_2 - x_1}{x_4 - x_3} < \frac{T_{12}}{T_{34}}$ is satisfied.

Two intervals method (TIW)

If it is considered that from MPP the following are obtained: the cumulative average time T_{12} required to produce units from x_1 to x_2 and the cumulative average time T_{34} required to produce units from x_3 up to x_4, and since the cumulative average time only is used, then the W model can be used. If $x_1 < x_2 \le x_3 < x_4$, then LC parameters α, β are uniquely defined as a solution of the system

$$\begin{cases} \dfrac{\beta\left(x_2^{1-\alpha} - x_1^{1-\alpha}\right)}{x_2 - x_1} = T_{12} \\ \dfrac{\beta\left(x_4^{1-\alpha} - x_3^{1-\alpha}\right)}{x_4 - x_3} = T_{34} \end{cases} \tag{3.64}$$

The system of equations (3.64) has a unique solution $0 < \alpha_T < 1,\ \beta_T > 0$, if

$$1 < \frac{T_{12}}{T_{34}} < \frac{(x_4 - x_3)}{(x_2 - x_1)} \frac{\ln x_2 - \ln x_1}{\ln x_4 - \ln x_3} \tag{3.65}$$

The system (3.64) is equivalent to two independent equations

$$\frac{x_4 - x_3}{x_2 - x_1} F(\alpha, x_1, x_2, x_3, x_4) = \frac{T_{12}}{T_{34}} \tag{3.66}$$

$$\beta = \frac{T_{12}(x_2 - x_1)}{x_2^{-\alpha+1} - x_1^{-\alpha+1}} = \frac{T_{34}(x_4 - x_3)}{x_4^{-\alpha+1} - x_3^{-\alpha+1}} \tag{3.67}$$

where the function $F(\alpha, x_1, x_2, x_3, x_4)$ is the same as (3.60). The proof of the uniqueness and existence of positive solutions of equations (3.66) and (3.67) is the same as in the TIC method.

Invariant method (INC, INW)

Introducing the LC approximation method based on only one sample

$$x_i, y_i, x_i < x_{i+1}, i = 1, 2, ...N \tag{3.68}$$

obtained from MPP and on invariants of the function

$$y(x) = \beta x^{-\alpha} \tag{3.69}$$

Invariants are the quantities that must be constant at all the pairs of points $(x_i, y_i), (x_j, y_j),\ i < j$ at the curve (3.69). From (3.69), it follows that

$$\begin{cases} -\alpha \ln x_i + \ln \beta = \ln y_i \\ -\alpha \ln x_j + \ln \beta_j = \ln y_j \end{cases} \tag{3.70}$$

After solving the system (3.70) in respect to two unknowns α and $\ln \beta$, two zero degree invariants are obtained:

$$I_1(x_i, y_i, x_j, y_j) = \alpha = -\frac{\ln(y_j) - \ln(y_i)}{\ln(x_j) - \ln(x_i)}, \tag{3.71}$$

$$I_2(x_i, y_i, x_j, y_j) = \ln \beta = \frac{\ln(y_i)\ln(x_j) - \ln(y_j)\ln(x_i)}{\ln(x_j) - \ln(x_i)}. \tag{3.72}$$

Note that $(\alpha, \ln \beta)$ is a unique solution of (3.70), because of the determinant of the system (3.70) equals to $\ln x_j - \ln x_i > 0$ for all $i < j$.

The empirical formula for approximation of the function (3.70) is chosen; $\bar{y}(x) = \bar{\beta} x^{-\bar{\alpha}}$ where

$$\bar{\alpha} = \frac{1}{M}\sum_{j>i}^{M} I_1(x_i, y_i, x_j, y_j) \text{ and } \bar{\beta} = \frac{1}{M}\sum_{j>i}^{M} \exp\{I_2(x_i, y_i, x_j, y_j)\} \tag{3.73}$$

if $I_1(x_i, y_i, x_j, y_j)$ and $I_2(x_i, y_i, x_j, y_j)$ are almost constant. Where $M = (N^2 - N)/2$ is the number of summands in sum (3.73) for all $i < j$. This method can be applied to both the W or C models, because the relationships of both models are the same (see (1.1) and (1.2)).

In all the presented methods, the adequacy is measured by a percent relative error:

$$\delta = \frac{100}{N}\sum_{i=1}^{N} \left| \frac{y_i - \bar{\beta} x_i^{-\bar{\alpha}}}{y_i} \right|, \tag{3.74}$$

where (x_i, y_i) is the measured production data (3.68), $\bar{\alpha}$ and $\bar{\beta}$ are the estimated parameters of a certain method.

3.6. Modelling of process interruptions and recursive learning

From the product perspective, if its assembly with current technology and without any production interruptions follows the learning model (1.1) or (1.2), the effect of the learning factors is minimal, even if the steady-state time is not reached. Any production interruption causes re-occurring learning, i.e. when the same learning factors occur several times for the same product. The mathematical formulation of such a re-occurring learning model is presented below. Please note that due to short time intervals between interruptions, forgetting factors are not considered.

Let **m** be a vector denoting product shifts from one department or operator to another and n is the total number of such shifts:

$$\mathbf{m}^{\mathrm{T}} = \begin{pmatrix} m_1 & m_2 & m_3 & \dots & m_n \end{pmatrix}. \tag{3.75}$$

Let **β** be a vector denoting the processing time for the first unit at each re-occurring learning.

$$\boldsymbol{\beta}^{\mathrm{T}} = \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 & \dots & \beta_n \end{pmatrix}. \tag{3.76}$$

Then then re-occurring learning curve is expressed by using Heaviside function $H(x)$:

$$\begin{gathered} y(x,\alpha,\boldsymbol{\beta},\mathbf{m}) = \left[\sum_{i=1}^{N-1} F(x, m_{i-1}, m_i) \beta_{i-1} \left(\frac{x}{m_{i-1}} \right)^{-\alpha} \right] + \\ + H(x - m_{n-1}) \beta_{N-1} \left(\frac{x}{m_{n-1}} \right)^{-\alpha} \end{gathered} \tag{3.77}$$

and:

$$F(x, m_{i-1}, m_i) = H(x - m_{i-1}) - H(x - m_i).$$

In order to calculate the assembly time for the whole production quantity N, the derived expression (3.77) is summed:

$$T = \sum_{i=1}^{n-1} \left[\sum_{j=1}^{m_i - m_{i-1}} y_\beta(j) \right] + \sum_{j=1}^{N - m_{N-1} + 1} y_\beta(j). \tag{3.78}$$

Slope coefficient α is supposed to be the same for each re-occurring learning.

3.7. Modelling of process splitting

Process splitting

Let n be the number of fully completed products (batch size) and $p = 1, 2, ..., P$ is the number of process divisions. The complex process splitting is depicted in Fig. 2.5 and Fig. 2.6. If the process is not divided, all assembly operations are being performed at one working station (Fig 2.4). If the process is divided into several work stations (i.e. assembly line), instead of one complex assembly, several more simple processes are apparent. The simplicity of each process affects the total assembly time regarding the number of parts p and total production quantity n. In the following subsection, these effects are presented.

Learning curve and parameters

The Plateau learning curve with break (3.3) as the basic model for the LC application in this research will be applied. A simplified expression of this LC model has the following form:

$$y(x)=\begin{cases}\beta x^{-\alpha}, & x\le x_c\\ T_c, & x>x_c\end{cases}, \qquad (3.79)$$

where x is the number of assembled unit, y is the assembly time of x'th unit, β is the assembly time of the first unit, α is the slope coefficient, T_c is the steady-state assembly time, x_c is the cycle number where steady-state assembly time is reached. These parameters fully define the learning curve. In a general case, when product assembly is performed at only one separate work center, LC parameters are constant.

Slope coefficient

There are many reports claiming that assemblies with less different operations results in lower complexity. From the literature review, it is clear that increasing the number of different components resulted in slower operator thinking and decision making time. Component grouping leads to reduced learning time of the assembly process. The assembled product structure also has a large impact on assembly performance; the simpler the structure - the less learning time is needed.

To sum up, more operations and more components require more thinking, more learning and vice versa, less operations; less thinking and learning. Therefore, it is obvious that the splitting of the complex wiring harness assembly would lead to a lower slope coefficient value for each divided work center (this was also reported by researchers). Therefore, in this research the expression to define the slope coefficient as a function of a number of divisions' p is proposed:

$$\alpha(p)=c_\alpha/p, 0<c_\alpha<1. \qquad (3.80)$$

Steady-state assembly time and stabilization cycle number

It is proved, by many researches that after a certain number of repetitive cycles x_c, steady-state assembly time T_c is reached. This means that after this number the repetitive cycle assembly improves no more. Obviously, if the assembly process is split, it also splits the steady-state operating time. By using the methodology presented in section 2 it is possible to divide the steady-state assembly time equally for the wiring harness. In addition, the steady-state assembly time can be calculated prior to assembly by summing time norms of certain operations. Thus, the steady state assembly time is expressed as follows:

$$T_c(p)=c_T/p, c_T>0. \qquad (3.81)$$

Stabilization existence is proved by many authors. It is assumed in this research that the stabilization number remains the same in the spite of divisions:

$$x_c(p)=c_{xc}>0. \qquad (3.82)$$

Assembly time of the first unit

Stabilization cycle number, slope coefficient (which is already known) and steady state assembly time (which is calculated) enable the calculation of the assembly of the first unit prior to the start of assembly:

$$\beta(p) = T_c(p)x_c(p)^{\alpha(p)}. \tag{3.83}$$

Aggregation time

Since the process is divided, additional time to aggregate separate parts of the assembly is needed i.e. the more divisions, the more aggregation time is needed and thus it is also a function based on the number of divisions:

$$T_e(p) = c_{Te}\left(p-1\right), c_{Te} > 0. \tag{3.84}$$

Total assembly time optimization problem

The time needed to produce n products when the process is divided into p parts

$$T(p,n) = p\int_0^n y(x,p)dx + T_e(p)p, \tag{3.85}$$

where

$$y(x,p) = \begin{cases} \beta(p)x^{-\alpha(d)}, \ x \le x_c(p) \\ T_c(p), \ x > x_c(p) \end{cases}; \tag{3.86}$$

by integrating total time is:

$$T(p,n) = T_1(p,n) + T_2(p), \tag{3.87}$$

where

$$T_1(p,n) = c_T\frac{pnt(p,n)}{p-c_\alpha}, \tag{3.88}$$

$$T_2(p) = c_{Te}p\left(p-1\right), \tag{3.89}$$

$$t(p,n) = \left(\frac{c_{xc}}{n}\right)^{\frac{c_\alpha}{p}}. \tag{3.90}$$

When $x \le x_c(p)$ (see sub-section 3.2), moreover $T_1(p,n) > 0$ and $T_2(p,n) \ge 0$, hence $T(p,n) > 0$, when

$$(p,n) \in D = \left\{p > 1, \ n > 1\right\} \tag{3.91}$$

and $0 < c_\alpha < 1, \ c_{Te}, c_T, c_{xc} > 0$.

Now the assembly time optimization problem of several variables can be stated. This optimization problem can be posed as a constrained optimization problem in the same form as a huge number of common design problems in engineering:

$$\text{minimize } T(\mathbf{r}), \tag{3.92}$$

subject to

$$(p,n) \in G \bigcap \{p < f(n)\}, \tag{3.93}$$

$$c_{\alpha} \in (0,1), \tag{3.94}$$

$$c_{xc}, c_{Te}, c_{T} \in \mathbf{R}_{+}^{1}, \tag{3.95}$$

where $T(\mathbf{r})$ is objective function, $\mathbf{r} = (p, n, c_{\alpha}, c_{Te}, c_{xc}, c_{T})$ is a vector of decision variables, and (3.93), (3.94), (3.95) are the constraints which define the convex and fully connected feasible domain $G \subset \mathbf{R}_{+}^{6}$, i. e., the number $T_{\min}$ and vector $\mathbf{r_0}$ have to be found that

$$T_{\min} = \min_{\mathbf{r} \in G} T(\mathbf{r}) \ , \tag{3.96}$$

$$\mathbf{r}_0 = \operatorname{Arg} \min_{\mathbf{r} \in G} T(\mathbf{r}) \tag{3.97}$$

However, such a problem can be very hard to solve in general, especially when the number of decision variables is large. There are several reasons (Hindi, 2004) for this difficulty:

- the problem "terrain" may be riddled with local optima;
- it might be very hard to find a feasible point (*i.e.*, $\mathbf{r_0}$ which satisfy all equalities and inequalities (3.93), (3.94), (3.95)), in fact, the feasible set which need not even be fully connected, could be empty;
- stopping criteria used in general optimization algorithms are often arbitrary;
- optimization algorithms might have very poor convergence rates;
- numerical problems could cause the minimization algorithm to stop all together or wander

It has been known for a long time (Hiriart-Urruty, Lemarechal, 2001), (Ben-Tal, Nemirovski, 2001), that if the $T(\mathbf{r})$ is convex, then the first three problems disappear: any local optimum is, in fact, a global optimum; feasibility of convex optimization problems can be determined unambiguously, at least in principle; and very precise stopping criteria are available using duality. However, the convergence rate and numerical sensitivity issues still remain a potential problem.

Due to reasons stated before, this paper will analyze problems (3.92)-(3.95) analytically when parameters $c_{\alpha}, c_{Te}, c_{xc}, c_{T}$ are received from process monitoring i.e. they are constants and satisfy constrains (3.95) and (3.95).

Considering the one dimensional optimization problem:

$$\begin{cases} \min_{p} T(p,n), \\ (p,n) \in G \subseteq D \end{cases}, \tag{3.98}$$

i.e., it is found that $T_{\min}(n) = \min_{p} T(p,n)$ and $p_{\min}(n) = \operatorname{Arg}\min_{p} T(p,n),\ p_{\min}(n) \in R_{+}^{1}$.

If $c_\alpha < 1, c_{xc} > n, c_{Te} > 0$ are constant, then for every n and for the sufficiently large P there exists a unique minimum of the function $T(p,n)$ by p, i. e.:

$$p_{\min}(n) = \operatorname{Arg}\min_{1\le p\le P} T(p,n) \tag{3.99}$$

and

$$\min_{1\le p\le P} T(p,n) = T(p_{\min}(n),n)\,. \tag{3.100}$$

From the equation (3.87):

$$\frac{\partial T}{\partial p} = \frac{\partial T_1}{\partial p} + \frac{\partial T_2}{\partial p}, \tag{3.101}$$

where

$$\frac{\partial T_1}{\partial p} = -c_T \frac{nt(p,n)}{(p-c_\alpha)}\left[\ln\big(t(p,n)\big) + \frac{c_\alpha}{(p-c_\alpha)}\right], \tag{3.102}$$

$$\frac{\partial T_2}{\partial p} = c_{Te}\left(2p-1\right). \tag{3.103}$$

For (3.98) it can be concluded that $\left.\frac{\partial T_1}{\partial p}\right|_{p=1} < 0$ and function $\frac{\partial T_1}{\partial p} < 0$ is strictly monotonously increasing for every n by p. In addition, $\frac{\partial T_1}{\partial p}$ have zero horizontal asymptote, because

$$\lim_{p\to\infty} \frac{\partial T_1}{\partial p} = 0. \tag{3.104}$$

For (3.99) it is concluded that $T_2'(1,n) > 0$ and $T_2'(p,n) > 0$ is strictly monotonously increasing for every n by p, therefore the solution of the optimization problem (3.98) is simplified to a solution of equation:

$$-\frac{\partial T_1}{\partial p} = \frac{\partial T_2}{\partial p}, \tag{3.105}$$

which has a unique solution $p_{\min}(n)$ if P is sufficiently large and then the minimal time is:

$$T_{\min}(n) = T(p_{\min}(n), n). \qquad (3.106)$$

Note that $T(p,n)$ is strictly monotone increasing by n because

$$\frac{\partial T}{\partial n} = c_T t(p,n) > 0. \qquad (3.107)$$

Hence function (3.106) is strictly monotone increasing too.

Considering the two dimensional optimization problem:

$$\begin{cases} \min\limits_{p,n} T(p,n) \\ (p,n) \in G \subseteq D \end{cases}, \qquad (3.108)$$

i.e., it can be found that $T_{\min} = \min\limits_{p,n} T(p,n)$ and $\mathbf{r}_1 = \operatorname{Arg}\min\limits_{p,n} T(p,n), \mathbf{r}_1 = (p_1, n_1) \in R_+^2$.

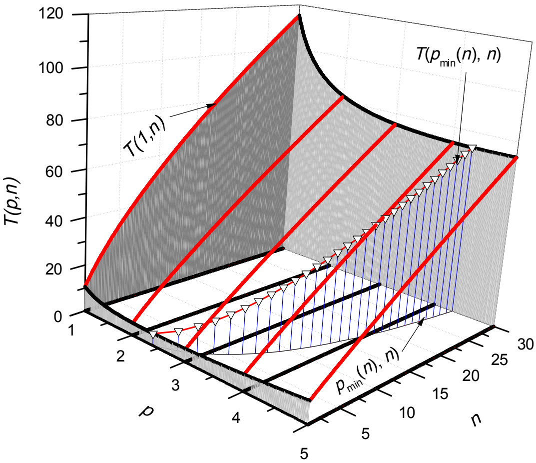

Fig. 3.3. Trajectory plot of the functions $T(p,n)$, $T_{\min}(n) = T(p_{\min}(n), n)$ and $p = p_{\min}(n)$, when $c_\alpha = 0.3, c_{Te} = 0.3, c_{xc} = 80, c_T = 2$

From (3.107) it is identified that

$$|\operatorname{grad} T| = \sqrt{\left(\frac{\partial T}{\partial p}\right)^2 + \left(\frac{\partial T}{\partial n}\right)^2} > 0, \ \forall (p,n) \in G, \qquad (3.109)$$

because $\frac{\partial T}{\partial n} \neq 0$. Detailed calculation shows that $\frac{\partial^2 T}{\partial p^2}$ and Hessian (determinant of Hesse matrix) of function $T(p,n)$ have different signs

$$\frac{\partial^2 T}{\partial p^2} > 0\,,\ \mathrm{H}[T] = \left\| \begin{pmatrix} \dfrac{\partial^2 T}{\partial p^2} & \dfrac{\partial^2 T}{\partial p \partial n} \\ \dfrac{\partial^2 T}{\partial p \partial n} & \dfrac{\partial^2 T}{\partial n^2} \end{pmatrix} \right\| < 0,\ \forall (p,n) \in G\,. \tag{3.110}$$

Hence function $T(p,n)$ is concave and has no global minima at interior of convex domain G. From the concavity of function $T(p,n)$ and (3.109) follows that the point at which $T(p,n)$ reached minimal value $T_{\min} = \inf_{\bar{G}} T = T(p_g, n_g)$ is located at contour ∂G of domain G ($(p_g, n_g) \in \partial G$) (Hiriart-Urruty, Lemarechal, 2001), i. e., $\mathbf{r}_1 = (p_g, n_g)$. This situation is demonstrated in Fig. 3.3.

The time considered in previous equations encompasses total assembly time, but the average time for each assembled part remains unknown and cannot be compared. Therefore, considering the normed time function which addresses the average time required for each assembled unit with different *p* and *n*:

$$T^{(n)}(p,n) = \frac{T(p,n)}{n} = c_T \frac{pt(p,n)}{p - c_\alpha} + c_{Te} \frac{p(p-1)}{n}\,, \tag{3.111}$$

then

$$\frac{\partial T^{(n)}}{\partial p} = \frac{1}{n} \frac{\partial T}{\partial p}\,, \tag{3.112}$$

$$\frac{\partial T^{(n)}}{\partial n} = -c_T c_\alpha \left[\frac{c_{Te} p(p-1)}{n^2 c_T c_\alpha} + \frac{t(p,n)}{n(p - c_\alpha)} \right] < 0\,. \tag{3.113}$$

It can be proved (proof is analogical) that the function (3.111) has a unique minimum for every *n*

$$T_{\min}^{(n)}(n) = \min_{1 \le p \le P} T^{(n)}(p,n) = T^{(n)}(p_{\min}^{(n)}(n), n), \tag{3.114}$$

$$p_{\min}^{(n)}(n) = \operatorname{Arg} \min_{1 \le p \le P} T^{(n)}(p,n) \tag{3.115}$$

and from (3.112) follows that $p_{\min}^{(n)}(n) \equiv p_{\min}(n)$. Function $T^{(n)}(p,n)$ (for every p) is strictly monotone decreasing (3.113), but then the function (3.115) is strictly monotone decreasing too.

Stating and analyzing the second two dimensional optimization problem:

$$\begin{cases} \min\limits_{p,n} T^{(n)}(p,n), \\ (p,n) \in G \subseteq D \end{cases}, \tag{3.116}$$

i.e. it can be found that $T^{(n)}_{\min} = \min_{p,n} T^{(n)}(p,n)$ and $\mathbf{r}_2 = \operatorname{Arg}\min_{p,n} T^{(n)}(p,n)$, $\mathbf{r}_2 = (p_2, n_2) \in R^2_+$.

From (3.113) is obtained

$$\left|\operatorname{grad} T^{(n)}\right| > 0,\ \forall (p,n) \in G, \qquad (3.117)$$

because $\frac{\partial T^{(n)}}{\partial n} \neq 0$. Detailed calculation shows that

$$\frac{\partial T^{(n)2}}{\partial p^2} > 0, \forall (p,n) \in G \qquad (3.118)$$

and that Hessian of function $T^{(n)}(p,n)$ is positive

$$\mathrm{H}\left[T^{(n)}\right] > 0,\ \forall (p,n) \in G, \qquad (3.119)$$

hence function $T^{(n)}(p,n)$ is convex and has no global minima at interior of convex domain G.

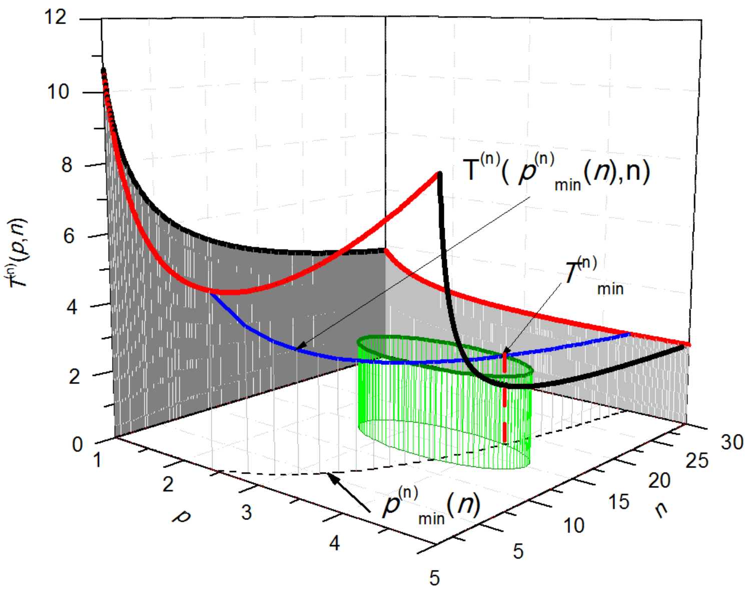

Fig. 3.4. Trajectory plot of the functions $T^{(n)}(p,n)$, $T^{(n)}_{\min}(n) = T^{(n)}(p^{(n)}_{\min}(n), n)$ and $p = p^{(n)}_{\min}(n)$, when $c_\alpha = 0.3, c_{Te} = 0.3,\ c_{xc} = 80, c_T = 2$ (solved by Conjugate gradient method).

From the convexity of function $T^{(n)}(p,n)$ and (3.117) it follows that the point at which $T^{(n)}(p,n)$ reached minimal value $T^{(n)}_{\min} = \inf_{\bar{G}} T^{(n)} = T^{(n)}(p_g, n_g)$ is located at contour ∂G of domain G ($(p_g, n_g) \in \partial G$) (Hiriart-Urruty, Lemarechal, 2001), i. e., $\mathbf{r}_2 = (p_g, n_g)$. This situation is demonstrated in Fig. 3.4.

Further, several modeled problems are solved to demonstrate solutions of the optimization problems (3.108) and (3.116) by the Conjugate gradient method proposed by Snyman (2005). Also, two other methods (Levenberg-Marquardt and Quasi-Newton) were considered. The solutions of the optimization problems in MathCad are presented in Appendix 1 and Appendix 2. Two different domains are analyzed to illustrate the presented analytical proofs.

In the first case, the feasible domain *G* is an ellipse:

$$\frac{(p-2.5)^2}{0.25} + \frac{(n-2.5)^2}{1} = 1 . \tag{3.120}$$

Then the optimization problems (3.108) and (3.116) become

$$\begin{cases} \min\limits_{p,n} T(p,n), \text{ or } \min\limits_{p,n} T^{(n)}(p,n) \\ 2 \le p \le 3,\ f_1(p) \le n \le f_2(p) \end{cases}., \tag{3.121}$$

where $f_1(p) = -\sqrt{1 - \frac{(p-2.5)^2}{0.25}} + 2.5,\ f_2(p) = \sqrt{1 - \frac{(p-2.5)^2}{0.25}} + 2.5$.

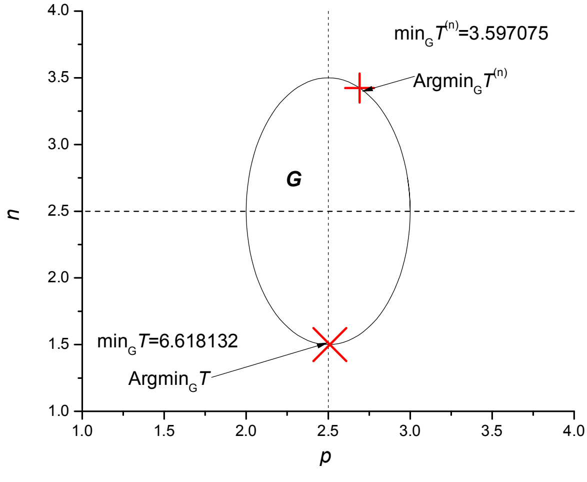

Fig. 3.5. Convex and connected domain *G* and the minimum points of both optimization problems (3.123)

Since the domain is convex and connected, the ellipse (3.120) can always be presented in parametric equations:

$$\begin{cases} p(t) = 0.5\cos t + 2.5 \\ n(t) = \sin t + 2.5 \end{cases}. \tag{3.122}$$

Then the optimization problems (3.108) and (3.116) becomes

$$\begin{cases} \min_t T(p(t),n(t)), \text{ or } \min_t T^{(n)}(p(t),n(t)) \\ 0 \le t \le 2\pi \end{cases}. \quad (3.123)$$

The solutions of optimization problems are presented in Fig. 3.5 and Fig. 3.6.

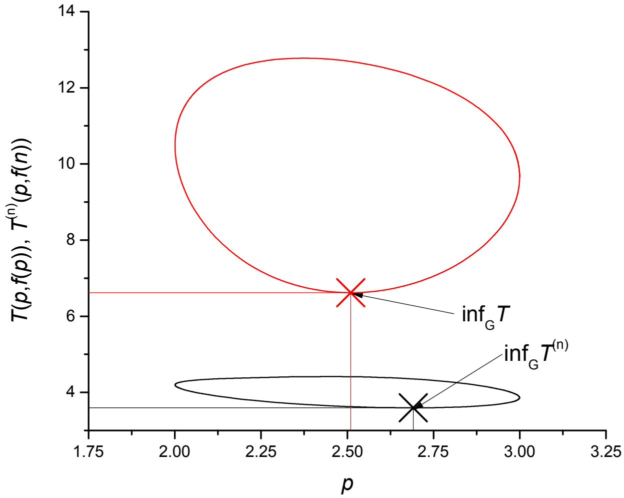

Fig. 3.6. Values of the goal functions for both optimization problems

It is clear from Figure 3.5 that both problems reached minimum points on a contour of feasible domain *G*. These points are also the lowest values of the function ant the feasible domain (see Fig. 3.6)

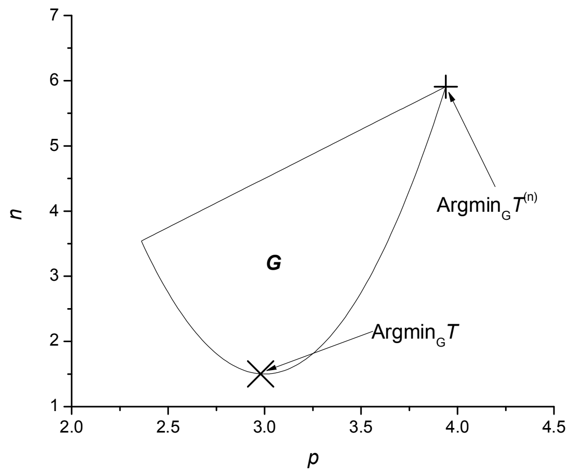

Fig. 3.7. Convex and connected domain *G* and the minimum points of both optimization problems (3.124)

The other case calculated is when the feasible domain *G* is delimited by parabola $f_1(p) = 5(p-3)^2 + 1.5$ and straight line $f_2(p) = 1.5n$. Then the optimization problems (3.108) and (3.116) are posed:

$$\begin{cases} \min_{p,n} T(p,n), \text{ or } \min_{p,n} T^{(n)}(p,n) \\ p_1 \le p \le p_2,\ f_1(p) \le n \le f_2(p) \end{cases}, \tag{3.124}$$

where $p_1 < p_2$ and $f_1(p) < f_2(p)$. The solution of problem (3.124) is presented in Fig. 3.7 and Fig. 3.8.

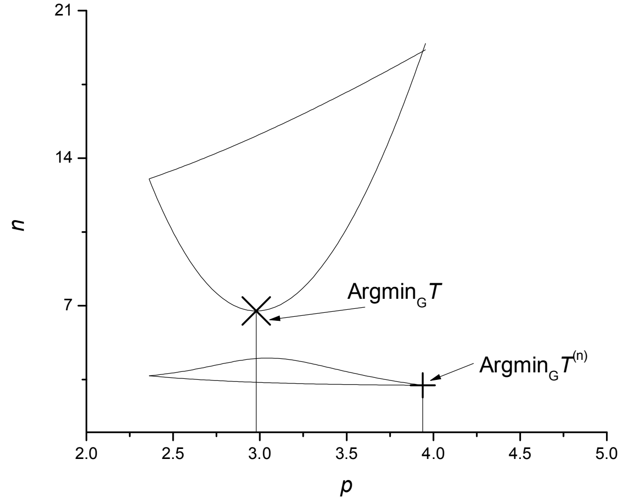

Fig. 3.8. Values of the goal functions for both optimization problems

The second modeled example also has the minimum points of the functions on the contour of feasible domain *G*. Therefore both examples illustrate and prove the previously performed analytical analysis.

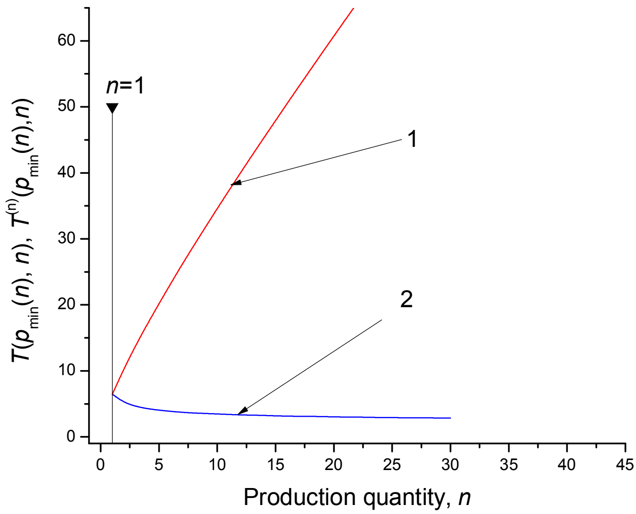

Fig. 3.9. Minimum point dependency on production quantity *n*. 1 – minimum of the function $T(p,n)$, 2 – minimum of the function $T^{(n)}(p,n)$

The other interesting result comes from (3.107) and (3.113). Please note that from (3.107) and (3.113) it follows that the minimum of the function $T(p,n)$ is monotonously increasing and the minimum of the function $T^{(n)}(p,n)$ is monotonously decreasing (Fig. 3.9). In the other words, the higher the number of units assembled; illustrates different results of these two functions. Obviously, the increased number of units demands a higher value on the total assembly time, but also results in lower average assembly time of the normed assembly time function.

Regarding the presented proofs, it is obvious that both optimization problems have solutions.

3.8. Conclusions and main results of the section

From the analytical part, the following conclusions and generalizations regarding this section are derived:

1. LC model with stabilization parameter derived. This Plateau model enables the prediction of assembly time stabilization after a certain number of repetitive assembly cycles.
2. The new LC model developed. This almost learning curve model is based on the solution of the differential equation and it possesses more versatility options than the classical LC models.
3. Another unique LC model that directly connects assembly time, repetitive production cycles and the injury development due to assembly vibrations and cumulative trauma disorders is derived.
4. Six different deterministic LC parameter recovery methods have been developed to estimate LC parameters from a limited production data.
5. A methodology to analyze manual assembly process interruptions and recursive learning is presented.
6. Manual assembly process splitting model has been created. This model enables to state optimization problem and find an optimal set of process parameters and minimize the total assembly time.
7. Created Plateau LC model, parameter estimation methods and process splitting methods are directly related to the goal of research. The proposed almost learning curve models, learning-fatigue model and reoccurring learning models are additional results of this research, however they play an important indirect role to the final results.

4. EXPERIMENTAL RESEARCH

4.1. Introduction

To test the adequacy of the proposed methods, a company performing manual operations has been selected. Measurement tests were performed on a Scandinavian company, which has production facilities in Central Europe, North America and Eastern Asia. During the research period, the company allowed researchers to collect production data at a specific facility. Therefore, the study is based on the working environment of a particular wiring harness manufacturer and all references apply to this specific manufacturer. This company does not design or create its own product; it belongs to the automotive industry supply chain, so it must manufacture the wiring harnesses strictly according to customer drawings, specifications and standards, and has no possibility of changing the product structure. The manufacturing is order-based. Obviously, the company suffers from the flexibility issue commonly met in order-based manufacturing.

The company produces an enormous variety of different harnesses (more than four thousand) for the automotive industry and customer demand is fluctuating and changing rapidly for each product. In addition, the demand of particular wiring harnesses sharply differs from one piece per year, to several hundred per month.

Three major issues will be addressed in this section:

- To apply the proposed LC parameter recovery methods to limited production data and prove their adequacy
- Evaluate the adequacy of all new LC models proposed
- Test and approve production optimization in a real manufacturing situation.

These tasks will be completed in the following sub-sections.

4.2. Adequacy test of parameter estimation methods

Using the methods presented in the 3rd section, calculations were performed on a series of different products and their assembly times based both on the Wright's and/or Crawford's data. To present the comparison of different method applications, two representative calculations are selected: one for the Wright's and one for the Crawford's model.

The first graphical comparison for the Wright model is given in Fig. 4.1 and LC parameters are given in Table 4.1. The invariant method (INW) encompasses all the production data points; the two point method (TPW) is based on the first and the last point and the two-interval method (TIW) is based on random selected intervals (see Fig. 4.1).

As it can be seen from (Fig. 4.1) and (Table 4.1), all the methods show fairly similar and adequate results (relative error is smaller than 5 %). Even though, quite accurate LC is estimated from only two provided data points. Since the INW method encompasses all the production data points, it mostly provided the best results when a sufficient number of points were presented, though in the example (Table 4.1) TPW shows a smaller relative error value calculated by equation (3.74) than INW.

The method of two intervals (TIW) also provided quite accurate results of LC estimation.

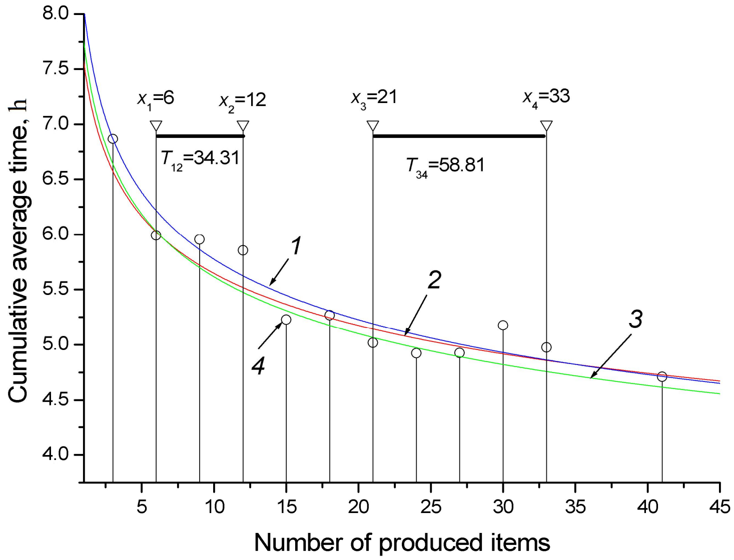

Fig. 4.1. Comparison of calculated LC based on the proposed methods for Wright data. 1 – TPW, 2 – INW, 3 – TIW. 4 – production data

Comparison of parameters are presented in Table 4.1.

Table 4.1. Comparison of LC parameters and relative error values (Wright model)

Data type	Method	α	β	δ, %
Wright	TPW	0.144	8.046	2.475
	INW	0.126	7.548	2.636
	TIW	0.139	7.735	2.821

The second comparison is given for the Crawford's data (Fig. 4.2, Table 4.2). The same as in Wright's case, the invariant method (INC) encompasses all the production data points; the two point method (TPC) is based on the first and the last point, the interval methods PIC and TIC are based on randomly selected intervals and the starting point for PIC is the first data point (see Fig. 4.2). Calculated LC parameters and relative error values by equation (3.74) are presented in Table 4.2.

The comparison shows that the results from the Crawford's data analysis with different models somewhat differs from each other (the difference is larger than in the Wright's data analysis, and the TIC method exceeded the limit error value of 5 %). The smallest relative error value calculated by equation (3.74) was provided by the invariant method (INC), thus representing the most accurate LC. Other methods show worse fitting results than in Wright's data; however, they are not dramatically different. These differences occur due to the dissimilar origin of Crawford's and Wright's data.

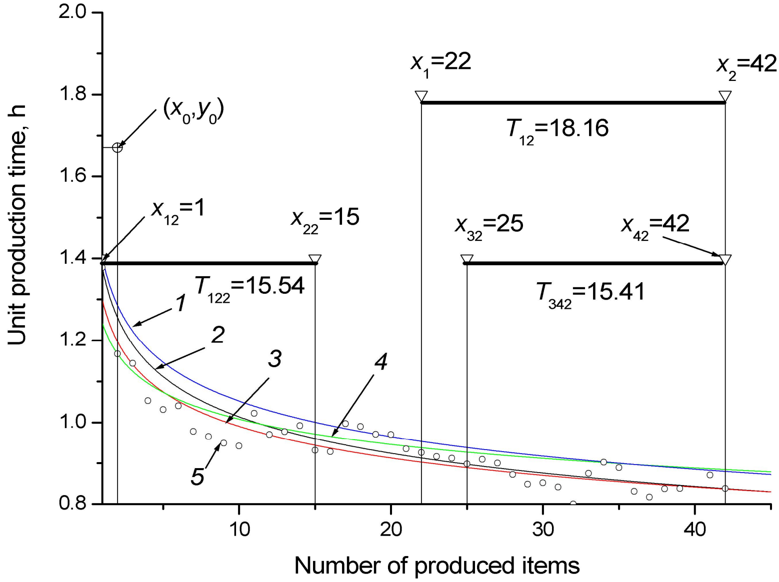

Fig. 4.2. Comparison of calculated LC based on the proposed methods for Crawford data. 1 – TIC, 2 –TPC, 3 –INC, 4 – PIC 5 – production data

The Crawford's model uses a particular processing time of each separate unit and the Wright's model uses cumulative average time. The particular unit time could be significantly impacted by random error.

Table 4.2. Comparison of LC parameters and relative error values (Crawford model)

Data type	Method	α	β	δ, %
Crawford	TIC	0.125	1.402	5.965
	TPC	0.133	1.377	3.670
	INC	0.118	1.299	3.289
	PIC	0.091	1.243	4.391

The Wright's data is already smoothed to some extent and therefore, the impact of random errors is reduced in such a way. For the invariant methods (INW and INC) there is no difference which data is analyzed, but for the other deterministic methods this difference exists. All the Crawford's data based methods (TIC, PIC, TPC) are more sensitive to errors and points and intervals choice than the Wright data based methods. At any rate, using deterministic methods (TPW, TPC, TIW, TIC, PIC) are highly affected by random error. However, a particular manufacturing situation determines the available dataset. Additionally, during the analysis some data selection tips were emphasized:

If possible, using more than the minimum required data is necessary for the robust LC estimation

Apply different methods to the same limited production data for the best results.

Table 4.3. Recovered LC parameters from limited production data

Part number	α	β	Units produced	Data points measured	Number of circuits	Method
WH01600PR145	0.283	88.446	552	5	600	INW
WH01526PR997	0.268	62.072	432	7	526	INW
WH01546PR111	0.208	113.625	62	4	546	TPW
WH01556PR356	0.185	97.238	664	3	556	TPW
WH01588PR789	0.186	93.421	453	7	588	INW
WH01471PR501	0.219	58.031	8	3	471	TIW
WH01467PR447	0.193	20.449	421	4	467	TIW
WH01347PR125	0.134	28.54	211	4	347	TIW
WH01212PR126	0.175	27.926	237	9	212	INW
WH01163PR978	0.177	25.642	110	3	163	TPW
WH01141PR215	0.157	19.627	7	3	141	TPW
WH01074PR187	0.128	3.838	110	5	74	INW
WH01071PR478	0.144	8.046	41	8	71	INW
WH01064PR663	0.098	1.362	42	7	64	INW
WH01046PR154	0.144	5.116	1294	4	46	TIW
WH01037PR178	0.081	3.014	128	4	37	TIW
WH01019PR101	0.092	2.354	330	4	19	TPW
WH01010PR005	0.064	0.207	30	7	10	INW
WH01004PR078	0.079	0.104	70	3	4	TPW
WH01002PR774	0.086	0.103	2200	10	2	INW

In addition to the presentation of models, a series of various wiring harness production data was analyzed in order to calculate the LC parameters. Regarding the time study, the Wright's data is easier to obtain and are also less sensitive to a random error. Therefore, the estimated parameters will be presented only for the Wright's data applying TPW, TIW and INW methods. 20 representative products with different complexity were measured during production. The results are presented in Table 4.3.

The results from the limited production data analysis show that LC parameters differ sharply from each other: α from 0.064 to 0.283 and β from 0.103 to 113.625 for the same process (manual wiring harness assembly). The number of wires in the particular wiring harness means that more circuits correspond to a more complex product (with more assembly operations) and it could be seen from the increased LC parameters α and β (see Table 4.3).

4.3. Experimental research of proposed Plateau model

In this section the adequacy of Plateau LC with stabilization parameter will be evaluated. In order to verify the adequacy of the method proposed $k=11$ (2.25), different experiments and 11 respective measurements have been performed in which only (x_0, y_0) and T_c have been obtained.

Table 4.4. Experimental data

Data	i	x_i	y_i, h
$k=11$	1	2	6.88
	2	44	3.078
	3	100	2.423
	4	163	2.047
	5	172	2.128
	6	204	2.224
	7	236	2.212
	8	268	2.178
	9	301	2.233
	10	349	2.114
	11	396	2.101
	12	428	2.136
	13	460	2.140

In line with this parameters; the parameters α_ε and x_c have been calculated. According to the manufacturer recommendations, $\varepsilon = 0.0016 \cdot T_c$ in all the calculations. A comparison of the LC $Y(x, x_0, y_0, \alpha_\varepsilon)$ obtained values with the complete experimental data (2.25) was chosen as the adequacy criterion (average percent relative error):

$$\Delta(\varepsilon) = \frac{100}{n_k} \sum_{i=1}^{n_k} \left| \frac{y_i^{(k)} - Y(x, x_0, y_0, \alpha_\varepsilon)}{y_i^{(k)}} \right|, \qquad (4.1)$$

where *k* is the number of experiments, n_k is the number of the *k-th* experiment data.

Some additional data is needed to perform the calculations. This data includes the number of chosen points, stabilization time T_c and other. Data is provided in Table 4.4.

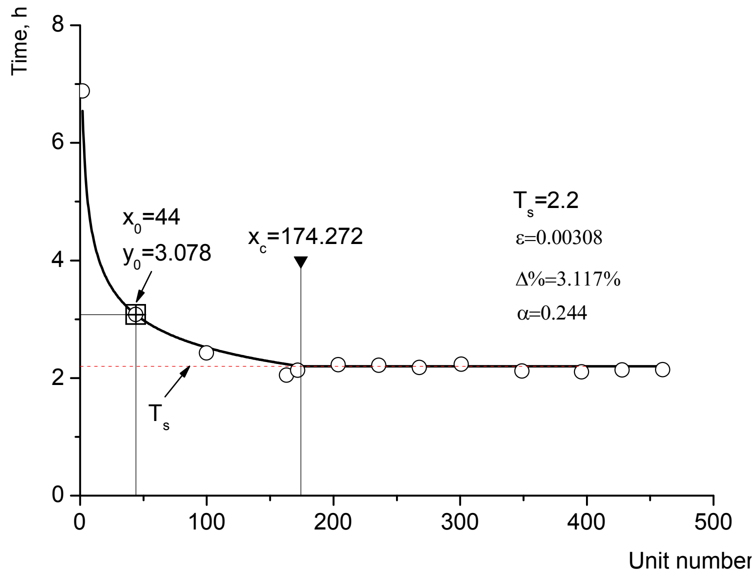

Fig. 4.3. Learning curve based on the proposed model (line) and a graph of the data set from Table 4.6, $k = 1$

In Table 4.4, the data of the first experiment are presented. The calculation results are given in Fig. 4.3.

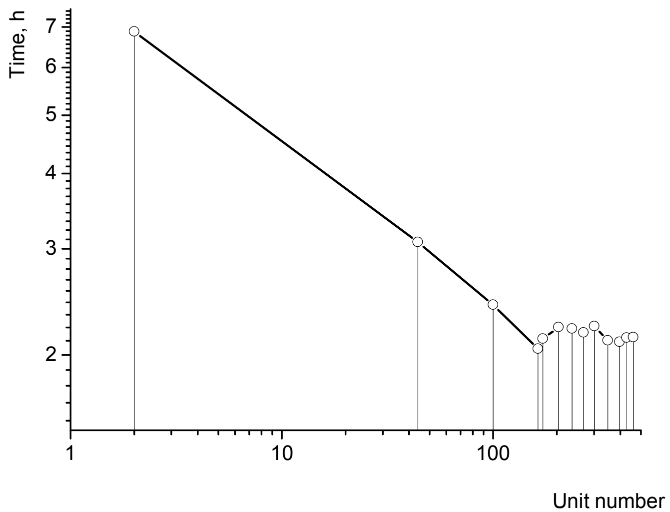

Fig. 4.4. Graph of the data set from Table 4.6, $k = 1$. (log-log scale)

In addition to this, Fig. 4.4. shows the experimental data in log-log scale. Logarithmic scale shows a clear linear relationship.

The same calculations were made for all the experiments provided in Table 4.4 and all necessary parameters were obtained, including learning curve parameters, stabilization point and relative error values. Calculation results are provided in Table 4.6. In the majority of cases, relative error did not exceed the feasible value (5%).

Table 4.5. Provided data for calculation

k	q	x_0	y_0, h	T_c, h	n_k	min x_i	max x_i	min y_i, h	max y_i, h
1	2	44	3.078	2.200	13	2	460	2.047	6.880
2	1	20	2.545	0.755	10	20	240	0.867	2.545
3	3	3	1.230	0.900	42	1	42	0.940	1.377
4	3	13	3.448	2.980	8	2	47	3.047	4.000
5	2	80	1.177	0.800	21	40	631	0.864	1.307
6	1	90	0.731	0.550	18	90	586	0.537	0.731
7	1	10	17.061	11.000	11	10	111	11.262	17.061
8	1	17	11.847	6.650	17	17	421	6.326	11.847
9	3	59	14.977	6.765	20	19	430	6.792	19.635
10	1	2	6.880	2.100	13	2	460	2.047	6.880
11	1	19	19.630	6.750	20	19	430	6.792	19.635

Table 4.6. Calculation results

k	ε	α_ε	x_c	Δ%
1	0.00308	0.244	174.272	3.117
2	0.00124	0.46011	280.55498	5.96799
3	0.00126	0.0989	70.63373	0.74863
4	0.00489	0.09657	58.88177	3.39804
5	0.00131	0.37123	226.35869	1.065052
6	0.00090	0.34043	207.57814	1.65911
7	0.01804	0.18226	111.13235	2.27506
8	0.01091	0.25905	157.95853	3.96111
9	0.01109	0.48996	298.75474	4.81239
10	0.00344	0.26923	164.16352	2.48049
11	0.01107	0.41313	251.90585	6.46187

The numerical experiments have shown that function (4.1) is unimodal and has a minimum approximate to proposed ε, i.e. $\varepsilon_m = \arg\min_{\varepsilon} \Delta(\varepsilon) \approx 0.0016 \cdot T_c$. The values of ε_m are illustrated in Table 4.7 and the graph of this unimodal function is presented in Fig. 4.5.

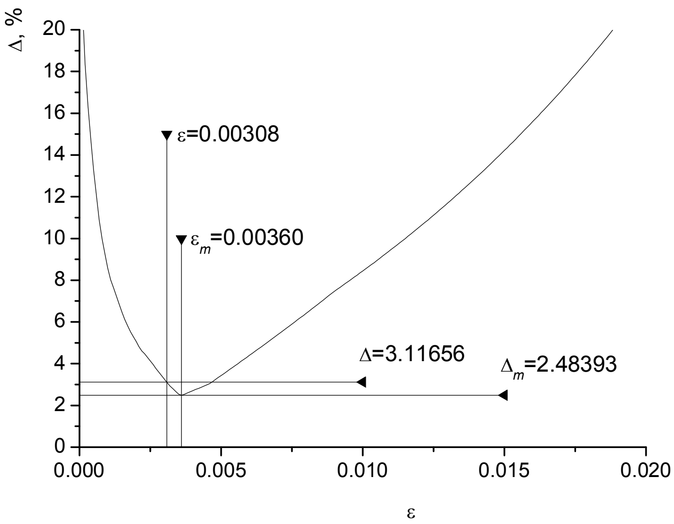

Fig. 4.5. Dependency of the average relative error on ε for the data set from Table 4.7

Function (3.3) is smooth everywhere except for the point x_c, where it has break of the derivative equal to ε, which is entirely defined as the angle $\varphi = -\arctan(\varepsilon)$, (if $\varphi = 0$ then function (3.3) is smooth everywhere). The values of angle φ in degrees are presented in Table 4.7.

Table 4.7. Calculated values of angle φ

Data		ε_m	φ_m *deg*
k	1	0.0036	-0.2063
	2	0.0013	-0.7445
	3	0.0014	-0.0080
	4	0.0048	-0.2750
	5	0.0001	0.0057
	6	0.0009	-0.0516
	7	0.0233	-13.348
	8	0.0082	-0.4698
	9	0.0120	-0.6875
	10	0.0033	-0.1891
	11	0.0079	-0.4526

Note that the proposed method can be applied to predict currently not stabilized learning processes. This is illustrated in Fig. 4.6. The condition

$\max_i x_i < x_c$ can be treated here as a criterion of unsettlement and predict the steady-state point as x_c. The calculation has shown that there is a steady-state of the learning rate in the experiments $k = 1,5,6,8,9,10,11$; while in the rest of them; stabilization is just predicted (see Table 4.6 and Fig. 4.6).

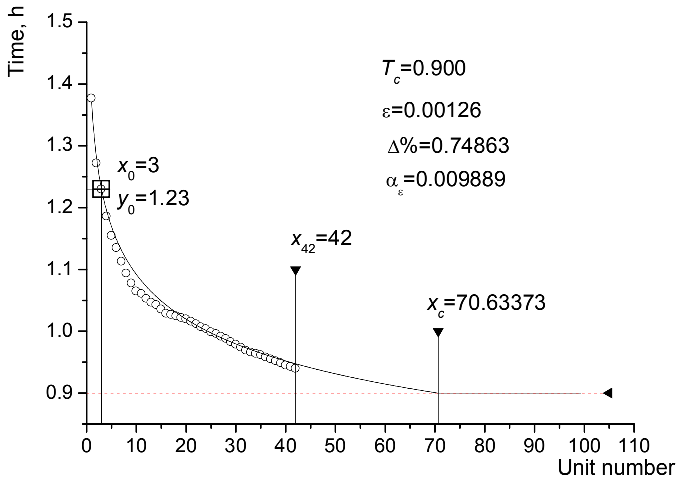

Fig. 4.6. Learning curve based on the proposed model (line) and a graph for the data set from Table 4.6, $k = 3$ (circles). Point of the stabilized learning rate x_c

To sum up, the ability to predict the stabilization of the assembly time is very important in an unstable manufacturing environment, since it opens the possibility to group production orders and to reduce learning time in such a way.

4.4. Adequacy of ALC models

Created ALC models were tested on certain production data that was monitored at the manufacturing company. There are two main tasks of this adequacy test:

- Approve model adequacy by relative error values (≤ 5%)
- Compare the ALC model with traditional power model

In this particular company, several types of manufacturing layouts are applied, from assembly line to singular prototype production. Data was collected at all of them for ALC model fitting. The data provided is in the form presented in (2.25) equation. Relative error calculations will be performed according to (2.28).

The proposed ALC model was tested on numerous production data sets. Three situations of the production data analyzed can be defined:

- Repetitive orders taking place in long time and random intervals
- Orders with low volume, prototype production
- Orders with high volume

Since many data sets were analyzed, to show the performance of the ALC model; three different examples are presented for each group of production data.

The largest group of all of the company's products is the repetitive orders arriving in random and longtime intervals. This means that operators are familiar with the product, therefore they do need time to remember the assembly at the beginning of the production cycle. The typical data set representing this situation is depicted in Fig 4.7.

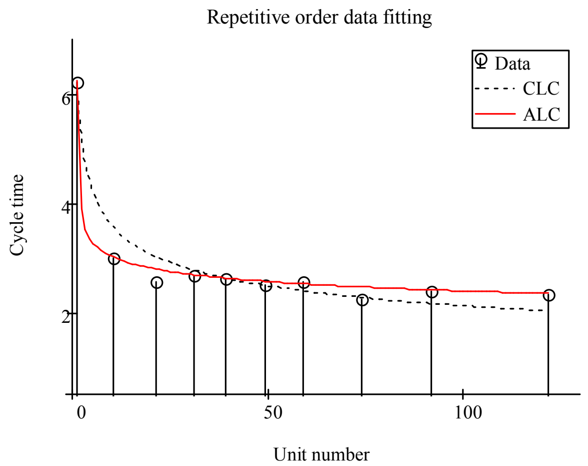

Fig. 4.7. CLC and ALC fitting for repetitive orders taking place in long time and random intervals

Fig. 4.7 shows, how the CLC model results are a fairly poor approximation when compared with ALC (see Table 4.8). CLC improves gradually, however the perturbation parameter in ALC enables an approximate steeper improvement of the operating time.

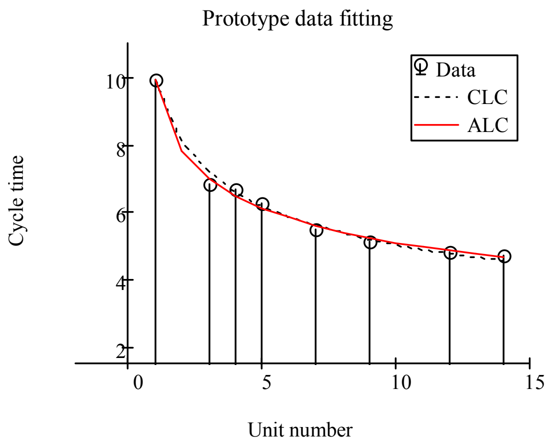

Fig. 4.8. CLC and ALC fitting for orders with low volume (prototype production)

Other groups of production orders are small order production and prototype production. These orders are mostly singular, with quantities up to 20 pieces. The example in Fig. 4.8 presents such production data with both models, ALC and CLC, applied to this data.

Please note that on such data, both methods show fairly good results and provide accurate approximation, although ALC shows slightly better result than CLC (see Table 4.8).

Table 4.8. Comparison of CLC and ALC approximation results

Experiment	Fig. 4.7	Fig. 4.8	Fig. 4.9
N	10	8	21
n	5	1	10
Prod. qty.	122	14	2210
α_y	0.2277	0.2961	0.5057
α_w	0.0998	0.2657	0.0000
ε_w	-6.5885	-2.4143	-28.9483
r_w	3.7900	5.5750	2.0770
δ_y	8.80 %	2.21 %	14.91 %
δ_w	3.47 %	1.73 %	3.73 %
δ_y / δ_w	2.54	1.28	4.00

The last group of significant wiring harnesses is those with high volume orders. These harnesses possess orders up to several thousand pieces. A typical example of such production data is presented in Fig. 4.9.

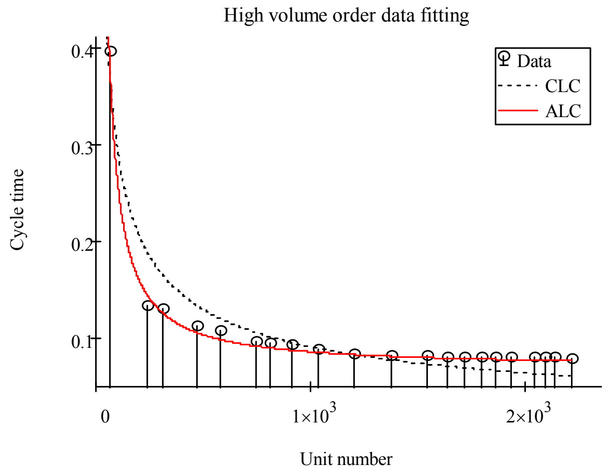

Fig. 4.9. CLC and ALC fitting for orders with high volume

It is needed to emphasize that in this group the stabilization of operating time exists. This stabilization is known as the plateauing phenomenon and can obviously be identified in the high volume orders of this company's production. In addition, CLC is unsuitable for such data approximation (relative error is higher than 5 %); however ALC shows quite accurate results and confirms the adequacy by falling within the limits of the feasible relative error value. (see Table 4.8). CLC again improves gradually and ALC enables approximation of assembly time stabilization.

4.5. Experimental research of proposed learning-fatigue model

In order to evaluate and prove the adequacy of the proposed model, the model was tested on production data from the manufacturer performing the manual assembly of the automotive wiring harnesses. This manufacturer is encountering major global market challenges for cost reduction; therefore operational efficiency is the major factor defining the company's competitiveness. This situation results in increased working rates, increased working time (operators have to work overtime in order to complete deliveries on time) and finally leads to boosted occurrences of hand-arm vibration syndrome and the other CTD.

Production data monitoring was performed in a particular assembly department, where operators assemble several different products in high volume. The production time was monitored for a half year period. In most of the cases, the monitored assembly time after initial improvement starts to climb as the assembly operation continues.

To present the model, one simple product was selected. A typical example of such a wiring harness product is depicted in Fig. 4.10.

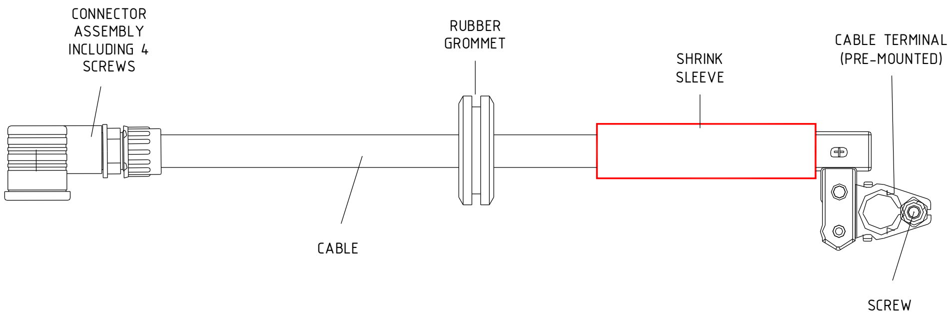

Fig. 4.10. Simple wiring harness with vibrations and repetitive motions used in assembly

The product assembly constitutes a number of standard operations. Some of these operations cause risk of vibrations (v) and repetitive motions (r) for an operator's health, while the others have no risk. Below, the list of operations needed to complete the product with standard time norms are presented:

- Placing cable onto assembly jig and assembling a connector – 1·0.6 min
- Fastening screws with impact wrench (v) – 5·0.1 min
- Piercing grommet with high force and pulling to exact position (r) – 1·0.3 min
- Heating the shrink sleeve with glue (v) – 0.6 min
- Packing the harness (r) – 0.1 min

To sum up, the total time of operator exposure to vibrations and hazard repetitive motions are 1.5 min or 0.025 h of the total 0.035 h of standard assembly time. This time will be used to calculate the perturbation parameter ε for the model. Perturbation parameter is calculated according to the formula:

$$\varepsilon = t_r (t_c / t_r)^{-1} = t_r^2 t_c^{-1} = (0.025)^2 900^{-1} = 6.944 \cdot 10^{-7}, \qquad (4.2)$$

where t_r is risk time, t_c is cumulative working time, when operator starts to feel uncomfortable with the assembly task.

Data is provided in the form presented in (2.25) equation. Relative error calculations will be performed according to (2.28).

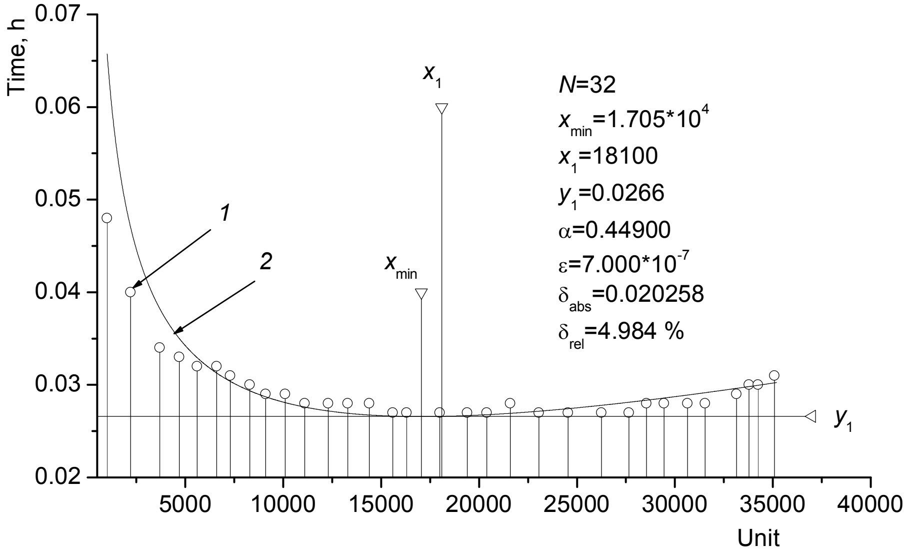

Fig. 4.11. Production data and ALC for performance prediction. *1* - monitored production data, *2* – function calculated by the model

In Fig. 4.11 the monitored production data and calculations by the proposed analytical model (3.39) are presented. It is clear from the graph in Fig. 4.11 that ALC approximates manual assembly performance quite accurately. The relative error of 4.98 % is sufficient to prove the model's adequacy. In addition, the analytical calculation shows that there is a minimal assembly time (3.48) after which the operator improves no more due to injury development.

The main outcome of this experiment is that LC with perturbation parameter can be used as an analytical tool to evaluate the risks to a human operator's health and safety. By using this tool it is not only possible to predict the dynamic of the assembly time, but also forecast an injury development. These measures are very important for job rotation, since it enables the movement of operators to other tasks before injury occurs. Additionally, the model is still quite simple; only one additional parameter is introduced.

Therefore, the proposed model is important in regard to the gap in scientific literature and it clearly supplements currently existing models.

4.6. Recursive learning model

Order-based manufacturing without a completely implemented JIT (just-in-time) technique makes planning an extremely complicated issue. When manufacturing orders are released only on customer demand and the company does not intend to produce from stock, then stochastic demand, fluctuating order quantities and delivery times, unplanned orders, priority orders and other uneven situations from more than one company's customers create chaotic planning (except from weekly demand for selected complex product depicted in Fig. 4.12). As a result, orders with late delivery are being terminated, shifted, delayed and new orders released to any department having free capacity.

The most complicated manufacturing and planning situations appear when the actual delivered production quantity is decreasing, but the operators are forced to work overtime. It will be later shown that the main reason causing this situation is assembly interruption and order shifting from one department to another for the most complex wiring harnesses (500 circuits and more which require the most learning time at the beginning).

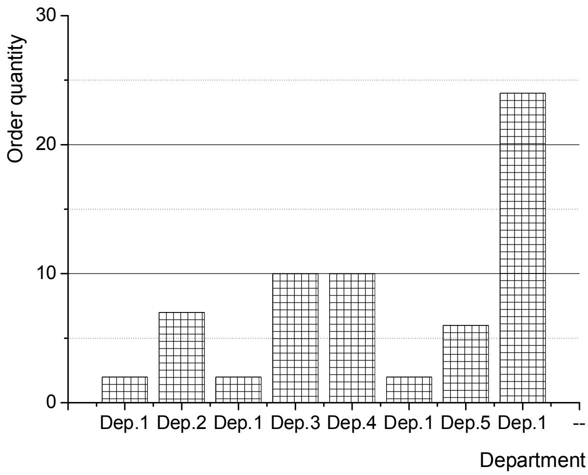

Fig. 4.12. Manufacturing order log representing department change for a selected wiring harness

To illustrate the extent of which the production disorders effect the processing time due to learning factors, one complex product, the assembly of which suffered many re-occurring learning phases, was selected.

After the last department change, the assembly of this particular product was thoroughly studied, the assembly time measured and shifting prevented for a half year period, despite continuing fluctuations in customer demand. The production data of this product is presented in Fig. 4.13.

Using the INC analysis, Crawford learning curve model (1.2) was used on the collected assembly data points. The calculation results confirm that the data follow Crawford learning.

Before this product was taken to account, it was treated like any other wiring harness at the company and it was moved several times from one department to another. The continuing order log for this wiring harness is depicted in Fig. 4.14.

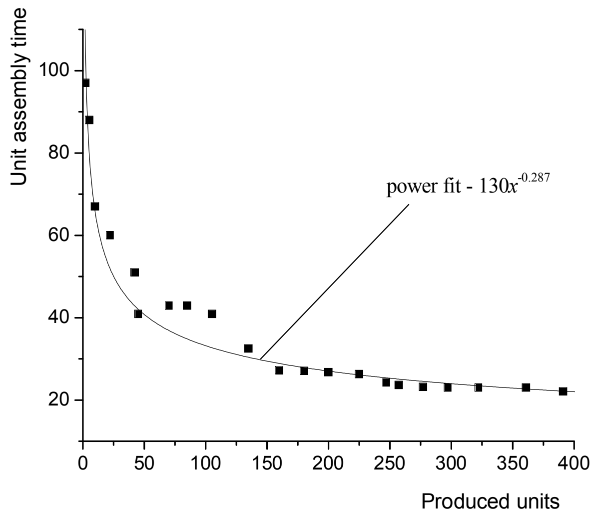

Fig. 4.13 Crawford model fit to selected production data by INC method. Obtained values α = -0.287; β = 130

Using formula (3.77), the re-occurring learning curve will be calculated for this product; as it is clear how many department changes were made and quantities that each department has produced. The vector **m** is filled according to the product order log:

$$\mathbf{m}^{\mathrm{T}} = \begin{pmatrix} 1 & 3 & 10 & 12 & 22 & 32 & 34 & 40 & 64 \end{pmatrix}.$$

For the first unit time at the vector **β,** the same number obtained by regression analysis will be used (except in shifts back to the same department). Forgetting factors were not considered. Therefore, when the product was returned to the same department it is assumed that the learning process is resumed and only the new department change sets the product assembly time on the top of the LC.

$$\boldsymbol{\beta}^{\mathrm{T}} = \begin{pmatrix} 130 & 130 & 95 & 130 & 130 & 90 & 130 & 89 \end{pmatrix}.$$

For simplicity in this calculation, slope parameter remained the same for all the departments; however there is also a possibility in the proposed model to include different slope coefficient values for different departments.

The calculated model is depicted in Fig. 4.14. The graph contains both the re-occurring learning curve (Fig. 4.14 curve 2) and the learning curve with a single initial learning (Fig.4.14 curve 1) for comparison.

Finally, the total learning time T is calculated for both cases: with re-occurring learning (T_{RL}) and conventional learning (T_{CL}) by (3.78):

$$T_{CL} = 3443 \text{ h}$$

$$T_{RL} = 6683 \text{ h}$$

Both calculation results from the graphical and numerical comparisons clearly indicate a significant difference between synchronized production (one initial learning), and chaotic production (re-occurring learning). Even a single department change would unnecessarily increase the processing time and can create a bottleneck. The calculated example shows an extreme extent when processing time due to learning increased by 1.94 times. The less complex wiring harness with less learning factors will be less sensitive to department and operator change, but the impact still remains significant.

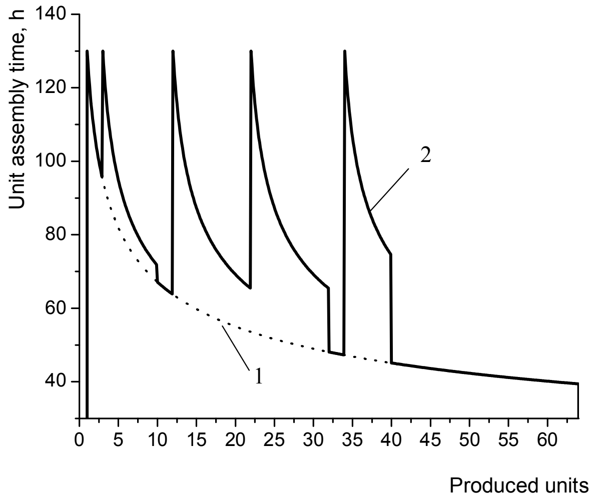

Fig. 4.14. Comparison of general unit learning curve (1.2) and the re-occurring learning curve (3.77) models

These results are very significant. The processing time is actually part of the technical side of production. However, non-technical factors (fluctuating order quantities, unstable demand and planning mistakes) affect this major technical parameter. Therefore, thoroughly applied production time reduction techniques (DFA, VAVE) might be turned into nothing for a variety on non-technical reasons. As can be seen from this experiment, planning mistakes cause time increments of assembly time and assembly time reductions by using technical measures results in percent's only. So learning time reduction appears to be extremely desirable in such a production environment to reduce cost and reduce the impact of planning mistakes.

4.7. Experimental research of process splitting

To test the presented technique practically, an experiment was performed at the same wiring harness company. A certain complex wiring harness product was selected. This product contains 570 different circuits as well as other components to be assembled onto the assembly jig. Prior to the experiment, some initial experimental data needed to be collected:

- Steady-state assembly time
- Stabilization point
- LC parameters

To calculate steady-state assembly time, a list of operations to complete this product was selected. Usually, these are the following assembly operations to be completed (please see table 4.9)

Table 4.9. Common assembly operations of the wiring harness.

ID	Operation
1	*Preparing of wires (number of different circuits)*
2	*Total amount of wires in the harness*
3	*Placing of different components in the harness*
4	*Placing of labels for assembly*
5	*Placing of braches/wires*
6	*Total length of placed branches/wires*
7	*Number of hoses to be pulled*
8	*Total amount of hose pulling*
9	*Assembling of large label*
10	*Assembling of small label*
11	*Plugging terminal into housing*
12	*Assembling of shrink sleeve*
13	*Assembling of cable ties*
14	*Pick-place of component*
15	*Inserting of additional component*
16	*Assembling of connector for electrical test*
17	*Number of packing's*
18	*Packing per meter harness*

Some of the operations defined in table 4.9 are not used in this particular product assembly. Although there are only 18 different operations (should be regarded as groups of operations), the complexity of each single operation attached to certain operation group has a unique mounting procedure. In other words, there might be 100 terminal insertions; however each insertion has its own mounting information (pin number, certain wire, etc.), which has to be learnt by assembly operation. Therefore, complexity is finally defined not only by the number of common (generalized) operations, but by the total number of all operations performed to assemble the final product.

In order to enable process modeling, the mathematical model of this product's assembly operations is created according to (2.7) and (2.9):

$$D = \begin{pmatrix} 1 & 0.9 & 1 & 1 & 0 \\ 2 & 0.1 & 1 & 1 & 0 \\ 3 & 0.5 & 1 & 1 & 0 \\ 4 & 0.15 & 1 & 1 & 0 \\ 5 & 0.06 & 1 & 2 & 0 \\ 6 & 0.03 & 1 & 2 & 0 \\ 7 & 0.1 & 1 & 2 & 0 \\ 8 & 0.05 & 1 & 2 & 0 \\ 9 & 0.12 & 1 & 2 & 0 \\ 10 & 0.1 & 1 & 2 & 0 \\ 11 & 0.04 & 1 & 2 & 0 \\ 12 & 0.2 & 1 & 2 & 0 \\ 13 & 0.06 & 1 & 2 & 0 \\ 14 & 0.04 & 1 & 2 & 0 \\ 15 & 0.06 & 1 & 2 & 0 \\ 16 & 0.1 & 1 & 2 & 0 \\ 17 & 0.5 & 1 & 2 & 0 \\ 18 & 0.07 & 1 & 2 & 0 \end{pmatrix}, \; P = \begin{pmatrix} 1 & 570 \\ 2 & 290.08 \\ 3 & 53 \\ 4 & 53 \\ 5 & 570 \\ 6 & 290.08 \\ 7 & 0 \\ 8 & 0 \\ 9 & 13 \\ 10 & 81 \\ 11 & 883 \\ 12 & 16 \\ 13 & 133 \\ 14 & 89 \\ 15 & 139 \\ 16 & 83 \\ 17 & 1 \\ 18 & 1.48 \end{pmatrix} \tag{4.3}$$

Following this, the necessary times can be calculated according to (2.10). The setup time is calculated first:

$$T_{Setup} = T(1) = \sum_{i=1}^{18} D_{i,2} D_{i,3} P_{i,1}^{1}, \text{ for } i: \; g_i = 1 = 9.6 \text{ h} \;, \tag{4.4}$$

$$T_{Run} = c_T = T(2) = \sum_{i=1}^{18} D_{i,2} D_{i,3} P_{i,1}^{1}, \text{ for } i: \; g_i = 2 = 2.0 \text{ h} \;. \tag{4.5}$$

These assembly times apply when single station assembly is employed. The main idea is to split the assembly process of this certain complex assembly into a more simple process flow (Fig. 4.15). However, each wiring harness is continuously divisible, i.e. if in a full wiring harness there are 800 terminals to be plugged into housings, this number can easily be distributed into two, three or more working stations. The process splitting distributes operations between assembly stations, but results in additional movement and sub-assembly time (calculation techniques of steady state assembly time of divided process and additional time were presented in the analytical part). Setup-time has a significant number of hours in this case. However, in this research only the cycle time is addressed. Setup time reductions can be performed by employing SMED techniques which are not the task of this research. Therefore, setup time will not be considered in further calculations. Also, in (2.7) there are additional definitions (number of operators and ergonomic factors), however they were not considered in this case.

Following necessary parameters collected by using other methods developed in this research and following values it was estimated that: $c_T = 2.0$ $c_\alpha = 0.32$, $c_{xc} = 1000$, $c_{Te} = 0.3$.

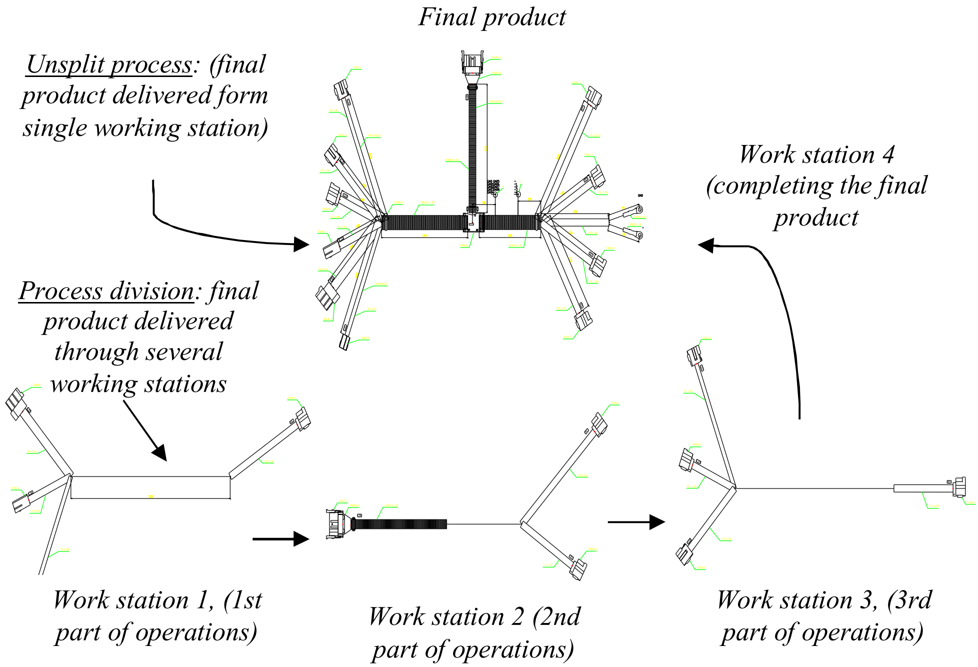

Fig. 4.15. Complex process splitting into 4 work stations

After the values of the parameters were estimated or calculated, several assembly experiments were performed to obtain the total production time, with the different number of divisions. There were five production runs performed to complete 50 pieces of wiring harnesses. In addition, for this certain product, the presented production time of a one dimension minimization procedure (3.98) was performed and optimal division number calculated for the selected batch size of n = 50. Calculation results and monitored production data is depicted in Fig. 4.16.

Additionally, measured production runs were used to track additional predictions of the model for different batch sizes. With the other production volumes the results are more or less the same, therefore, several values of n (5, 15 and 30) were selected for graphical representation. Production data and calculated minimums are presented in Fig. 4.16. The solution of the optimization problem in MathCAD is presented in Appendix 3.

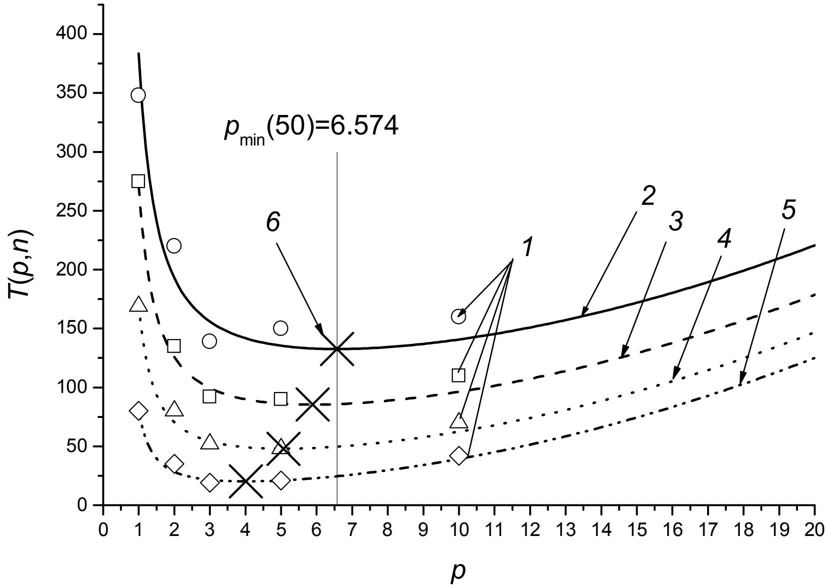

Fig. 4.16. Calculated total assembly time and production data. *1 – production data*, *2 -* $T(p,50)$ *; 3 -* $T(p,30)$ *; 4 -* $T(p,15)$ *; 5 -* $T(p,5)$ *; 6 – calculated minimum points.*
Parameters*:* $c_\alpha = 0.32$, $c_T = 2$, $c_{xc} = 1000$, $c_{Te} = 0.3$, $P = 20$

It can be seen that for this particular product the optimal division number exists for any different n, however only one of the experiments (*n* = 15) hit the minimum point. Although some of the data points lies on the model lines, in general, the proposed model is only roughly consistent with the measured production data. This complicates the main conclusion that monitored production data empirically validates the proposed model. Therefore, additional insights are needed to justify these discrepancies. When studying results from the Fig. 4.16, it is clear that higher values of *p* and *n* are resulting in worse actual datasets. This proposes an explanation that with higher labor divisions and production quantity the impact of random error increases. With higher labor divisions cause some discrepancies between work stations – more divisions, more possibilities for variable c_{Te} to vary. Also, higher production volumes n result in less learning time, therefore any abnormal operations (dropped housing during assembly, jammed terminal, tangled wires and etc.) are visible. Consequently, in a real setting the assumptions (3.80), (3.81), (3.82) and (3.84) might vary across individual divisions, some operators learn faster than others, some operations might be more complicated than another, although they are continuously divided. The outcome of these factors are learning slope discrepancies. Therefore, all these reasons lead to the result that in the general proposed model it is only roughly consistent with the real world production data. On the other hand, with all the simplifications and assumptions made, a huge amount of manual work and random errors, the model provided fairly good results.

Nevertheless, it is obvious that assembly of the complex products at a single working station is inefficient due to the large learning time and even splitting into two parts reduces the total production time significantly, however too many work stations ends up with increased total production time, therefore proving the existence of an optimum.

Further, detailed discussion regarding model applicability in general is presented. The main optimization problem (generalized) is presented in (3.92) with constrains (3.93), (3.94) and (3.95). However, this problem is quite difficult to solve and its application in manufacturing might be even more complicated. In addition, real setting variables appear to be estimated (or calculated) from production process monitoring rather than optimized by solving the optimization problem. Therefore, the optimization problem (3.92) can be simplified into at least a three different issues:

- one dimension optimization (to find optimal number of process divisions p)
- two dimensional optimization problem (to find optimal number of process divisions p and optimal production volume n)
- two dimensional optimization problem with normed function (to find p and n)

The simplest problem regarding its solution and applicability is the one dimensional optimization; to find the optimal number of divisions p. Since assumptions (3.80), (3.81), (3.82) and (3.84) are made, the results might be artefactual to some extent. One of the potential sources of artifact is the simplification made in (3.82). However, only if x_c is largely affected (hardly realistic, marginal situation) by labor division, such artifact would exist. Otherwise, slight variations of x_c will not significantly affect the final results. This optimization problem was tested in a real manufacturing situation. While ultimate results proposed only rough consistency with real datasets, it can on the other hand, be concluded that with such a simple model provided fairly good results. It is a very important result when considering short cycle production lines in an unstable manufacturing environment where there is simply less time for complex calculations and difficult combinatorial problems.

The two dimensional optimization problem is actually an extension of the one dimensional optimization. As it can be seen in Fig. 3.3, the time function is monotone and no minimum regarding production size n exist. In other words, minimum point exists only for certain production quantity n.

The second two dimensional optimization problem (with normed time) shows optimization possibility if a certain set of feasible combination of p and n exists (see Fig. 3.4). There is no global minimum, however the lowest value of the function $T_{\min}^{(n)} = \inf_{\bar{G}} T^{(n)} = T^{(n)}(p_g, n_g)$. The existence of the feasible subset G is also a very important result of this research, since it enables a combined proposed methodology with other models and methods, i.e. with mixed-model line balancing and etc. In other words, the feasible set G can be estimated by other methods and then additionally evaluated with the proposed model regarding the complexity aspects. This could be a background for further research.

4.8. Conclusions and main results of the section

In the section of experimental research, the created analytical models were tested on real production data from the manufacturing company. Following conclusions and generalizations regarding are derived:

1. Experimental results confirm that LC parameters can be accurately estimated by using deterministic methods.
2. Production data from the manufacturing company confirms the existence of steady state assembly time phase and proves the adequacy of proposed Plateau model (relative error does not exceed 5 %)
3. Different measurements of assembly situations confirm ALC models to be more versatile than CLC models
4. Experimental research proves the adequacy of the proposed learning-fatigue model (relative error value less than 5 %)
5. The proposed recursive learning model appears to be an effective measure to analyze and calculate impact of the product shifting from department to department.
6. Experimental research of the created process splitting model confirms the concept of learning time reduction by the simplification of complex products.

5. CONCLUSIONS

1. Appropriate research methodology is defined to perform research tasks. The wiring harness component, as a typical manual assembly product, was defined and its main terminology and components presented. In addition to this, different wiring harness assembly technologies were introduced and discussed in detail. It was found that the most important problem arises due to the selection of the appropriate number of process divisions for wiring harness assembly. Moreover, a mathematical model connecting manual wiring harness assembly operations and total assembly time was created. This model enables the modelling of an assembly process of wiring harness, helps to distribute assembly operations between process divisions and acts as a standard assembly time calculation methodology. The standard assembly time is the main parameter of the wiring harness and will be used for modelling. Further, the efficiency optimization problem is stated according to OEE methodology and in this problem the learning time is organized to be reduced, not only calculated. The general optimization problem will be used as a basis for further calculations. Finally, a methodology for raw production data monitoring is presented and three different monitoring techniques are introduced.

2. In this dissertation several newly created mathematically proved LC models have been developed. First of all, an adequate mathematically grounded Plateau LC with stabilization parameter is proposed. All the propositions that ground the method application correctness are proved. Having only one measured data point and stabilized time, LC can be unequivocally recovered. This will enable the prediction of the stabilization point in future. Currently presented method enables researchers to predict sequential production development. In order to predict the initial learning phase from the known stabilized time T_c LC parameters are also needed to be known, this can be achieved by using parameter estimation methods already presented in this dissertation.

The next proposed LC model is an almost learning curve model (ALC). The proposed ALC model is based on the solutions of special (with perturbation parameter) differential equations. The additional variable (perturbation parameter) enables much more versatility for production data fitting. Sufficient conditions for the perturbation parameter and other parameters of ALC that enable ALC to have all the necessary CLC properties are determined. In addition, usage of the ALC showed an additional insight into the analysis of learning and skill development analysis and modeling.

Another new LC model presented in this research is the learning-fatigue model. In the same way as the ALC model, the learning-fatigue was derived by perturbed differential equation, however perturbation parameter in this case represents the impact of the vibrations and repetitive motions to human performance. Additionally, the presented model enables the prediction of working loss of the operator and suggests rotating the assembly operator before injury starts to develop.

The last proposed LC model is the re-occurring learning model, which was used to evaluate planning impact to the total assembly time. Poor and chaotic planning leads to product shifting from one assembly department to another. In manual assembly this leads to unnecessary and re-occurring learning. Analytical tools help to avoid costly planning mistakes by calculating assembly time prior to product shift. Another important conclusion is that a technically adjusted and synchronized process can be distorted by poor planning.

3. In this research, LC deterministic parameter estimation methods are proposed. Application conditions of statistical and deterministic methods were clearly defined. In previous research, authors did not make a difference between statistical and deterministic methods. Implementation of the proposed methods is quite simple and straightforward; therefore they can be easily applied for real situations, since the learning curve differs for different operators, proposed methods can be applied to calculate and identify such variances between individuals quite easily with small data sets.

4. Manual assembly process efficiency optimization problem was stated and solved by employing the presented process splitting model. The model is based on the Plateau learning curve developed in this dissertation. Analytical research proves that there is a certain optimal process division when total assembly time would be minimal. Several optimization problems were stated and two of them have significant applicability. The first optimization problem is the one dimension optimization problem to find an optimal number of process divisions p. It was mathematically proved that this problem has a global minimum point. The second important optimization problem is the two dimension optimization problem with normed time function to find an optimal number of process divisions p and batch size n. Mathematical analysis showed that the objective function is monotone and there is no global minimum point. However, the lowest function value which is within feasible set G can be found. This opens an important possibility to combine this method with other assembly line balancing methods and lays the background for further research.

5. To test the adequacy of the proposed methods, production data monitoring was performed at a certain manufacturing company. First of all, the results confirm the premise of the plateauing phenomena, i.e. that the assembly time is decreasing to a certain steady-state limit T_c and thus proved the adequacy of the proposed Plateau LC with stabilization parameter. Acceptable relative error values (≤ 5 %) after the comparison with real production data proves adequacy of the model.

The proposed ALC model was tested on the wiring harness manufacturer production data. Three different groups of products were analyzed: repetitive orders taking place over a long time and random intervals; orders with low volume, prototype production; orders with high volume. For repetitive orders and high volume orders with steeper operating time improvement and further stabilization, the developed ALC model approximates data definitely better than classical LC. For small order production (prototypes, small series) both models LC and ALC delivered much the same approximation results. This proves the ALC model versatility. Therefore, the ALC model adequacy is approved (relative error does not exceed 5 %) and also the ALC allows a more accurate approximation of production data than the traditional CLC model.

Regarding the learning-fatigue model, the application results show that calculations by the proposed model approximates the manual assembly performance quite accurately, the relative error is quite small, i.e. 4.98 % and also does not exceed the 5 % limit. Therefore, this result approves the adequacy of the model.

Parameter estimation from the limited production data was performed and the results of estimation show that parameters can still be estimated by using mathematically grounded deterministic methods. The examples (for Crawford's and Wright's models) of parameter estimation were presented to illustrate how deterministic methods provided fairly adequate results by using a single random sample of production data (the majority of application results fell within the feasible relative error limit of 5 %). A comparison between both models showed that the Wright's data based methods provide more accurate results than the Crawford's data based ones, due to the fact that the Wright's model smoothens the data initially to some extent. Using the methods presented, LC parameters were estimated for a series of different wiring harnesses. It was found that those LC parameters for the same products differ sharply from each other, i.e. the LC parameters (slope coefficient and production time of the first unit) increase for more complex harnesses.

The one dimension optimization problem was tested in a real manufacturing environment. After fitting analysis, it is clear that the proposed model is only roughly consistent with the measured production data. On the other hand, with all the simplifications and assumptions made, huge amount of manual work and random errors, the model provided fairly good results. Therefore, it is obvious that an assembly of the complex products at a single working station is inefficient, due to the large learning time and therefore splitting into two parts reduces total production time significantly, however too many work stations results in an increased total production time, which finally, proves the existence of an optimum.

Since all the tasks of the dissertation are completed the goal of the dissertation is achieved.

REFERENCES

1. ABDEL-MALEK, L. L., and RESARE, L. J. Algorithm based decision support system for the concerted selection of equipment in machining/assembly cells. *International Journal of Production Research*, 2000, 38(2), 323–339.

2. ABDULLAH, T. A., POPPLEWELL, K. and PAGE C. J. A review of the support tools for the process of assembly method selection and assembly planning. *International Journal of Production Research*, 2003, 41(11), 2391–2410.

3. AGUIRRE, E. and RAUCENT, B. Economic comparison of wire harness assembly systems. *Journal of Manufacturing Systems*, 1994, 13(4), 276-288.

4. AGUIRRE, E., FERRIERE, L. and RAUCENT, B. Robotic Assembly of Wire Harnesses: Economic and Technical Justification. *Journal of Manufacturing Systems*. 1997, 16(3), 220-231.

5. AINSA, I., GONZALEZ, D., LIZARANZU, M. and BERNAD, C. Experimental evaluation of uncertainty in hand-arm vibration measurements. *International Journal of Industrial Ergonomics*. 2011, 41, 167-179.

6. ALLWOOD, J. M. and LEE, W. L. The impact of job rotation on problem solving skills. *International Journal of Production Research*. 2004, 42(5), 865–881.

7. ANZANELLO, M. J. and FOGLIATTO, F. S. Learning curve modelling of work assignment in mass customized assembly lines. *International Journal of Production Research*. 2007, 45(13), 2919-2938.

8. ANZANELLO, M. J. And FOGLIATTO, F. S. Learning curve models and applications: Literature review and research directions. *International Journal of Industrial Ergonomics*, 2011, 41, 573-583.

9. ANZANELLO, M. J. and FOGLIATTO, F. S. Scheduling learning dependent jobs in customized assembly lines. *International Journal of Production Research*. 2010, 48(22), 6683-6699.

10. BADIRU, A. B. Computational survey of univariate and bivariate learning curve models. *IEEE Transactions on Engineering Management*. 1992, 39(9), 176–188.

11. BALOFF, N. Extension of the Learning Curve - Some Empirical Results. *Operational Research Quarterly*. 1971, 22(4), 329-340.

12. BEN-TAL, A. and NEMIROVSKI A. *Lectures on Modern Convex Optimization. Analysis, Algorithms, and Engineering Applications*. SIAM, 2001.

13. BEVIS, F. V., FINNIEAR, C., TOWILL, D. R. Prediction of Operator Performance during Learning of Repetitive Tasks. *International Journal of Production Research.* 1970, 8(4), 293-305.

14. BLACK, J. T. Design rules for implementing the Toyota Production System. *International Journal of Production Research*, 2007, 45(16), 3639–3664.

15. BONZANI, P. J., MILLENDER, L., KEELAN, B. and MANGIERI, M. G. Factors prolonging disability in work-related cumulative trauma disorders. *The Journal of Hand Surgery*, 1997, 22(1), 30-34.

16. BOOTHROYD, G. and DEWHURST, P. *Product design for manufacture and assembly*. New York: CRC Press, Taylor & Francis Group, 2002.

17. CEDERLUND, R., ISACSSON, A. and LUNDBORG, G. Hand Function in Workers with Hand-Arm Vibration Syndrome. *Journal of Hand Therapy*. 1999, 12, 16-24.

18. COHEN Y., Absenteeism as a major cause of bottlenecks in assembly lines. *International Journal of Production Research*. 2012, 50(21), 6072-6080.

19. COHEN Y., VITNER G., SUBHASH C. and SARIN, S. C. Optimal allocation of work in assembly lines for lots with homogenous learning. *European Journal of Operational Research*. 2006, 168, 922–931.

20. CRAWFORD, J. R. *Learning curve, ship curve, ratios, related data*. Lockheed: Aircraft Corporation, 1944.

21. CROWLEY, A. Construction as a manufacturing process: Lessons from the automotive industry. *Computers & Structures*, 1998, 67(5), 389-400.

22. DAR-EL, E. M. *Human Learning: from Learning Curves to Learning Organizations*. Massachusetts: Kluver Academic Publishers, 2000.

23. DAR-EL, E. M., AYAS, K. and GILAD, I. A dual-phase model for the individual learning process in industrial tasks. *IIE Transactions*. 1995, 27(3), 265-271.

24. DENCE, T. P. A Brief Look into the Lambert W Function. *Applied Mathematics*. 2013, 4, 887-892.

25. DEWHURST, P. and BOOTHROYD, G. Early Cost Estimating in Product Design. *Journal of Manufacturing Systems*. 1988, 7(3), 183-191.

26. DONG, R. G., WELCOME, D. E., MCDOWELL, T. W., and WU, J. Z. Measurement of biodynamic response of human hand–arm system. *Journal of Sound and Vibration*, 2006, 294, 807–827.

27. DOOLEN, T. L. and HACKER, M. E. A review of lean assessment in organizations: An exploratory study of lean practices by electronics manufacturers. *Journal of Manufacturing Systems*, 2005, 24(1), 55-67.

28. ELKINS, D. A., HUANG, N. and ALDEN, J. M. Agile manufacturing systems in the automotive industry. *International Journal of Production Economics*. 2004, 91, 201-214.

29. ELMOSELHY, S. A. M. Hybrid lean–agile manufacturing system technical facet, in automotive sector. *Journal of Manufacturing Systems*. 2013, 32, 598-619.

30. ENGSTRÖM, T., JONSSON, D. and JOHANSSON, B. Alternatives to line assembly: Some Swedish examples. *International Journal of Industrial Ergonomics*, 1996, 17(3), 235-245. ISSN 0169-8141.

31. FOGLIATTO, F. S. and ANZANELLO, M. J. Learning Curves: The State of the Art and Research Directions. In M.Y Jaber (Ed.), *Learning Curves: Theory and models, and applications.* FL, Baco Raton: CRC Press, Taylor and Francis Group, 2011. pp. 3-23

32. FREIBOTH, M., FRIELING, E., HENNIGES, D. and SAAGER, C. Comparison of different organizations of assembly work in the European automotive industry. *International Journal of Industrial Ergonomics*, 1997, 20(5), 357-370. ISSN 0169-8141.

33. FUKUSHIMA, T. Precise and fast computation of Lambert W-functions without transcendental function evaluation. *Journal of Computational and Applied Mathematics*. 2013, 244, 77-89.

34. FULLERTON, R. R., KENNEDY, F. A. and WIDENER, S. K. Lean manufacturing and firm performance: The incremental contribution of Lean management accounting practices. *Journal of Operations Management*. 2014, 32, 414-428.

35. GABEL, T. and RIEDMILLER, M. Distributed policy search reinforcement learning for job-shop scheduling tasks. *International Journal of production research*. 2012, 50(1), 41-61.

36. GLOBERSON, S. and GOLD, D. Statistical attributes of the power learning curves model. International Journal of production research. 1997, 35(3), 699-711.

37. GLOBERSON, S., LEVIN, N. and SHTUB, A. The impact of breaks on forgetting when performing a repetitive task. *IIE Transactions*.1989, 21(4), 376-381.

38. GLOCK C.H., JABER M.Y. and ZOLFAGHARI, S. Production planning for a ramp-up process with learning in production and growth in demand. *International Journal of Production Research*. 2012, 50(20), 5707-5718.

39. GOLDBERG, M. S. and TOUW, A. *Statistical Methods for Learning Curves and Cost Analysis*. CNA Corporation, 2003.

40. GOOYERS, C. E. and STEVENSON, J. M. The impact of an increase in work rate on task demands for a simulated industrial hand tool assembly task. *International Journal of Industrial Ergonomics*, 2012, 42, 80-89.

41. GRIFFIN, M. J. *Handbook of Human Vibration*. Amsterdam: Elsevier, 1996.

42. GUNASEKARAN, A. Agile manufacturing: A framework for research and development. *International Journal of Production Economics*. 1999, 62, 87-105.

43. GUNAWAN, I. Implementation of Lean Manufacturing Through Learning Curve Modeling for Labor Forecast. *International Journal of Mechanical & Mechatronics Engineering*. 2010, 9(10), 46-52.

44. HANSON, R. and MEDBO, L. Kitting and time efficiency in manual assembly. *International Journal of Production Research*. 2012, 50(4), 1115–1125.

45. HAZIR, O. and DOLGUI, A. A decomposition based solution algorithm for U-type assembly line balancing with interval data. Computers & Operations Research, 2015, 59, 126-131.

46. HEATHCOTE, A., BROWN, S. and MEWHORT, D. J. K. The power law repealed: the case for an Exponential law of practice. *Psychonomic Bulletin and Review*. 2000, 7(2), 185–205.

47. HEIZER and RENDER. *Operations Management*. Pearson: Prentice Hall, 2006.

48. HELANDER, M. *Guide to human factors and ergonomics*. Boca Raton: CRC Press, Taylor & Francis Group, 2006.

49. HINDI, H. A Tutorial on Convex optimization. *Proceedings of the 2004 American Control Conference Boston*, Massachusetts, 2004, .

50. HIRIART-URRUTY, J. B. and LEMARECHAL C. *Fundamentals of Convex Analysis*. Springer, 2001.

51. HOLWEG, M. and PIL, F. K. *The Second Century, Reconnecting Customer and Value Chain through Build-to-Order*. Cambridge: The MIT Press, 2004.

52. HOLWEG, M. The genealogy of LEAN production. *Journal of Operations Management*, 2006, 25(2), 420-437.

53. IPA Magazine. Fraunhofer IPA Slovakia. 2015. [Viewed on 2015-05-18]. Access via www.ipaservis.sk: http://www.ipaservis.sk/slovnik_view.aspx?id_s=83/

54. JABER, M. Y. and BONNEY, M. A comparative study of learning curves with forgetting. *Applied Mathematical Modelling*. 1997, 21, 523–531.

55. JABER, M. Y. and BONNEY, M. Production breaks and the learning curve: The forgetting phenomenon. *Applied Mathematical Modelling*. 1996, 20(2), 162–169.

56. JABER, M. Y. and GLOCK, C. H. A learning curve for tasks with cognitive and motor elements. *Computers & Industrial Engineering*. 2013, 64(3), 866–871

57. JABER, M. Y. and GUIFFRIDA A. L. Learning curves for processes generating defects requiring reworks. *European Journal of Operational Research*. 2004, 159(3), 663-672.

58. JABER, M. Y. and GUIFFRIDA, A. L. Learning Curves for Imperfect Production Processes with Reworks and Process Restoration Interruptions. *European Journal of Operational Research*. 2008, 189, 93–104.

59. JABER, M. Y. and KHER, H. V. The dual-phase learning–forgetting model. *International Journal of Production Economics*. 2002, 76(3), 229–242.

60. JABER, M.,Y. and SIKSTROM, S. Numerical comparison of three potential learning and forgetting models. *International Journal Production Economics*. 2004, 92(3), 281–294

61. JANIAK, A. and RUDEK, R. A new approach to the learning effect: Beyond the learning curve restrictions. *Computers & Operations Research*. 2008, 35(11), 3727–3736.

62. JENAB, K. and LIU, D. A graph-based model for manufacturing complexity. *International Journal of Production Research*, 2010, 48 (11), 3383-3392.

63. KANNAN, S. and JAYABALAN, V. A new grouping method to minimize surplus parts in selective assembly for complex assemblies. *International Journal of Production Research*. 2001, 39(9), 1851-1863.

64. KHANCHANAPONG, T., PRAJOGO, D., SOHAL, A. S., COOPER, B. K., YEUNG, A. C. L. and CHENG, T. C. E. The unique and complementary effects of manufacturing technologies and Lean practices on manufacturing operational performance. *International Journal of Production Economics*. 2014, 153, 191-203.

65. KONZ, S. and JOHNSON, S. *Work Design: Industrial Ergonomics*. Fifth Edition. Scottsdale: Holcomb Hathaway Publishers, 2000.

66. KREINOVICH, V. Data Processing Beyond Traditional Statistics: Applications of Interval Computations. A Brief Introduction. *APIC-95 El Paso, Extended Abstracts*, A Supplement to the International Journal of Reliable Computing. 1995.

67. KRISTENSEN, B. J., FALLENTIN, N. and EKDAHL, C. Criteria for classification of posture in repetitive work by observation methods: A review. *International Journal of Industrial Ergonomics*. 1997, 19, 397-411.

68. KUCUKKOC I. and ZHANG, D. Z. Type-E parallel two-sided assembly line balancing problem: Mathematical model and ant colony optimization based approach with optimized parameters. *Computers & Industrial Engineering*. 2015, 84, 56-69.

69. KUMAR, R. S. and GOSWAMI A. EPQ model with learning consideration, imperfect production and partial backlogging in fuzzy random environment. *International Journal of Systems Science*. 2015, 46(8), 1486–1497.

70. LEE, D. A. *The Cost Analyst's Companion*. Virginia: McLean, 1997.

71. LI, G. and RAJAGOPALAN, S. A learning curve model with knowledge depreciation. *European Journal of Operational Research*. 1998, 105(1), 143-154.

72. LOWENTHAL, F. Learning Curves-an Axiomatic Approach. *Managerial and Decision Economics*. 1987, 8, 195-200.

73. MADAN, M., BRAMORSKI, T. and SUNDARRAJ, R. P. The effects of grouping parts of ready-to-assemble products on assembly time: an experimental study.

International Journal of Operations & Production Management, 1995, 15(3), 39-49. ISSN 0144-3577.

74. MATHIESON, J. L., WALLACE, B. A. and SUMMERS, J. D. Assembly time modelling through connective complexity metrics. *International Journal of Computer Integrated Manufacturing*, 2013, 26(10), 955-967.

75. MCCREERY, J. K. and KRAJEWSKI, L. J. Improving performance using workforce flexibility in an assembly environment with learning and forgetting effects. *International Journal of Production Research*. 1999, 37(9), 2031-2058.

76. MERRITT, T. W. and GOPALAKRISHNAN, S. An application of fuzzy set theory to the prediction of cumulative trauma disorders of the upper extremity. *International Journal of Industrial Ergonomics*. 1994, 13, 95-105.

77. MILTENBURG, J. U-shaped production lines: A review of theory and practice. *International Journal of Production Economics*. 2001, 70, 201-214.

78. MOLLEMAN, E., SLOMP, J. and ROLEFES, S. The evolution of a cellular manufacturing system – a longitudinal case study. *International Journal of Production Economics*. 2002, 75, 305-322.

79. MONFARED, S. K. and JENAB, K. Complexity analysis of an operation in demand-based manufacturing. *International Journal of Production Research*. 2011, 49(17), 5303-5315.

80. MONTANO, A., VILLALOBOS, J.R., GUTIERREZ, M. A. and MAR, L. Performance of serial assembly line designs under unequal operator speeds and learning. *International Journal of Production Research*. 2007, 45(22), 5355-5381.

81. MOSCHIONI, G., SAGGIN, B. and TARABINI, M. Prediction of data variability in hand-arm vibration measurements. *Measurement*. 2011, 44, 1679–1690.

82. MOULTRIE, J. and MAIER, A. M. A simplified approach to design for assembly. *Journal of Engineering Design*. 2014, 25(1-3), 44-63.

83. MURPHY, P., MCGORRY, R., TEARE, P. and BROGMUS, G. Design and performance of a manual task evaluator. *International Journal of Industrial Ergonomics*. 2000, 25, 257-264.

84. NEESE, R. and KONZ, S. Vibrometry of Industrial Workers: A case study. International *Journal of Industrial Ergonomics*, 1993, 11, 341-345.

85. NEIDIGH, R.O. and HARRISON, T.P. Optimizing lot sizing and order scheduling with non-linear production rates. *International Journal of Production Research*. 2010, 48(8), 2279-2295.

86. NEMBHARD, D. A. and OSOTHSILP N. Task complexity effects on between-individual learning/forgetting variability. *International Journal of Industrial Ergonomics*. 2002, 29(5), 297-306. ISSN 0169-8141.

87. NEMBHARD, D. A. The Effects of Task Complexity and Experience on Learning and Forgetting: A Field Study. *Human Factors*. 2000. 42(2), 272-286.

88. NEMBHARD, D. and UZUMERI, M. Experiential learning and forgetting for manual and cognitive tasks. International Journal of Industrial Ergonomics. 2000, 25, 315-326.

89. NEWELL, A. and ROSENBLOOM, P. S. Mechanisms of skill acquisition and the law of practice. In J. R. Anderson (Ed.), *Cognitive skills and their acquisition / 16th Symposium on Cognition*, Lawrence Erlbaum Associates, 1981, 1-51.

90. OHNO, T. *Toyota Production System: Beyond Large Scale Production*, Cambridge: Productivity Press, 1988.

91. OLVER, F. D. and LOZIER. *NIST Handbook of Mathematical Functions*. Cambridge University Press, 2010.

92. ONG, N. S., and BOOTHROYD, G. Assembly times for electrical connections and wire harnesses. *The International Journal of Advanced Manufacturing Technology*. 1991, 6, 155-179.

93. ORTEGA, J. Job rotation as a learning mechanism. *Management Science*. 2001, 47(10), 1361–1370.

94. OXENBURG, M. *Increasing Productivity and Profit Through Health and Safety*. Chicago: CCH International, 1993.

95. PAMPANELLI, A. B., FOUND, P. and BERNARDES, A. B. A Lean & Green Model for a production cell. *Journal of Cleaner Production*. 2014, 85, 19-30.

96. PAN, C. and SMITH, S. Case study: the impact of assembly reorientations on assembly time. *International Journal of Production Research*. 44(21), 2006, 4569–4585.

97. PANANISWAMI, S. and BISHOP, R. C. Behavioral implications of the learning curve for production capacity analysis. *International Journal of Production Economics*. 1991, 24(1-2), 157-163.

98. PANHALKAR, N., PAUL, R. and ANAND, S. Optimization of Automobile Assembly Process to Reduce Assembly Time. *Computer-Aided Design & Applications*. 2014, 11(S1), S54–S60.

99. PILLER, F. T. Mass Customization: Reflections on the State of the Concept. *The International Journal of Flexible Manufacturing Systems*. 2004, 16, 313–334.

100. PRABHU, G. V., HELANDER, M. G. and SHALIN, V. L. Effect of Product Structure on Manual Assembly Performance. *International Journal of Human Factors in Manufacturing*. 1995, 5(2), 149-161. ISSN 1045-2699.

101. PRYCE, J. D. A New Measure of Relative Error for Vectors. *SIAM Journal on Numerical Analysis*. 1984, 21, 202-215.

102. PUTZ-ANDERSON, V. *Cumulative Trauma Disorders*. London: Taylor & Francis, 1988.

103. RAMSEY, F., HAMADEA, R., F., JABER, M.,Y. and SIKSTRÖM, S. Analyzing CAD competence with univariate and multivariate learning curve models. *Computers & Industrial Engineering*. 2009, 56(4), 1510–1518

104. RICHARDSON, M., JONES, G. and TORRANCE, M. Identifying the task variables that influence perceived object assembly complexity. *Ergonomics*. 2004. 47(9), 945-964. ISSN 0014-0139.

105. RICHARDSON, M., JONES, G., TORRANCE, M. and BAGULEY, T. Identifying the Task Variables That Predict Object Assembly Difficulty. *Human Factors*. 2006, 48(3), 511-525. ISSN 0018-7208.

106. ROBERTS, C. *Ordinary Differential Equations – Applications, Models, and Computing*. Taylor & Francis Group, 2010.

107. SAMY, S. N. and ELMARAGHY, H. A model for measuring products assembly complexity. *International Journal of Computer Integrated Manufacturing*. 2010, 23(11), 1015-1027.

108. SETA, M. D., GRYGLEWICZ, S. and KORT, P. M. Optimal investment in learning-curve technologies. *Journal of Economic Dynamics & Control*. 2012, 36, 1462-1476.

109. SHINGO, S. *A Study of the Toyota Production System: From an Industrial Engineering Viewpoint*. Productivity press, 1989.

110. SILVEIRA, G. D., BORENSTEIN, D. and FOGLIATTO, F. S. Mass customization: Literature review and research directions. *International Journal of Production Economics*. 2001, 72, 1-13.

111. SINGER, G. GOLAN, M. and COHEN, Y. From product documentation to a 'method prototype' and standard times: a new technique for complex manual assembly. *International Journal of Production Research*, 2014, 52(2), 507–520.

112. SMUNT, T. L. and WATTS, C. A. Improving operations planning with learning curves: overcoming the pitfalls of 'messy' shop floor data. *Journal of Operations Management*. 2003, 21(1), 93-107.

113. SMUNT, T. L. Log-linear and non-log-linear learning curve models for production research and cost estimation. *International Journal of Production Research*. 1999, 37(17), 3901-3911.

114. SNYMAN, J. A. *Practical Mathematical Optimization: An Introduction to Basic Optimization Theory and Classical and New Gradient-Based Algorithms*. Springer. 2005.

115. SU, Q. Applying case-based reasoning in assembly sequence planning. *International Journal of Production Research*, 2007, 45(1), 29–47.

116. TANAKA, S. and MCGLOTHLIN, J. A conceptual quantitative model for prevention of work-related carpal tunnel syndrome (CTS). *International Journal of Industrial Ergonomics*, 1993, 11, 181-193.

117. TEPLITZ, C. J. *The Learning Curve Deskbook: A Reference Guide to Theory, Calculations and Applications*. New York: Quorum Books, 1991.

118. THOMOPOULOS, N. T. *Assembly Line Planning and Control*. New York: Springer, 2014.

119. TIACCI, L. Simultaneous balancing and buffer allocation decisions for the design of mixed-model assembly lines with parallel workstations and stochastic task times. *International Journal of Production Economics*. 2015, 162, 201-215.

120. UZUMERI, M. and NEMBHARD, D. A population of learners: A new way to measure organizational learning. *Journal of Operations Management*. 1998, 16, 515–528.

121. WAKKER, P. P. Explaining the Characteristics of the Power (CRRA) Utility Family. *Health Economics*. 2008, 17, 1329-1344.

122. WANG, A., KOC, B. and NAGI, R. Complex assembly variant design in agile manufacturing. Part I: System architecture and assembly modelling methodology. *IIE Transactions*, 2005a, 37, 1–15

123. WANG, A., KOC, B. and NAGI, R. Complex assembly variant design in agile manufacturing. Part II: Assembly variant design methodology. *IIE Transactions*, 2005b, 37, 17–33

124. WATERWORTH, C. J. Relearning the learning curve: a review of the derivation and applications of learning-curve theory. *Project Management Journal*. 2000, 31(1), 24-31.

125. WEBER, A. The Moving Assembly Line Turns 100. *Assembly Magazine*. 2013. [Viewed on 2015-05-18]. Access via www.assemblymag.com: http://www.assemblymag.com/articles/91581-the-moving-assembly-line-turns-100

126. WEIGERT, G., HENLICH, T. and KLEMMT, A. Modelling and optimization of assembly processes. *International Journal of Production Research*. 2011, 49(14), 4317–4333.

127. WELLS, R., MATHIASSEN, S. E., MEDBOD, L. and WINKELE, J. Time—A key issue for musculoskeletal health and manufacturing. *Applied Ergonomics*. 2007, 38, 733–744.

128. WOMACK, J. P., JONES, D. T. and ROOS, D. *The Machine That Changed the World*. New York: Rawson Associates, 1990.

129. WRIGHT, T. P., Factors affecting the cost of airplanes. *Journal of the Aeronautical Sciences*. 1936, 3(4), 122-128.

130. YANG, T., KUO, Y., SU, C. T. and HOU C. L. Lean production system design for fishing net manufacturing using lean principles and simulation optimization. *Journal of Manufacturing Systems*. 2015, 34, 66-73.

131. YELLE, L. E. The learning curve: historical review and comprehensive survey. *Decision Science*. 1979, 10(2), 302-328.

132. YOUNG, A. Volkswagen Is Hiring Engineers For New US Car Development Center At Factory Where Passat Is Made. *International Business Times*. 2015. [Viewed on 2015-05-18]. Access via www.ibtimes.com: http://www.ibtimes.com/volkswagen-hiring-engineers-new-us-car-development-center-factory-where-passat-made-1828052/

133. ZETTERBERG, C. and OFVERHOLM, T. Carpal tunnel syndrome and other wrist/hand symptoms and signs in male and female car assembly workers. *International Journal of Industrial Ergonomics*. 1999, 23, 193-204.

134. ZULCH, G., BRAUN, W. J. and SCHILLER, E. F. Analytical approach of determining job division in manual assembly systems. *International Journal of Production Economics*. 1997, 51, 123-134.

APPROBATION OF THE RESEARCH RESULTS

PUBLICATIONS

List of publications in ISI Web of Science

1. Kleiza, Vytautas; Tilindis, Justinas. Log-linear learning model for predicting a steady-state manual assembly time // Nonlinear analysis: modelling and control / Lithuanian Association of Nonlinear Analysis (LANA), Lithuanian Academy of Sciences. Vilnius : Institute of Mathematics and Informatics. ISSN 1392-5113. 2014, Vol. 19, no. 4, p. 592-601. DOI: 10.15388/NA.2014.4.5. [Science Citation Index Expanded (Web of Science); Index Copernicus; Inspec; Zentralblatt MATH], [IF (E): 1,099 (2014)]. [0,500]

2. Tilindis, Justinas; Kleiza, Vytautas. Impact of vibrations to human fatigue and manual assembly performance // Journal of vibroengineering / Vibromechanika, Lithuanian Academy of Sciences, Kaunas University of Technology, Vilnius Gediminas Technical University. Kaunas: Vibroengineering. ISSN 1392-8716. 2015, vol. 17, no. 7, p. 3919-3927. [Science Citation Index Expanded (Web of Science); Inspec; Academic Search Complete; Central & Eastern European Academic Source (CEEAS); Computers & Applied Sciences Complete; Current Abstracts; TOC Premier]. [IF (E): 0,617 (2014)]. [0,707].

List of publications in other databases (ISI Proceedings and etc.)

1. Kleiza, V; Tilindis, J. Application of Lean Production System Tools for Manual Assembly Process Improvement // MECHANIKA: proceedings of the 17th International Conference, April 12-13, 2012, Kaunas, Lithuania / Kaunas University of Technology. Kaunas : Technologija, 2012. ISSN 1822-2951. p. 130-133. [Conference Proceedings Citation Index]. [0,500]

2. Kleiza, Vytautas; Tilindis, Justinas. Problems of new technology and systems introduction into production: LEAN case // Intelligent technologies in logistics and mechatronics systems, ITELMS'2012 : proceedings of the 7th international conference, May 03-04, 2012, Panevėžys, Lithuania / Kaunas University of Technology Panevezys Institute, Panevezys Technology and Science Park, Intelligent Transport Systems, Poland, Tallin University of Technology, Riga Technical University. Kaunas : Technologija, 2012. ISBN 9786090205716. p. 105-109. [Conference Proceedings Citation Index]. [0,500]

3. Tilindis, Justinas; Kleiza, Vytautas. The effect of learning factors due to low volume order fluctuations in the automotive wiring harness production // Procedia CIRP : Robust manufacturing conference (RoMaC 2014). Amsterdam : Elsevier Science. ISSN 2212-8271. 2014, Vol. 19, 129-134. DOI: 10.1016/j.procir.2014.05.019. [Conference Proceedings Citation Index; Science Direct]. [0,500]

4. Tilindis, Justinas; Kleiza, Vytautas. Mathematical modeling of production operations and processes for product design and manufacturing // Applied Mechanics and Materials : Materials Engineering for Advanced Technologies (ICMEAT 2013). Stafa-Zurich : Trans Tech Publications Ltd. ISSN 1660-9336. 2014, Vol. 510, p. 243-247. DOI: 10.4028/www.scientific.net/AMM.510.243. [Conference Proceedings Citation Index]. [0,500]

List of publications in other peer-reviewed scientific journals

1. Kleiza, Vytautas; Tilindis, Justinas. The optimization of the overall learning dependent manual assembly efficiency // Intelligent technologies in logistics and mechatronics systems, ITELMS'2014 : proceedings of the 9th international conference, May 22-23, 2014, Panevėžys, Lithuania / Kaunas University of Technology. KTU Panevezys Faculty of Technologies and Business, Military University of Technology, Tallin University of Technology, Riga Technical University. Kaunas : Technologija. ISSN 2345-0088. 2014, p. 159-161. [Conference Proceedings Citation Index]. [0,500]
2. Tilindis, Justinas. Optimization of the total production efficiency by manual assembly processes modeling // Intelligent technologies in logistics and mechatronics systems, ITELMS'2015 : proceedings of the 10th international conference, May 21-22, 2015, Panevezys, Lithuania / Edited by Z. Bazaras, V. Kleiza. Kaunas: Technologija. ISSN 2345-0088. 2015, p. 270-272. [Conference Proceedings Citation Index]. [1,000].

LIST OF CONFERENCES

1. 4th International Conference “Changes in Social and Business Environment”. November 3-4, 2011, Kaunas University of Technology, Panevėžys Institute, Panevėžys, Lithuania. Presentation – “Implementation of LEAN Tools for Manual Assembly Process Management”
2. 17th International Conference "MECHANIKA-2012”. April 12-13, 2012, Kaunas University of Technology, Kaunas, Lithuania. Presentation – “Application of Lean Production System Tools for Manual Assembly Process Improvement”
3. 7th International Conference Intelligent Technologies in Logistics and Mechatronics Systems (ITELMS'2012). May 3-4, 2012, Kaunas University of Technology, Panevėžys Institute, Panevezys, Lithuania. Presentation – “Problems of New Technology and Systems Introduction into Production: LEAN Case”
4. 8th International Conference Intelligent Technologies in Logistics and Mechatronics Systems (ITELMS'2013). May 23-24, 2013, Kaunas University of Technology, Panevėžys Institute, Panevezys, Lithuania. Plenary presentation – “Manufacturing cost forecasting based on learning curve models”
5. 3rd International Conference on Materials Engineering for Advanced Technologies (ICMEAT 2013) December 31, 2013-January 2, 2014, Brisbane

Australia. Presentation – "Mathematical Modelling of Production Operations and Processes for Product Design and Manufacturing"

6. 9th International Conference Intelligent Technologies in Logistics and Mechatronics Systems (ITELMS'2014). May 22-23, 2014, Kaunas University of Technology, Panevezys, Lithuania. Presentation – "The Optimization of the Overall Learning Dependent Manual Assembly Efficiency"
7. 2nd CIRP Sponsored Robust Manufacturing Conference (RoMaC 2014) - Innovative and Interdisciplinary Approaches for Global Networks. July 7-9, 2014, Jacobs University, Bremen, Germany. Presentation – "The Effect of Learning Factors Due to Low Volume Order Fluctuations in the Automotive Wiring Harness Production"
8. 10th International Conference Intelligent Technologies in Logistics and Mechatronics Systems (ITELMS'2015). May 21-22, 2015, Kaunas University of Technology, Panevezys, Lithuania. Presentation – "Optimization of the Total Production Efficiency by Manual Assembly Processes Modelling"
9. 56th Conference of the Lithuanian Mathematical Society. June 16-17, 2015, Lithuanian Mathematical Society, Kaunas University of Technology, Faculty of Mathematics and Natural Sciences, Kaunas, Lithuania. Presentation – "An Almost Learning Curve Model"

APPENDICES

Appendix 1. Solution of the two dimensional optimization problem in MathCAD (normed function).

Definition of the parameters

$c_{\alpha} := 0.3 \qquad c_{Te} := 0.3 \qquad c_{xc} := 80 \qquad c_T := 2$

Formulation of the goal function

$\alpha(p) := \frac{c_{\alpha}}{p} \qquad T_{ICT}(p) := \frac{c_T}{p} \qquad T_A(p) := c_{Te}(p-1) \qquad xc(p) := c_{xc}$

$\beta(p) := T_{ICT}(p) \cdot xc(p)^{\alpha(p)}$

$y(x,p) := if\left(x < xc(p), \beta(p) \cdot x^{-\alpha(p)}, T_{ICT}(p)\right)$

The goal function (normed)

$$T(p,n) := \frac{p \cdot \left(\int_0^n y(x,p)\, dx + T_A(p)\right)}{n}$$

Solution of the optimization problem

Guess values

$x := 2 \qquad y := 2$

Constrains

Given

$2 < x < 3$

$$-\sqrt{1 - \frac{(x-2.5)^2}{0.25}} + 2.5 < y < \sqrt{1 - \frac{(x-2.5)^2}{0.25}} + 2.5$$

Solution

$$XY := Minimize(T, x, y) = \begin{pmatrix} 2.691 \\ 3.424 \end{pmatrix}$$

Function value

$T\left(XY_1, XY_2\right) = 3.597$

Appendix 2. Solution of the two dimensional optimization problem in MathCAD.

Definition of the parameters

$c_{\alpha} := 0.3 \qquad c_{Te} := 0.3 \qquad c_{xc} := 80 \qquad c_{T} := 2$

Formulation of the goal function

$\alpha(p) := \frac{c_{\alpha}}{p} \qquad T_{ICT}(p) := \frac{c_T}{p} \qquad T_A(p) := c_{Te}(p-1) \qquad xc(p) := c_{xc}$

$\beta(p) := T_{ICT}(p) \cdot xc(p)^{\alpha(p)}$

$y(x,p) := \text{if}\left(x < xc(p), \beta(p) \cdot x^{-\alpha(p)}, T_{ICT}(p)\right)$

The goal function

$T(p,n) := p \cdot \left(\int_0^n y(x,p)\,dx + T_A(p) \right)$

Solution of the optimization problem

Guess values

$x := 2 \qquad y := 2$

Constrains

Given

$2 < x < 3$

$-\sqrt{1 - \frac{(x-2.5)^2}{0.25}} + 2.5 < y < \sqrt{1 - \frac{(x-2.5)^2}{0.25}} + 2.5$

Solution

$XY := \text{Minimize}(T, x, y) = \begin{pmatrix} 2.506 \\ 1.5 \end{pmatrix}$

Function value

$T\left(XY_1, XY_2\right) = 6.618$

Appendix 3. Solution of the one dimensional optimization problem in MathCAD.

Definition of the parameters

$c_{\alpha} := 0.32$ $c_{Te} := 0.3$ $c_{xc} := 1000$ $c_T := 2$ $n := 50$

Formulation of the goal function

$\alpha(p) := \frac{c_{\alpha}}{p}$ $T_{ICT}(p) := \frac{c_T}{p}$ $T_A(p) := c_{Te}(p-1)$ $xc(p) := c_{xc}$

$\beta(p) := T_{ICT}(p) \cdot xc(p)^{\alpha(p)}$

$y(x,p) := \mathrm{if}\left(x < xc(p), \beta(p) \cdot x^{-\alpha(p)}, T_{ICT}(p)\right)$

The goal function

$T(p) := p \cdot \left(\int_0^n y(x,p)\, dx + T_A(p) \right)$

Solution of the optimization problem

Guess value

$x := 15$

Constrains

Given

$1 < x < 20$

Solution

$X := \mathrm{Minimize}(T, x) = 6.574$

Function value

$T(X) = 132.612$

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