LITHUANIAN ENERGY INSTITUTE

TOMAS IEŠMANTAS

RELIABILITY OF ENERGY NETWORKS CONSIDERING UNCERTAIN AND TIME-DEPENDENT DATA

Doctoral dissertation
Technological Sciences, Energetics and Power Engineering (06T)

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ABBREVIATIONS

ASME – The American Society of Mechanical Engineers

BMA – Bayesian Model Averaging

CAIDI - Customer Average Interruption Duration Index

CDP – Criteria – Dependent Poisson model

DIC – Deviance Information Criterion

EGIG – European Gas pipeline Incident data Group

FORM – First Order Reliability Method

GPD – Generalized Poisson Distribution

IAEA – International Atomic Energy Agency

LINEX – Linear Exponential loss function

MCMC – Markov Chain Monte Carlo

MLE – Maximum Likelihood Estimate

MTTF – Mean Time To Failure

NEB – National Energy Board (Canada)

OPS – Office of Pipelines Safety (USA)

PRA – Probabilistic Risk Assessment

SAIDI – System Average Interruption Duration Index

SORM – Second Order Reliability Method

TTF – Time To Failure

UKOPA – United Kingdom Onshore Pipelines operators' Association

NOTATIONS

$f(\cdot)$ - Probability density function

$F(\cdot)$ - Cumulative distribution function

$R(\cdot)$ - Reliability function

t - time (years, months, etc.)

θ - vector of unknown parameters

$\pi(\cdot)$ - posterior (or prior) probability distribution

$\lambda(t)$ - time-dependent failure rate

$L(X \mid \theta)$ - likelihood function given parameter values θ

Y - observed data sample

X - random variable

INTRODUCTION

Energy networks are highly complex systems which has a critical role in the development of the society. Reliability of complex systems is a characteristic of special importance. The proper functioning of the society is based on the premise that the systems supporting it are not supposed to fail in the near future. The cascading outages rolling through entire power network, explosions of natural gas transportation networks, bottlenecks of the main roads – these are but few examples of the infrastructure, which, if failed, severely affects the proper way of life of society.

Surely, the scientific community has taken great measures and devoted large amount of time to analyse and modify complex systems in order to increase their reliability. However, the practices and assumptions developed several decades ago still persist, interfering with the adequate and more realistic analysis of infrastructures. The assumption that systems or elements of those systems are independent of age (or more generally, of time spent in operating state) is almost omnipresent in the literature of reliability modelling of complex systems, even though the understanding of the role of time dependent characteristics increases.

Such situation has formed due to the inherent difficulties of mathematical models when time dependencies are taken into account. In addition, the dynamics of data uncertainty has its part in the inadequate reliability assessment of complex systems in current practices.

In this thesis, the author undertakes a task to resolve the aforementioned issues. The Bayesian statistical framework is taken as a starting point as well as a general view enveloping the whole work done in the following pages.

The aim and objectives of the work

The aim of this thesis is to develop and demonstrate the methodology that would enable comprehensive assessment, analysis and application of reliability of energy networks, when taking into consideration uncertain and time-dependent data.

Objectives:

1. To develop a methodology and create necessary mathematical models, enabling reliability assessment, when heterogeneous failure data and their uncertainty depend on time;
2. To examine and demonstrate the potential of methodology for the purpose of assessment of time-dependent reliability of energy networks;
3. By applying the methodology, to investigate the gas network reliability and its effect on the energy consumption at gas compressor stations;
4. To assess the time-dependent reliability of Lithuanian power and gas transmission networks, taking into account data heterogeneity.

Relevance of the thesis

1. In order to guarantee that the constantly increasing requirements for energy consumption efficiency and reliability of supply are optimally fulfilled, more comprehensive mathematical models are needed;

2. Currently available models for the reliability assessment are not sufficient for the case of network reliability, when taking into consideration the dependency on time and uncertainty of data;
3. Until now, little effort was devoted to the assessment of network reliability, while jointly investigating its effect on energy consumption in different parts of the network.

Scientific novelty of the thesis

The principles of energy network reliability assessment were established, when failure or fault data are dependent on the age of system, uncertain and heterogeneous.

Network reliability theory was supplemented by the mathematical models, which enable to take into consideration change of incident registration criteria and to assess more adequately sever cascading outages in the network.

Statements to be defended

1. The proposed methodology enables more comprehensive assessment of time-dependent reliability and uncertainty for heterogeneous energy networks;
2. Hierarchical generalization of Borrel-Tanner and data registration criteria-dependent Poisson models provides more accurate estimates of reliability characteristics for gas and power transmission networks;
3. Energy consumption of gas network compressor station is directly proportional to the failure rate of network pipelines;
4. Line outage and pipeline failure rates of Lithuanian power and gas transmission networks are decreasing and may be modelled as time-dependent even for the small data samples and under different data registration criteria.

Practical significance of the dissertation

The developed methodology enables to perform more complete network reliability analysis and its application, when characteristics of network elements change over time and failure data are heterogeneous. Estimates of time-dependent line outage and pipeline failure rates for Lithuanian power and gas transmission networks might be used in order to optimize maintenance and inspection programs. In addition, prediction of gas network reliability shall be used for the assessment of gas explosion risk dynamics and identification of its main factors.

Approbation of dissertation results

The results of this thesis were presented in seven international conferences and published in three journals indexed in the “Thomson Reuters” database “Web of Science Core Collection”. In addition, four publications were published in periodicals refereed in international scientific information databases; eleven publications were published in other scientific journals or conference proceedings.

1 REVIEW OF RELEVANT SCIENTIFIC LITERATURE AND METHODS

1.1 Time-dependent reliability of complex energy systems

This chapter of the thesis serves the purpose of putting the reader into a context where the questions of this research reside. First of all, complex systems and infrastructures are discussed: what is meant when speaking about infrastructures, their importance, reliability and its role in the development of such structures as well as the Lithuanian context.

Once the objects of this study are introduced, questions about how those structures tend to behave when the time aspect is placed into picture will be taken on. This is where the notion of "time-dependent" reliability is discussed.

As infrastructures are usually deployed over large area, spatial dependencies necessarily manifest. One of such manifestations results in heterogeneity in data from similar systems and components operating in different locations.

This chapter will be concluded by the discussion of relation between power consumption of infrastructures and reliability, the problem largely neglected in the research of systems.

1.1.1 Complex energy systems and infrastructures

Many of the systems that function around can be described as complex, having many parts that are related to each other in a nontrivial manner. Some of those parts can be described as simple components; others might be multicomponent systems. Some parts can have crucial role in a proper functioning of the system, some may be of a less significant influence, e.g. when failure has not even been noticed for a very long time. Surely, naming a certain system a complex one and the other one as non-complex is a subjective problem. However, when talking about complex systems, the following will be kept in mind:

Definition. Systems composed of many components or subsystems, which are interconnected and act in a nontrivial way, are called *complex systems*.

This definition although still quite loose is sufficient for the purposes of the work. It is also consistent with how the complex systems are understood in other disciplines [96], and how they are defined in [25] (see also a recent work on cascading outages in power network treated as a complex infrastructure [42]). For example, a light bulb is not considered a complex system, as it is composed of small amount of components. On the other hand, a nuclear reactor or computers are complex systems, as they contain many components and subsystems that are highly interrelated. Various networks are by definition also complex systems. For example, power network, gas transmission network, etc.

Some complex systems may be given a name of infrastructures, as they span over large areas of the countries and drive the development of the society itself. When appropriate, it will be switched from complex systems to networks or to infrastructures, depending on the kind of properties in mind. In this thesis, the main concern will be complex energy systems (a power network, a natural gas transmission network, etc.).

However, the importance of complex systems cannot overshadow the risks they pose: Fukushima disaster (2011), Gulf oil spill (2010), Egypt power outage (2014) – these are all very recent events, caused by failures in complex systems. Surely, disruptions of the operation of systems do come from other sources rather than just from internal faults. For example, a recent work in a Lithuanian context was done by Augutis, *et al.* [14], where they devised a system of energy security indicators, covering technical, economic and socio-political aspects. From the point of view of the European Union policy, a study [3] was performed on the electricity transmission network with the emphasis on the problems resulting due to the integration of different power networks into a so called pan-European system. This study identified three main aspects that needed a special attention: capacity to transmit electrical energy, maximum benefit from a given infrastructure, the development of the transmission technologies for environmental, operational, energy efficiency and investment considerations. Also, studies of energy infrastructures exist in terms of vulnerability (criticality), see, for example, [13, 12, 15]. Another important line of work is the analysis of ageing network systems [82].

However, the questions of system security or the economic benefits are not the main focus of this research. Rather, reliability (in its technical sense) will be the principal motif of this thesis. At this stage, only a “loose” definition of reliability will be given postponing the exact definition for later chapters:

Definition. *Reliability* of the system is the ability to carry out its intended function for a specified length of time.

Reliability of complex (energy) systems is very difficult to evaluate numerically, and various approaches exist. Those approaches could be classified as either physics-based or statistical data-based models. Both kinds have their respective advantages and disadvantages. Physics-based reliability assessment is an approach where models are based entirely on physical laws. For example, fracture mechanics (an extensive review of main principles is very well covered in [68]) employs notions like elasticity, material strength to infer about the possibility of crack occurrence in the system (such as the pipelines carrying natural gas in gas transmission network). Such models when applicable may very accurately predict faults. However, for large systems, it is not physically possible to analyse the entire system in terms of how material of components behaves under different conditions.

Statistical data-based models, on the other hand, enable analysing the entire system, no matter how large it is. Statistical observations reflect the stochastic nature of system failures and therefore provide a probabilistic view of the entire system reliability. Despite that, due to probabilistic nature of the models, a large degree of uncertainty is brought into the predictions of future failures. Therefore, there is a trade-off between the easiness of the analysis and its accuracy.

In addition, data-based models are more universal due to their generality as compared to physics-based approaches. Hence,

the assessment approach selected in this thesis is a statistical data-based analysis.

A specific tool selected for the inference will be presented and discussed in the next chapter.

There is one major assumption about the systems that is employed almost exclusively in the research regarding reliability of large systems. It is the hypothesis that those systems are time-independent, i.e., the intensity of faults or failures occurring in the system does not depend on how long the system has been operating. Yet, the reality is that the time that systems spend in operating state matters and has a significant impact on reliability. A large part of this thesis is focused on relaxation of this simplifying assumption in the general theory of complex system reliability.

1.1.2 Specification of "time-dependent" term

The preceding section was concluded with the almost omnipresent assumption that systems are independent of time that they spent in the operating state. This section is devoted to explanation and elaboration on the notion of time-dependent systems and time-dependent inference.

First of all, it should be clarified that this thesis does not engage in a discussion about time-dependent dynamics of the functions that system is "responsible" for. Rather, the focus is on the dynamics of failures of that system, i.e., how the rate of failures changes with time. Hence, the definition:

Definition. System can be classified as *time-dependent* if the data (or their certain explicit function), representing some particular aspects of the system, manifest a trend-like behaviour, or change points of the data with substantially different values than the previous ones exist. The validation of the trend can be carried out by appropriate statistical tests, or by expert insights (i.e., visual data inspection).

The trend of the data may be increasing, decreasing or a mixture of both. If the data do not depend on the time variable, it is said that the system is time-independent. Since failure rate, a measure of how often the system fails (it will be defined more rigorously later), is one of the most important features of the system, it will be almost exclusively referred to when talking about the time-dependent systems.

An increasing failure rate is usually the result of system deterioration, a continuous decrease of the system quality. Some components may deteriorate faster than others. For example, the pump of some fluid will age faster than the pipeline, through which that fluid is pumped (given that the fluid is not too corrosive). In fact, it is a general phenomenon: active systems age faster than passive systems. This is particularly important in nuclear power plant industry. Active components are much easier to analyse in terms of reliability, while passive components due to small failure rates provide much less data and therefore are much more difficult to analyse. In fact, it is an active research area in nuclear power plant engineering community (47, IAEA Safety Glossary (Version 2.0, 2006)).

A decreasing failure rate lies on the other side of the time-dependency line. This might occur in two instances. First, components, which are completely new, when put into operation, may exhibit a decreasing failure rate trend, known as burn-in phase [24]. It is the period when faults due to manufacturing processes are fixed by maintenance personnel. This stage continues for a certain period of time, when it settles down and failure rate becomes constant or time-independent, and then again after some time, it starts increasing due to degradation processes resulting in the so-called bathtub curve representing failure rate changes of the system lifetime.

The second instance, when failure rate may drop over time, is due to increasing efficiency of maintenance, materials and the technology itself. This is particularly significant in natural gas pipeline infrastructure, as new methods of leakage detection enable earlier maintenance. The increase of the efficiency of maintenance and detection overshadow the deterioration processes in pipelines.

Time-dependency is also exhibited when there is a significant change in the values of the failure rate. For example, failure rate of a system follows some constant value, when at some particular moment of time, it abruptly changes to another value. Even though such phenomena might be due to a certain physical change in the system, which goes unnoticed, it may be the case that the criteria of the system failure have changed. For example, a failure in the gas pipeline might be recorded when the leakage exceeds 3000 m^3 of gas, but after the change in regulations, only the spill of above 30000 m^3 of gas might be classified as a failure and recorded into a database. This results in the lower failure rate value, as it can be estimated from the information in the database.

The reader might wish to argue that time-dependent systems could be considered simply as *ageing* systems. However, the above-mentioned example of the changing data collection criteria clearly falls out of "ageing systems" definition scope. On the other hand, the burn-in period cannot be naturally defined as a reverse ageing, since during this period, there is an active interference into the system in order to fix the manufacturing flaws. Therefore, a time-dependent system notion has a wider scope/meaning as compared to the ageing system.

1.1.3 Reliability of complex energy systems

Since the main focus of this thesis is the power network and gas networks, the review of reliability assessment of complex systems will be concerned with (but not completely limited to) representation of research for those objects.

Although the risk or reliability assessment of network is not something very recent, it still lacks a coherent probabilistic treatment of uncertain data and parameter estimates. Failure data of separate nodes or connecting lines typically are pooled in one sample neglecting all the variability due to the difference in sources, and what is worse, raw estimates are plugged in to obtain the overall reliability assessment of the network. Lynn *et al.* started discussing it a while ago [80]; however, little effort has been made to advance in this matter.

Lynn *et al.* divided the state-of-the-art research of network reliability into those who compute reliability by combinatorial-like algorithms, assuming known failure probabilities but dealing with complex topologies, and those who apply statistical inference techniques to incorporate uncertainty of data sample, but work with a simple parallel and series configurations or some "easy" mixture of both. Extending the last case, another one could be added, a more recent, trend Complex Network Analysis

(CNA), when the reliability of the network is evaluated taking into account the topological relations between nodes and connecting lines. A compelling analysis of the view to power network as a complex network can be found in a survey by Pagani and Aiello [99].

The result of such scientific community division is that complex networks typically are left without any proper treatment of uncertainty. Having this in mind, Lynn *et al.* developed a methodology of any network reliability assessment by joining pivotal decomposition [16] together with Bayesian inference.

Following this work, reliability assessment of networks functioning in random environments started appearing, mostly due to Özekici [98, 97] (see also [29]). The main idea is that a system functions within fluctuating weather conditions, and the failure probability is conditioned on the state of the environment. Developed assessment methods allowed incorporating uncertainty due to different states of environment. However, the application areas were never extended to the power networks where various stochastic phenomena cause, e.g., random outages and cascades of outages.

Dobson carried out a more fruitful work regarding power network reliability, although just on the matter of cascading outages [40, 39, 37]. He suggested analysing cascades in a transmission network as a Galton-Watson branching processes [57]. Subsequent theoretical and practical works expanded this idea and analysed the behaviour of the branching propagation estimators [38, 110, 41]. Dobson and colleagues' work on transmission network outages enriched the available literature of probabilistic risk assessment of power networks. Although there are some recent works that deal with uncertainty in power system data (see [44, 34], where authors employ Bayesian approach to propagate uncertain states of systems to obtain importance ranking of individual system components), the uncertainty part in observed statistical reliability data somewhat left out of the main framework, i.e., the uncertainty of cascade propagation rate estimates or uncertainty introduced by the variability between different cascades was not addressed.

Natural gas pipeline networks, on the other hand, are still analysed mostly in terms of physics-based models, even though the state of the entire network cannot be reflected in the analysis of few specific samples of the pipeline material.

Reliability of pipelines carrying natural gas (or any other type of fluid) is often analysed in terms of cracks and leakages occurring through its entire body. External interference and material failure due to corrosion (48.4 % and 32.8, respectively, [46]) are the main causes of such "openings" in pipelines. Since external interferences cannot be analysed by physics-based approach, material failure is often taken as the central theme of pipeline reliability. It falls in fact under the notion of time-dependent reliability.

External interference, or third-party damage, refers to any accidental damage done to the pipe as a result of activities of personnel not associated with the pipeline.

Corrosion focuses mainly on a loss of metal from a pipe, although the concepts apply to many corrosion-like degradation mechanisms. While it is usually a very slow process, it does require the injection of energy to slow or halt the disintegration.

Corrosion is of concern because any decrease of pipe wall thickness invariably means a reduction of structural integrity and hence an increase in risk of failure [59]. Material can be put under strain also due to the environment within the pipeline; high pressures and chemically active fluid may add to the overall degradation of the pipeline and eventual crack, leakage or even rupture.

Pipeline corrosion is a well-researched topic, and there are plenty of publications as well as codes. The main line of the research is the determination of remaining strength in corroded pipelines (e.g., see ASME manual [9] or research in [1, 6, 133]). The methodology is based on the analysis of limit state by so-called FORM or SORM and is often under the umbrella of structural reliability (85, 43). Another line would be modelling of crack growth by various stochastic processes (see [73, 134, 112]). The main role here is given to stochastic processes and is often covered by the more general topic of probabilistic fracture mechanics [7].

At this point, it should be noted that even though power network and gas transmission network are both network-structured, the reliability evaluation is quite different. Since the purpose, materials, loads, operating and maintenance traditions are very different for these two networks, it is quite natural to expect different reliability assessment programs. However, this differentiation has its own disadvantages: reliability of gas transmission networks is focused on local analysis leading to the weakly understood global reliability aspects, while power network reliability assessment practices are devoid of uncertainty concept and its proper treatment.

There is an emerging line of reliability assessment for general complex systems, and is applicable for reliability evaluation of a power network or gas network, as they fall under the category of complex systems. The main idea is to decompose some particular complex system into larger units and apply the linear programming in order to obtain bounds on the system reliability in a reasonable time [36].

1.1.4 Heterogeneity of reliability data

There is an issue with system reliability data that deserves its own section. Consider a case when data are collected from components or systems operating in a wide area. For example, overhead electricity lines deployed through the entire area of the country, pipelines operating in varying soil and environmental conditions. Possibly, this is a certain class of components, for which the data are collected from nuclear power plants operating in different countries or continents. These are examples of cases when in order to assess reliability, the usual choice is to pool the data.

In reliability engineering applying statistical data analysis, it is a quite common practice to pool statistical information obtained from different sources. At a first sight, it seems quite natural to make such decision: if systems are similar and perform the same function, then samples should also be treated as similar. However, such aggregation causes loss of information about specificity of those systems and impact of their environment since assumption that they are identical or at least similar does not hold in every case, e.g., similar components can perform differing ageing behaviour when operating in different environmental and maintenance conditions, even though those components came from the same factory. However, here comes another issue, leading to decision to pool the data: highly reliable systems, especially if they are time-dependent, do not supply sufficient statistical information for long-term reliability investigation, so that aggregation would strengthen statistical inferences. For aggregated sample, one may get estimates with smaller uncertainty bounds than they would be in case between-source (separate samples) variability is considered. Those too optimistic uncertainty bounds will lead to less strict safety margins; this implies higher risk of exceeding safety limits.

This thesis will be especially concerned with the analysis of reliability data heterogeneity, since it has a significant influence on the overall system reliability and its prediction. Therefore, the definition is:

Definition. Failure data (or any other type of data expressing state of the system) obtained from different information sources are considered heterogeneous if different samples cannot be treated as produced by identically distributed random variables.

Two approaches could be used for the statistical analysis of heterogeneous data: mixed-effect and hierarchical Bayesian methods. The first one belongs to the classical frequentist method class, while the second one (as the name indicates) is a Bayesian method. Due to the reasons spelled in the next chapter, the hierarchical Bayesian methods will be dealt with.

1.1.5 Time-dependent reliability assessment

Reliability and safety of energy facilities, chemical factories, oil companies, etc. in many cases depends on their component reliability, which is mainly time-dependent. Component ageing is mainly caused by two impacts: operating conditions and technical inspection actions. Various ageing tests exists, see, for example, an excellent monograph by Lai and Xie [76], where they give lots of references about various aspects of aging identification. However, a case-specific consideration of ageing effects results into highly complex Probabilistic Risk Assessment (PRA), and the prevailing practice is to assume constant failure rate, sometimes even a non-conservative one. However, not taken into account at the time when safety margins are being estimated, ageing effect can cause failures or multiple damages at given non-standard operating conditions or breakdown situations.

The framework to deal with ageing in a coherent way depends on the type of data at hand. Statistical data can be represented as a pure failure sample, i.e., it can be failure counts in consecutive (not necessarily equal) time periods, records of component state (failed or not) at specific times, or it might be an evolution of component physical degradation characteristics, e.g., crack size.

There are a vast number of references, with models developed specifically to deal with the last type of data, known as degradation models (this class of models can be classified as physics-based). One of such comprehensive studies is a review of Singpurwalla *et al.* [123]; a part of this book is devoted to the stochastic diffusion-based state models and covariate induced hazard rate processes. Also, Wienke [128] reviewed a wide class of models called frailty models (see also Yashi and Mantan [135], where authors reviewed available literature on the likelihood construction for covariate induced hazard rate models when two cases are possible: unobserved and observed covariate processes – a class of time dependent frailty models).

Kelly and Smith [71] reviewed a state of the art of PRA with one of applications being related to ageing; also, valve leakage is a valuable addition to the topic. Moreover, some relevant papers are written by Colombo *et al.* [107], where authors present nonparametric estimation of time-dependent failure rate, or by Ho [59], where semi-parametric family of bathtub shaped failure rates is analysed. Although these approaches offer a rich class of failure trend models, it also requires larger samples of data. It is the price for the flexibility of these models.

Almost exclusively, time-dependent reliability assessment literature is concerned with ageing and deteriorating systems. The material strength changes as a component or the system ages; wear out and degradation are the notions referring to the ageing processes of systems. On the one hand, it is quite reasonable that scientific literature mostly covers this aspect of time-dependency; many systems in operation (power networks, gas networks, heating systems, power plants) are on the verge of their expected life, and a complete replacement would incur great financial costs; therefore, operators are looking for a way to deal with ageing and to extend reliability operation. On the other hand, the decreasing failure rate (as opposed to the increasing failure rate in case of ageing) is often neglected, and there is a lack of research done on it. The reasons may be improving materials, inspection or maintenance capabilities, etc.

Although there are many various ways of how to model ageing or degradation processed, just the most important and relevant to the study object will be discussed in more detail.

It is often important to observe the development of degradation processes characterized by gradual drift of the mean value. In such cases, the additivity assumption is often chosen as a base line. If the degradation measure (e.g., crack length) is denoted by $Z(t)$, then there is a model (see Figure 1):

$$Z(t) = z_0 + \sigma W(t - t_0) + \mu \cdot (t - t_0),\ t_0 \le t,$$

with z_0 being initial degradation, t_0 - beginning of the degradation, μ drift parameter, σ variance, and $W(t)$ standard Wiener process. For a more elaborate presentation of the model see [67] and references therein.

If the failure of the system is supposed to occur once degradation hits a certain level h, then the lifetime of the system is

$$T_h = \inf\{t \ge t_0 : Z(t) \ge h\},$$

with the probability distribution

$$f_{T_h}(t)=\frac{h-z_0}{\sqrt{2\pi\sigma^2(t-t_0)^3}}\exp\left(-\frac{\left(h-z_0-\mu(t-t_0)\right)^2}{2\sigma^2(t-t_0)}\right).$$

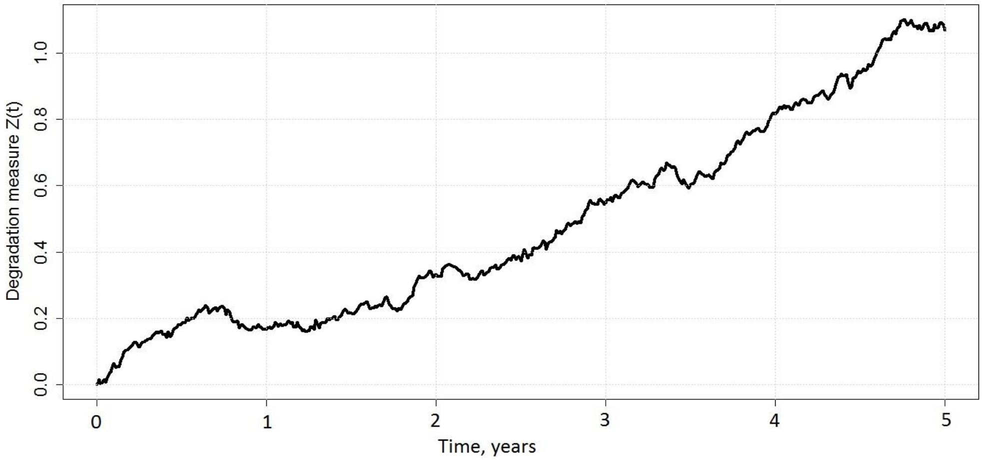

Figure 1. Example of the degradation stochastic process.

General degradation path, as considered in [84], may be analysed:

$$Z_r(t)=g(t,A),$$

where $A=(A_1,...,A_n)$ is a random vector with positive components and distribution function F_A and g is a specified increasing and continuously differentiable function of real time t. For example, let us assume that observed degradation $Z(t)$ deviates from real degradation $Z_r(t)$, then, the degradation model with noise $U(t)$ may be considered:

$$Z(t)=Z_r(t)U(t).$$

The noise is taken to be of the form $U(t)=\exp(\sigma W(t))$ (see Fig. 2).

However, these models allow the decrease of degradation measure, which is not likely. For example, crack size cannot decrease, i.e., crack cannot disappear without intervention with probability higher than zero. Therefore, one might consider a gamma process as a model for degradation measure. In this case, increments are independent and non-negative random variables with gamma distribution and identical scale parameter and a time-dependent shape function [100]. This process guarantees non-decreasing values of the measure. More information on the degradation models can be found in [93].

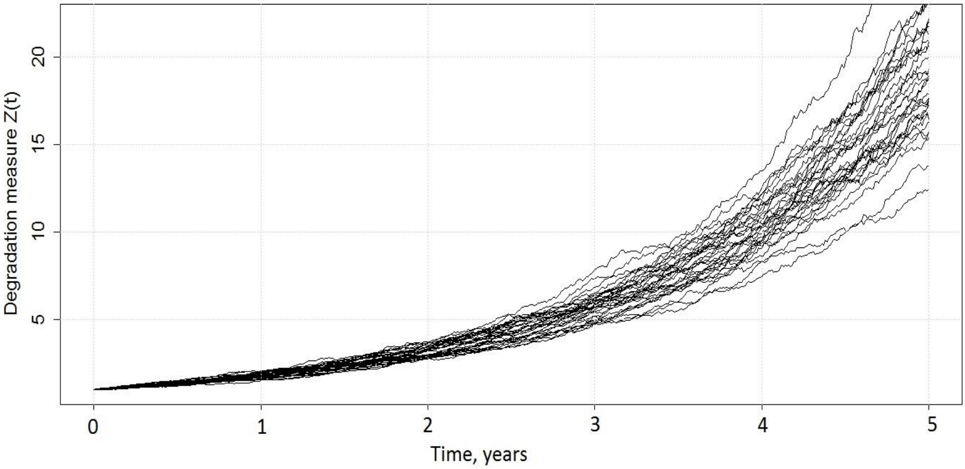

Figure 2. Example of the general path degradation process.

Considering the decreasing failure rate cases, i.e., cases when systems become more reliable with time, there are two types of such behaviour. The first one is the burn-in process. Burn-in process is a method of elimination of initial failures (infant mortality) of components before they are shipped to customers or put into field operation [62]. Another type is when operation, maintenance and inspection strategies improve resulting in less and less failures. For example, this is very clear in gas transmission pipelines: even though the material of pipelines deteriorates, increasing efficiency of inspection and maintenances controls failure rate in a decreasing manner. The literature on this second type of causes of decreasing failure rate is not researched, and there is no (at least, the author did not find any) literature addressing this issue. Therefore, burn-in analysis will be considered in more detail.

In case of insufficient burn-in, high initial failure rate causes high repair costs. On the other hand, excessive burn-in, the reduced failure rate will be at the cost of increased capital and recurring costs. The best time to stop the burn-in process for a given criterion to be optimized is called the *optimal burn-in*.

It is considered that one of the used for the aforementioned bathtub shaped failure rate distribution is that one can determine the optimum burn-in time when the initial failure rate is too high for the product to be released directly after the production.

There are many classes of probability distributions, exhibiting bathtub shaped failure rate. First, a rigorous definition of the bathtub shaped failure rate is necessary.

Definition. The failure rate function $\lambda(t)$ is said to have a *bathtub shape* if $0 \le t_1 \le t_2 \le \infty$ exist such that:

a) $\lambda(t)$ strictly decreases when $0 \le t \le t_1$,

b) $\lambda(t)$ is constant when $t_1 \le t \le t_2$,

c) $\lambda(t)$ strictly increases when $t_2 \le t$.

The time instants t_1 and t_2 are called the first and the second change points. The time interval $[0, t_1]$ is called the infant mortality period; the interval $[t_1, t_2]$, where $\lambda(t)$ is flat is called the normal operating life period (useful life period); the interval $[t_2, \infty]$ is called the wear-out period.

The following is a list of most commonly used distributions with bathtub shaped failure rate:

1. Gamma distribution:

$$f(t) = \frac{\beta^\alpha t^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta t}, \; \lambda^{-1}(t) = \int_0^\infty \left(1 + \frac{u}{t}\right)^{\alpha-1} e^{-\beta u} du.$$

Failure rate is decreasing when $0 < \beta \le 1$.

2. Weibull distribution:

$$f(t) = \alpha\beta(\alpha t)^{\beta-1} \exp\left(-(\alpha t)^\beta\right), \; \lambda(t) = \alpha\beta(\alpha t)^{\beta-1}.$$

Failure rate is decreasing whenever $\beta \le 1$.

3. Makeham distribution:

$$f(t) = \left(\alpha e^{\beta t} + \gamma\right) \exp\left(-\gamma t - \frac{\alpha}{\beta}\left(e^{\beta t} - 1\right)\right). \; \lambda(t) = \gamma + \alpha e^{\beta t}.$$

Failure rate is decreasing if $\beta \le 0$.

4. Burr XII distribution:

$$f(t) = \frac{kct^{c-1}}{\left(1 + t^c\right)^{k+1}}. \; \lambda(t) = \frac{kct^{c-1}}{\left(1 + t^c\right)}.$$

Failure rate is always decreasing if $c \le 1$, for $c > 1$ the slope is negative for $t^c > c - 1$.

There are many more classes of distributions with bathtub shaped failure rates, e.g., see a monograph of Lai and Xie [76].

1.1.6 Reliability and energy efficiency

In this thesis, an issue, which requires a broader context than the already provided one until this point, will be taken on. Part of this thesis, although of substantially smaller extent, will be devoted to the question of how concepts like system reliability and power consumption of that particular system are related to each other. Although this theme of the research is not something completely new, it is still mostly neglected, especially when it comes to larger systems.

Industrialized nations have become significantly more energy efficient since 1973. These countries adopted dozens of policies to improve energy efficiency in all sectors of their economies in the wake of the 1970s oil crises. These policies in turn contributed to a decline in energy intensity during the past 30 years. In addition, most industrialized countries have intensified their energy efficiency efforts in recent years as a part of their strategy to cut greenhouse gas emissions [52].

On the other hand, systems of nationwide importance and those with a potential to cause threats to the society are under the pressure to keep them as reliable as possible. For example, power network has to fulfil some agreed level of reliability. Otherwise, fatalities at gas transmission pipelines may be caused if the leakage evolves into an explosion.

Therefore, a problem, which will be just partially solved in this thesis, is formulated:

How to quantify the effect of a system reliability level on the energy efficiency characteristics of that system?

The research done in this direction, according to the authors' knowledge, is mostly confined to the electronic devices. For example, in [122], authors analyse the problem of optimization of reliability and power consumption in systems on a chip. The formulation of the optimization problem is based on semi-Markov decision processes. Assayad *et al.* [10] take on the problem of autonomous critical real-time systems (e.g., satellites) and use Pareto front methodology to optimize multiprocessor schedule in terms of its length, reliability and power consumption. Another example could be reliability and power consumption analysis in ultra-low power systems like pacemakers, defibrillators, etc. [81].

More in the spirit of this thesis, although still somewhat in another direction, is the discussion of Reynolds and Cowart [111], where they argue that the policies for power network energy efficiency, from the customer's point of view, are a least-cost reliability insurance.

The above references are as close to this thesis as it gets in the research of this topic. Further, it will be concentrated on the gas transmission pipeline network case, as a segment application part of this thesis will be devoted to the case of gas pipeline compressor station energy consumption and the influence of network reliability characteristics.

Gas pipeline networks are one of the most important infrastructures, counting their history since 1890, when leak-proof pipeline couplings were invented [89]. However, carrying gas from one point to another is not the only concern when planning construction of a new network or expanding the already existing one. There are questions like what size the pipelines should be, what kind and how many compressors are going to be needed, where to place them, what is the necessary topology of the network, etc. This is far from a complete list of planning questions, which beg to be answered. One attempt to answer them is through mathematical optimization, which is usually formulated as a cost minimization problem. The literature on this problem is numerous and quite diverse: spanning from simple single-sourced steady-state tree-like network [104] to time-dependent complex network with detailed models of compressors, valve stations, valves without remote access, electric motors, etc. [120]. Reviews of the optimization problem can be found in [28] and [117].

However, in author's knowledge, network reliability issue has never been considered explicitly in optimization problems, whether it is cost minimization, consumed fuel minimization, or another task. Questions like how severe is the influence of the network leakages over compressor power/fuel consumption, what will be the costs of maintenance works if one pipeline coating is chosen over another, how customers will be affected by failures in the network, etc. No research done to address these issues has been found except a brief mentioning of the importance of reliability, like in [66].

The effect of leakages on gas compressor energy consumption will be confined to. There is no *a priori* knowledge about this effect, as no research has been done, at least not that is known of. All that is known is that there is an increasing concern over the economic as well as environmental side effects of the gases released into the atmosphere due to cracks over the entire body of the network. It is just the nature of the beast: cracks appear and grow unnoticed causing a flow of natural gas out of its body. Methane, which is one of the main constituents of natural gas, has a hand in the climate change process by being more potent greenhouse gas than carbon dioxide is. As it has been speculated, few per cent of leakage of natural gas would offset the advantages of its lower CO_2 emissions [115].

On the other hand, consumers suffer from skittled-away gases as well. In the USA, it was estimated that consumers paid at least $20 billion from 2000 to 2011 for gas they never received [2, 5]. That is because utilities are not quick enough to replace old pipeline systems that are generously pouring gas into the environment through multiple cracks and holes in the network.

Having these points in mind, another related question can be raised: how much energy is wasted by pushing the additional amount of gas through the pipeline network that will eventually gush out of the system via one crack or another? Moreover, if this additional energy consumption is significant, how could it be reduced? The conclusions would be of great value in the network optimization problem that has been discussed before: the significant amount of additional energy means that objective functions in optimization have to take into account reliability of the pipelines.

1.2 Methods of Bayesian reliability assessment for energy systems

In this chapter, the theory, methods[1] and tools will be presented, as they are necessary to understand the methodology, validation and application parts of the thesis.

Heterogeneity, time-dependence, and uncertainty are three main factors considered for the selection of the paradigm, on which this thesis was built. The entire methodology and necessary tools will be built on the base of Bayesian paradigm. This chapter is devoted to presentation of the subjective probability paradigm, as employed by Bayesian methods, and how it translates into the theory of reliability. The information and mathematical models provided in the following sections can be found in various textbooks and monographs [56, 69, 94].

It should be noted that the author refrains himself from comparing frequentist and Bayesian paradigms, as it would require many more details about the axiomatic of the probability theory, and this would be out of the scope of this thesis.

1.2.1 Bayesian methods – a way to handle the uncertainty

The entire Bayesian statistical framework is based upon a simple and elegant theorem in probability theory. Bayes' theorem provides the mathematical means of combining information and data in the context of a probabilistic model in order to update a prior state of knowledge. This theorem modifies a prior probability, yielding a posterior probability via the expression:

$$\mathbb{P}(H \mid D) = \mathbb{P}(H)\frac{\mathbb{P}(D \mid H)}{\mathbb{P}(D)}. \tag{1}$$

The "dissection" of this equation leads to a four parts (see Table 1).

Table 1. Components of Bayes' theorem [69].

Term	Description
$\mathbb{P}(H \mid D)$	Posterior distribution, which is conditional upon the data D that are known related to hypothesis H
$\mathbb{P}(H)$	Prior distribution, from knowledge of hypothesis H that is independent of data D
$\mathbb{P}(D \mid H)$	Likelihood, of aleatory model, representing the process of mechanism that provides data D
$\mathbb{P}(D)$	Marginal distribution, which serves as a normalization constant

In the context of system reliability, risk and safety assessment, probability distributions are used to represent a state of knowledge regarding parameter values in the models, and Bayes' theorem gives the posterior distribution for the parameter (or

[1] Method in this dissertation will be understood as a series of steps taken to acquire knowledge. Model, on the other hand, should be considered as an approximation/simplification based on several assumptions about the phenomena at hand.

multiple parameters) of interest, in terms of the prior distribution, failure mode, and the observed data, which in the general continuous (the discrete version is not used in this thesis) form is written as:

$$\pi_1(\theta \mid x) = \frac{L(x \mid \theta)\pi_0(\theta)}{\int L(x \mid \theta)\pi_0(\theta)d\theta}. \qquad (2)$$

In this expression, $\pi_1(\theta \mid x)$ is the posterior distribution for the parameter of interest, $L(x \mid \theta)$ is the likelihood and $\pi_0(\theta)$ is the prior knowledge that is available prior to the observation of the data and can be classified (somewhat arbitrarily) as *informative* and *non-informative*. The posterior distribution is the most important, as all the inferences about the reliability of the system are made by using it as a base.

Since the methods used in this thesis are quite complex, numerical approximations will be employed, in which the marginal distribution is not important. Therefore, posterior distribution will often be written as:

$$\pi_1(\theta \mid x) \propto L(x \mid \theta)\pi_0(\theta),$$

meaning that the posterior distribution is known up to normalization constant.

Quantitative assessment of uncertainty is almost omnipresent in the analysis of the functioning of systems (subsystems, components, etc.). What concerns reliability, availability, maintainability or reparability, essentially time-related probability assessments of successful functioning are studied.

Therefore, there is a need for concepts as well as a tool for handling uncertainty properly. What is more, for handling it in such a way that there is a natural linkage with rational decision-making. Moreover, such concepts and tools must embrace different kinds of information, familiar to those confronting uncertainty in complex systems: on the one hand, the background knowledge and judgements of the subject matter expert; on the other hand, experimental or online data in the form of counts or measurements of failures or failure times.

The observed data are usually dealt with by forming the likelihood function. However, the incorporation of expert judgement or any other prior information requires at least some knowledge of subjective probability.

Here, probability is taken for an expression by a person with specified knowledge about an uncertain quantity. The philosophical position is that one’s personal uncertainty is expressed through the probability of an uncertain quantity, given one’s state of knowledge, real or assumed. This is termed the subjective or personal attitude to probability.

A view that one’s knowledge about any aspect of the world may be insufficient to describe it with a complete certainty enables treating model parameters as random variables. In classical statistics, parameters are just fixed quantities, while in Bayesian statistics, they have probability distribution, just like the observed data. It does not mean that there is no true model, it just means that one’s knowledge about parameters is insufficient. Thus, probability distribution is put on its possible values.

1.2.2 Handling subjective information

The subjective Bayesian approach is based on a very simple collection of ideas. One is certain about many things concerning the problem at hand and can quantify uncertainty as probabilities for the quantities he or she is interested in and conditional probabilities for observations. When statistical data related to failures, faults in various fusion components arrive, Bayes' theorem tells how to move from the prior probabilities to new conditional probabilities.

Since many systems have more than one predecessor, there is a considerable amount of prior knowledge carried by the testers/operators, who are familiar with the ways in which systems and components at hand may fail, both from general considerations and from testing and field reports [56].

The expert judgement elicitation is of great importance in cases when there are no reliability data for the system, as it has not even been built yet. Besides, the statistical information from other similar systems is not very extensive. Hence, there are lots of uncertainties, and one way to reduce them is to obtain "soft" data by asking experts about the probable numbers of failures of one or another component, the expected time to failure, etc.

Any attempt to describe some aspects of the world in terms of mathematical apparatus has its own advantages and disadvantages. Prior distributions are not an exception.

Advantages [106]:

- Subjective prior distributions are proper (they integrate to one);
- A subjective prior distribution is generally well behaved analytically. The effect of a subjective prior distribution on the posterior is as if there were additional replications of the data;
- Subjective prior distributions may be used to introduce the informed understanding, beliefs, and experience of a scientist or decision maker into the Bayesian analysis of a problem to take advantage of this additional information about the phenomenon under study;
- Often there is insufficient information on a problem to solve it by objective Bayesian or frequentist methods. The subjective Bayesian approach may be the only way to find an acceptable solution, since it brings additional information to bear on the problem.

Disadvantages:

- It is not always easy to assess subjective prior distributions (those based upon earlier knowledge), because it is not always easy to translate the prior knowledge into a meaningful probability distribution. The parameters that index subjective prior distributions are called hyper-parameters. They must be assessed, and the assessment problem is sometimes difficult, especially in case of multi-parameter problems.

- Results of a Bayesian analysis that used subjective prior distribution are meaningful to the particular analyst, whose prior distribution was used, but not necessarily to other researchers, because their prior distribution might have been different.
- A subjective prior distribution might not be very tractable mathematically.

Suppose the expert has some prior beliefs about one of the reliability measures, e.g., failure rate of gas pipeline of particular size, and he would like to bring these prior beliefs into the analysis in a formal way. It will often suffice to permit one's prior beliefs to be represented by some smooth distribution that is a specific member of a family of subjective prior distributions.

One such family is called a natural conjugate family. This family is chosen largely on the basis of mathematical convenience and is closed under transformation from prior to posterior.

Another family of mathematically convenient subjective prior distributions is called the family of exponential power distributions. The richness of hyper-parameters within the class provides a great flexibility of expression of prior views. The third family of subjective prior distributions is the class of mixed prior distributions.

The recipe for developing an appropriate family of natural conjugate prior distribution in general is to first form the likelihood function, and then:

- Interchange the roles of the random variable and the parameter in the likelihood function;
- "Enrich" the parameters of the resulting density kernel of a distribution, that is, make their values perfectly general and not dependent upon the current data set;
- Identify the distribution corresponding to the resulting kernel and adjoin the appropriate normalizing constant to make the density integrate to on

Table 2. Some natural conjugate prior distribution families.

Sampling distribution		Natural conjugate prior distribution
1	Binomial	Failure probability follows a beta distribution
2	Negative binomial	Failure probability follows a beta distribution
3	Poisson	Mean follows a gamma distribution
4	Exponential with mean λ	λ follows a gamma distribution
5	Normal with known variance but unknown mean	Mean follows a normal distribution
6	Normal with known mean but unknown variance	Variance follows an inverted gamma distribution

The class of exponential power distributions for a random variable x has the density

$$g(x)=\frac{k}{\sigma}\exp\left(-0.5\left|\frac{x-\theta_0}{\sigma}\right|^{2/(1+\beta)}\right),$$

where $\theta\in R,\beta\in(-1,1],\sigma>0,k^{-1}=\Gamma(1+0.5(1+\beta))2^{1+0.5(1+\beta)}$. It can be found that

$$\mathbb{E}[x]=\theta_0$$

and

$$Var[x]=2^{1+\beta}\sigma^2\frac{\Gamma[3/2(1+\beta)]}{\Gamma[1/2(1+\beta)]}.$$

The parameter β reflects the degree of non-normality of the distribution.

1.2.3 Non-informative way of Bayesian modelling

In the subjective prior elicitation part, it has been mentioned that some sub-models of the larger model can be fed with sample of statistical data, while others require expert opinion elicitation. Now, let us assume the sub-models, which can be supplied with data sample, but the expert opinion elicitation is not possible. Such case could be in more complex reliability models, where ageing or other time-dependencies of failure rate are assumed. Hence, it can be said that there is no additional a priori information about the model parameters.

In order to perform a full Bayesian analysis, the prior distributions have to be selected so that the influence on the results is as small as possible. This can be achieved by employing so-called non-informative prior distributions. The justification of one prior distribution form over the other is basically technical considerations, like transformation invariant property in Jeffreys' prior.

In objective Bayesian analysis, prior distribution is selected by formal rules, which are described briefly in the following text. So, there are ten main principles or sets of formal rules, by which non-informative priors (also known as reference priors) may be constructed: Laplace's Principle of insufficient reason, invariance, data-translated likelihoods, maximum entropy, the Berger-Bernardo method, geometric considerations, coverage matching methods, Zellner's method, decision-theoretic and Rissanen's method.

- When the parameter space is finite, then Laplace's rule, known as the Principle of Insufficient Reason, is to use uniform distribution over the parameter support region. However, one could run into a so-called partitioning paradox, when inconsistently applying the rule to all coarsening and refining of the parameter space simultaneously. However, such issues arise rarely, and in reliability theory, due to the nature of problems, that should not occur frequently. A generalization to continuous spaces is a flat prior, which is an improper distribution, i.e., does not integrate to one.
- Invariance issues that occur while using flat priors led to the development of mathematical apparatus for constructing parameterization of invariant

prior distributions. One such group of invariant priors is Jeffreys' priors obtained as a negative expectation of log-likelihood Hessian matrix. Invariantness under group action on the model led to the left and right Harr measure. Right Harr measure is generally preferred in practice.

- Box and Tiao [104] introduced the notion of "data-translated likelihoods" in order to motivate the use of uniform priors. The likelihood is data-translated if it is of the form $L(\phi)=g\left[\phi-t(y)\right]$ for some real valued functions $g(\cdot)$ and $t(\cdot)$. Box and Tiao recommended using uniform prior distribution on the one-dimensional parameter space. However, this approach is quite restrictive, since only normal and gamma families of distributions yield exactly data-translated likelihoods.
- If the parameter space is discrete, then one can select a prior distribution by maximizing the so-called entropy functional, which represents the amount of uncertainty implied by the prior distribution, i.e., priors with larger entropy are regarded as less informative. However, this approach is prone to the same partitioning paradox as the Laplace's prior.
- Bernardo (followed by an extensive research by Bernardo and Berger) suggested measuring the missing information in the experiment by using Kullback-Leibler divergence and then finding the distribution that maximizes this measure when sample increases to infinity. Under certain regularity conditions, it turns out that Berger-Bernardo priors are identical to Jeffreys' prior.
- Yet, another way to choose a prior is to characterize it through the notion that they ought to "let the data speak for themselves". From this viewpoint, it may be considered desirable to have posterior probabilities agree with sampling probabilities. For example, Chang and Willegas proved that for group transformation models, repeated-sampling coverage probabilities and posterior probabilities agree when the prior on the group is right-Haar measure. However, sometimes it is not possible to get exact agreement and instead one might seek approximate agreement.
- Zellner suggested choosing prior that maximizes the difference. He called such prior the maximal data information prior (MDPI). In location-scale problems, it leads to right-Harr measure, while in binomial model, it results in distribution, which has tail behaviour between that of Jeffreys' and uniform priors. However, Zellner's method is not parameterization invariant.

There were some attempts to define prior distribution through the framework of decision theory. For example, Hartigan defined an unbiased decision for some loss function and in one-dimensional problems obtained general form for prior distribution that satisfies asymptotic unbiasedness. In particular, if the loss function is Hellinger distance, the prior assumes Jeffreys' prior form. Another approach is by Kashyap, who considered the selection of a prior as a two-person zero-sum game against nature.

For problems where parameters are discrete and assume values from the countable set, one can use Rissanen's method. It is based on coding theory. The following table lists common choices of non-informative prior distribution in some reliability assessment problems.

While theoretically justifiable, non-informative prior distributions in practice might give some problems (like non-integrable posterior distribution), in numerical Bayesian model realization, so-called diffuse priors are used. These diffuse priors usually are approximations of non-informative ones, with the property of being proper probability distribution functions.

Table 3. Common non-informative priors in various reliability problems.

Model	Non-informative prior
Success/failure data	
$Binomial(\pi)$	$Beta(0.5, 0.5)$
Failure count data	
$Poisson(\lambda)$	$\lambda^{-1/2}$
Time to failure data	
Exponential λ	λ^{-1}
Weibull (c, α)	$1/c\alpha$
Gamma (α, β)	$\pi(\alpha, \beta)$
Inverse-Gaussian (λ, ψ)	$1/\lambda\psi$
Normal (μ, σ)	σ^{-2}

1.2.4 Basic notions of Bayesian reliability

In order to present a strict understanding of reliability and related notions, this section is devoted to mathematical definitions and notations.

Assume a system whose state at time t is described by $X(t) = (X_1(t), ..., X_n(t))$, a vector-valued random variable. For example, $X(t)$ may be the one-dimensional variable taking on value 1 corresponding to the functioning state, and 0 if the system is in failed state. $X(t)$, being a random variable, will be governed by a distribution function.

$$F(x_1, ..., x_n; t).$$

Explicitly, $F(x_1, ..., x_n; t)$ equals the probability of the following event:

$$X_1(t) \leq x_1, ..., X_n(t) \leq x_n.$$

The *time to failure* (TTF) of an item means the time elapsing from when the item is put into operation until it fails for the first time. $t = 0$ is set as the starting point. TTF will be treated as a random variable T with probability density function $f(t)$ and distribution function (or *unreliability function*)

$$F(t) = \mathbb{P}[T \le t] = \int_0^t f(u)\,du.$$

The *reliability function* of an item is defined by

$$R(t) = 1 - F(t) = \mathbb{P}[T > t]. \tag{3}$$

Hence $R(t)$ is the probability that the item does not fail in the time interval $(0,t]$, or equivalently, the probability that the item survives the time interval $(0,t]$ and is still functioning at time t.

Failure rate function $\lambda(t)$ is defined as follows:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} \frac{1}{R(t)} = \frac{f(t)}{R(t)}. \tag{4}$$

The failure rate function is a function of the life distribution of a single item and an indication of the "proneness to failure" of the item after time t has elapsed. The failure rate in its most general form may be described by a so-called "bathtub" curve as in Fig. 3.

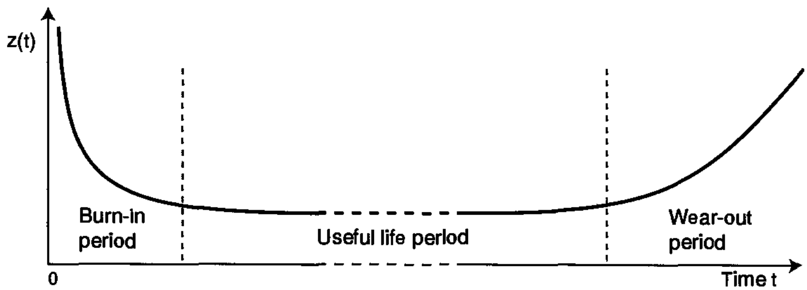

Figure 3. Bathtub curve describes the most general form of the failure rate function.

The following relation between reliability function and failure rate function can be proved:

$$R(t) = \exp\left(-\int_0^t \lambda(u)\,du\right). \tag{5}$$

Another important quantity in the theory of reliability is Mean Time To Failure (MTTF), and may be calculated by:

$$MTTF = \mathbb{E}[T] = \int_0^\infty t f(t)\,dt. \tag{6}$$

To put it in a Bayesian context, some parametric form of the failure rate parameterized by θ is considered:

$$\lambda(t) = \lambda(t;\theta).$$

Following Bayesians, θ is considered to be a random variable on its own with prior distribution $\pi_0(\theta)$. Updating it with observed data sample $X(t)$ posterior distribution is obtained:

$$\pi_1(\theta \mid X(t)) \propto \pi_0(\theta) L(X(t) \mid \theta). \qquad (7)$$

Since θ is a random variable, it follows that $\lambda(t)$ is a random variable as well. Its distribution is obtained just by the general rules of probability. This also means that the reliability function, which is a function of the failure rate and of the parameter θ, is also a random variable with its own probability distribution. Therefore, as random variable, reliability function may be prescribed credibility intervals at each time slice t. In other words, the fact that parameters are treated as random variables translates into advantage to have every notion in reliability theory treated as having a stochastic origin.

There are many other notions in the theory of reliability, like availability, maintainability, inspectability, etc. However, these and other are well covered in many well-written textbooks [22, 16]. In addition, there is a large amount of literature regarding reliability modelling in terms of Markov and semi-Markov processes. All the above-mentioned characteristics can be calculated by viewing the system state as a (continuous or discrete) stochastic process going through different states. A resourceful monograph was written on this particular topic by Limnios and Oprisan [78]. However, the theory would be hard to apply in case of networks as the number of states of the network increase very rapidly as the number of components grow. Therefore, in this thesis Markov chains will not be used except in a model of transformer deterioration.

However, since network reliability concept will be used in this thesis most extensively, it needs a greater elaboration.

Let S be a simple system, i.e., it can be considered a unit with regard to all relevant reliability and maintenance aspects. In addition, S can be in one of the two states: state 1 (available, operating, and functioning), state 0 (unavailable, failed, down). Hence, the indicator variable for the state of the system at time t is a binary variable:

$$Z(t) = \begin{cases} 1, & \text{if S is available at time } t, \\ 0, & \text{otherwise.} \end{cases}$$

The indicator variables of system S and its elements e_i, $i = 1, 2, ..., n$, are denoted as

$$Z_s = \begin{cases} 1, & \text{if S is available,} \\ 0, & \text{otherwise,} \end{cases} \quad z_i = \begin{cases} 1, & \text{if } e_i \text{ is available,} \\ 0, & \text{otherwise.} \end{cases}$$

Since z_s and z_i are random variables, probabilities p_s and p_i exist with

$$z_s = \begin{cases} 1, & \text{with probability } p_s, \\ 0, & \text{with probability } 1 - p_s. \end{cases} \quad z_i = \begin{cases} 1, & \text{with probability } p_i, \\ 0, & \text{with probability } 1 - p_i. \end{cases}$$

The states of the elements uniquely determine the state of the system. Hence, for different systems S different functions φ exist with

$$Z_s = \varphi(z_1, z_2, ..., z_n).$$

φ is called the *structure function* of S, which also expresses the probability of failure ($z_s = 0$) or success ($z_s = 1$) once element state variables z_i are exchanged with probabilities of those states.

Binary system S with structure function $\varphi(z)$ is called *coherent* if

a) Every element is relevant
b) $\varphi((0_i, z)) \leq \varphi((1_i, z))$, i.e., the state of the system does not improve if some elements fail.

The following function will be called the reliability of system S

$$R(t) = \varphi(p_1(t), p_2(t), ..., p_n(t)).$$

(8)

In Bayesian context, since probabilities $p_i(t)$ are treated as random variables, the reliability function is also a random variable; therefore, the expected reliability is calculated as follows:

$$\overline{R}(t) = \mathbb{E}_{\pi(p_1(t), p_2(t), ..., p_n(t)|X(t))}\left[\varphi(p_1(t), p_2(t), ..., p_n(t))\right], \qquad (9)$$

where the expectation is taken with respect to posterior probability distribution.

1.2.5 Loss function role in Bayesian reliability

Bayesian methods are widely used in reliability engineering problems. However, after posterior distribution are at hand, practitioner needs to make decision on how to summarize this posterior distribution, i.e., which estimator to employ, so that expressed characteristics would be justified. The usual practice is to employ posterior mean as estimator, which originates from squared error loss function. This choice is usually made due to resulting expression simplicity and maybe tradition. But this is not necessarily the best choice.

To resolve such vagueness, firstly, one needs to consider loss function, which evaluates penalty $W(\theta, \delta)$ associated with decision δ, when the parameter takes the value θ [114]. Although the choice of this function is not trivial, squared error loss function

$$W(\theta, \delta) = (\delta - \theta)^2$$

is the usual choice. The estimator under this loss function will be denoted by $\delta_{s.e.l}(\theta_i)$ and its expression is as follows:

$$\delta_{s.e.l}(\theta_i) = \mathbb{E}[\theta_i]. \qquad (10)$$

Nevertheless, this does not mean that this loss function is a reasonable solution.

In reliability assessment overestimation of failure rate causes a decision-maker to take actions to increase system reliability that in turn causes economic losses. However, in case of underestimation, reliability (and safety) level will be too optimistic and will result in higher risk of safety limits exceedance. Thereof overestimation is not as severe as underestimation and quantitatively should be penalized less. From these considerations, one could make a conclusion that loss function should be asymmetric when estimating failure rate.

One of the possibilities of asymmetric losses is so-called LINEX (abbreviations of LINear-Exponential function) function introduced by Varian [127]:

$$LINEX\left(\Delta\right)=be^{a(\Delta)}-c\left(\Delta\right)-b,$$

where $\Delta=\hat{\theta}-\theta,\ a,c\neq 0,b>0$ and $\hat{\theta}$ is an estimate of parameter θ. For multi-parameter problem [138]:

$$LINEX\left(\Delta\right)=\sum_{i=1}^{N}b_i\left(e^{a_i\Delta_i}-a_i\Delta_i-1\right),$$

where $\Delta=\left[\Delta_1,...,\Delta_N\right],\ N$ – number of parameters.

Then instead of usually applied posterior mean $E\left[\theta_i\right]$, the Bayes estimator under LINEX loss function for multi-parameter problem is

$$\delta_{LINEX(a)}\left(\theta_i\right)=-\frac{1}{a}\ln\mathbb{E}\left[e^{-a\theta_i}\right], \tag{11}$$

where values $a\in\{1;-1\}$ will be used in the further investigation. In case of $a=1$ (identified as $LINEX\left(1\right)$) underestimation is preferred, while for $a=-1$ (identified as $LINEX\left(-1\right)$) overestimation is a more acceptable choice.

Since introduction, it has also used in reliability theory. Some of references are: Schiibe [119] used LINEX loss function to estimate failure rate of integrated circuit, Basu and Ebrahib [17] considered the problem of estimating reliability function under various distributional assumptions, Singh and Srivastava [63] obtained estimates of reliability function and shaped parameter of finite range failure time model, Soliman [124] examined theoretical possibilities of reliability estimation in a generalized life-model with samples of various sizes, obtained from Burr – XII type distribution and compared maximum likelihood estimator (MLE) and LINEX estimators.

1.3 Author's contribution to the relevant scientific field

The main contribution of the author is the development of the methodology, which enables time-dependent assessment of reliability of energy networks when taking into consideration uncertain and time-dependent data. In addition, the author developed a Data Registration Criteria-Dependent Poisson model for the purpose of taking into account changes of event registration criteria when performing reliability assessment of natural gas transmission network. This resolved the issue when large parts of statistical data had to be dismissed due to the different rules used to record those parts of the sample. By enabling the use of all data, independent of the criteria in use, the author demonstrated the advantages of the model by several applications on different pipeline networks.

Borrel-Tanner model was generalized, so that heterogeneity of outages of overhead power network lines could be incorporated into the prediction of severity of cascading outages. This is particularly important since the model generated higher probabilities of large cascading outages. Current models provide too small probabilities, as compared to the real data, and therefore fail to properly model the phenomena of the cascading outages.

Another significant contribution of the author is the analysis of the effect of the network reliability to the gas network compressor station energy consumption. Even though the existence of correlation between the energy consumption and the gas transmission network reliability is *a priori* clear, the author showed that large amounts of gas wasted to the atmosphere due to various cracks in the overall body of the network pipelines and that additional gas compressor energy required to compensate for that wasted gas is directly proportional to the failure rate of gas pipelines.

2 METHODOLOGY FOR LONG-TERM RELIABILITY ASSESSMENT OF COMPLEX ENERGY SYSTEMS

2.1 Bayesian assessment of time-dependent complex systems

In order to assess the reliability of a given network, one should be able to correctly model different parts of it – edges, nodes. Therefore the first step in network reliability assessment is to construct a model as close to reality as possible to separate network elements. In this section, general steps necessary to perform assessment of systems' time-dependent reliability are described. Since power network or gas transmission networks are just separate cases of complex systems, those particular systems will not be referred to unless it is necessary to give some specific example. This approach enables the construction of assessment methodology in a more general manner.

If the evidence (e.g., trend tests, expert opinions, etc.) shows possible time-dependencies in statistical failure data, then one can consider a trend model for failure rate. Several examples for failure rate trend function $\lambda(t)$ can be:

1. piecewise constant $\lambda(t)=\lambda_i, t\in[\tau_i;\tau_{i+1}]$,
2. linear $\lambda(t)=\theta_1+\theta_2 t,$
3. exponential (log-linear) $\ln\lambda(t)=\theta_1+\theta_2 t,$
4. power-law (Weibull) $\lambda(t)=\theta_1 t^{\theta_2}$,
5. Xie and Lai model
   $\lambda(t)=\theta_1\theta_2(\theta_1 t)^{\theta_2-1}+\theta_3\theta_4(\theta_3 t)^{\theta_4-1}, 0<\theta_2<1, \theta_4>1,$ [132],
6. generalized Makeham $\lambda(t)=\theta_1 e^{\theta_2 t}+\dfrac{\theta_3}{1+\theta_4 t}$, [76].

where $\lambda(t)$ is time-dependent failure rate, t – time variable, θ – parameters, which influence the shape of failure rate trend function.

Linear ageing is the most simple and obvious natural way to give a first-order approximation to changes in the failure rate, but it does seem to have a practical disadvantage. Wolford *et al*. [131] analysed two data sets using several functional forms for $\lambda(t)$; one such analysis is reported by Atwood [11]. They found that a Bayesian posterior distribution for $\lambda(t)$ was approximately lognormal when a log-linear or power-law model was used for $\lambda(t)$, but this was not a case when a linear model was used. Apparently, the approximate lognormality required a much larger data set when linear ageing was assumed in comparison to the case when power-law or exponential ageing were assumed.

Usually, the timing for failure rate consideration is divided into three distinct periods: burn-in period, useful life, wear-out period. For such general trend, the linear, power-law or exponential distribution cannot provide desirable fit to the data. For this reason, models that have ability to shape-up the entire bathtub curve are needed, and Xie & Lai or generalized Makeham trend models can be applied (Lai and Xie [76]). Notwithstanding flexibility of these models, it can be quite difficult to apply them by using frequentist framework, because due to number of parameters and lack of sufficient data to estimate them. Classical statistical methods are ill-suited for this situation, leading in such cases to excessively wide confidence intervals.

Some authors (Radionov [116], Okazaki and Aldemir [95], Radulovich, Veseley and Aldemin [108]) introduce a threshold of age at which ageing is assumed to begin. Then $\lambda(t)$ is assumed to be a constant before the threshold of age is attained, and to increase according to one of the above formulas afterwards. The threshold is generally unknown and must be estimated from the data. Thresholds cause difficulty in classical statistics, because the assumptions for the asymptotic theory of maximum likelihood estimation are typically violated. Therefore, it is difficult to quantify the uncertainty in the estimate of the threshold. However, even in this case, the application of Bayesian modelling, for instance, using simulation package such as BUGS® (Lunn, Thomas and Best [64]), is still possible.

Time-dependent reliability (whether it is due to ageing, or due to increasing efficiency of maintenance technologies) can be thought of as time-dependent change of beliefs about reliability parameters (e.g., failure rate). Change of beliefs occurs not just due to new failure data or other information (mentioned above), which becomes available in time, but also it continuously changes due to flow of time and evolution of probabilistic beliefs.

One of the difficulties of Bayesian inference is inability to deal with changes of time-dependent parameter as a continuous process. This problem partially can be overcome by considering ageing (or degradation) as step-wise process, which is constant in certain period of time and has value jump in another period. Mathematically this can be expressed as a jump process:

$$d(t) = \sum_{i=1}^{N-1} 1_{\{t_i < t < t_{i+1}\}} d(t_i), \qquad (12)$$

where $d(t)$ is any model of characteristic (or reliability parameter) under consideration, and constant $d(t_i)$ is value of characteristics at each time period $t_i; N$ – number of time intervals.

Model of characteristic $d(t)$ can have any functional form. It can be linear, Weibull, or some other form. Depending on the adopted formula, $d(t)$ will be based on vector of parameters $\Theta = (\theta_1, ..., \theta_m)$:

$$d(t) = d(t;\theta). \qquad (13)$$

If an analyst considers more than one model, then indexation is used for different models, i.e. $d_i(t;\Theta_i)$, where d_i denotes i^{th} model with Θ_i vector of parameters.

Modelling conception introduced above allows interpreting distribution of parameters as time-dependent. If prior knowledge and beliefs about failure rate or another reliability parameter are represented by probability density distribution $\pi(\Theta)$ and statistical observations have likelihood $L(Y \mid d(t))$, where $Y=(y_1,...,y_N)$ is sample of observations, then, according to Bayes theorem, time-dependent beliefs about the reliability parameter are expressed as a posterior distribution:

$$\pi(\Theta \mid Y,t)=\frac{\pi(\Theta)L(Y \mid d(t,\Theta))}{\int_\Omega \pi(\Theta)L(Y \mid d(t,\Theta))d\Theta}. \tag{14}$$

Assume that parameters $\theta_1,...,\theta_m$ are a priori independent, then, according to the definition of independent random variables, a prior distribution of Θ can be expressed as:

$$\pi(\Theta)=\prod_{i=1}^{m}\pi_i(\theta_i),$$

where $\pi_i(\theta_i), i=\overline{1,m}$ are priors for components of vector Θ.

If data set contains n statistical observations, then posterior distribution is represented as:

$$\pi(\Theta \mid Y,t)=\frac{\prod_{i=1}^{m}\pi_i(\theta_i)\prod_{j=1}^{n}L(y_j \mid d(t_j,\Theta))}{\int_\Omega \prod_{i=1}^{m}\pi_i(\theta_i)\prod_{j=1}^{n}L(y_j \mid d(t_j,\Theta))d\Theta}. \tag{15}$$

Model checking may be performed either by considering a hypothesis testing with so-called posterior p-values or with Deviance Information Criterion (DIC). Posterior p – values are defined as follows:

$$p=\mathbb{P}\left[D(Y^{rep},\theta)>D(Y,\theta) \mid Y\right],$$

where Y^{rep} – is the replicated data that could have been observed or, to think predictively, as the data that would appear if the experiment that produced data Y were replicated in the future with the same model. The rule of thumb is that p value should be close to 0.5 [92, 86]. $D(Y,\theta)$ is a discrepancy measure. The following discrepancy measures (most often found in literature) will be used:

$$D_1(Y,\theta)=\sqrt{E\left(Y-E(Y \mid \theta)\right)^2},\ D_2(Y,\theta)=\sum\frac{\left(Y_i-E[Y_t \mid \theta]\right)^2}{Var[Y_t \mid \theta]}. \tag{16}$$

DIC is defined as follows [125]:

$$DIC=-2\mathbb{E}_{\Theta|X}\left[\log L(X \mid \Theta)\right]+p_D, \tag{17}$$

Θ denotes all model parameters, and $p_D=\frac{1}{2}Var\left[-2\log L(X \mid \Theta)\right]$ is a so-called effective number of parameters (not necessarily an integer), which is typically lower than the nominal number of parameters due to borrowing of strength under the hyper-density [32] and can be regarded as a measure of model complexity. DIC is a measure of fitness of predictive distribution, and since the prediction of reliability is one of the main goals for practitioners, it is an appealing model characteristic for present reliability assessment purposes. On the other hand, this criterion has some disadvantages. One of them is that it is not a normed criterion; therefore, there is no way to tell from the value alone whether the model is a good fit to the data or not. DIC for one model has to be compared to DIC of another model, and a significantly lower DIC value will be indicated a better fit. Since DIC is basically a function of log-likelihood function, it may have negative as well as positive values and (theoretically) of any magnitude. This poses a question of how to decide whether the values of DIC for a different model differ significantly. No answer could be provided. For example, if DIC values for two models are 35 and 20, then this is quite a significant difference, while values 1000 and 1015 are not so significantly different. However, the rule of thumb should be that the smaller the value, the better is the fit to the model as compared to another model choice. In addition, the visual inspection of replicated data should always be carried out together with quantitative measure while choosing between different measures.

In addition, usually it is the case when several trend models fit data almost equally well, i.e., possible set of "good" models contain more than one possibility

$$\mathrm{M} = \left(d_1\left(t,\Theta_1\right),\ldots,d_r\left(t,\Theta_r\right)\right),$$

where $d_i\left(t,\Theta_i\right), i=\overline{1,r}$ are models which were considered as having a good fit. In such circumstances, uncertainty of modelling cannot be handled appropriately within classical statistical framework. As noticed by Hoeting [60], standard statistical practice ignores model uncertainty. Data analysts typically select a model from some class of models and then proceed as if the selected model had generated the data. This approach ignores the uncertainty in model selection, leading to over-confident inferences and decisions that are more risky than one thinks they are. As an alternative approach, model averaging is more correct, because it takes into account a source of uncertainty that analyses based on model selection ignore (Kulinksaya, Morgenthaler and Staudte [74]). Also, according to Hoeting [60], Bayesian model averaging advantages include better average predictive performance than any single model that could be selected.

Then, let us denote $A(t)$ as failure rate averaged over set of models M. Considering the present notation, posterior probability of averaged time-dependent failure rate can be represented as:

$$p\left(A(t)\,|\,Y\right) = \sum_{j=1}^{r} p\left(A(t)\,|\,Y, d_j\left(t,\Theta_j\right)\right) p\left(d_j\left(t,\Theta_j\right)|\,Y\right), \tag{18}$$

where $p\left(d_j\left(t,\Theta_j\right)|\,Y\right)$ is posterior probability distribution of model $d_j\left(t,\Theta_j\right)$ given the set M of available models; $p\left(A(t)\,|\,Y, d_j\left(t,\Theta_j\right)\right)$ is posterior distribution of quantity $A(t)$ under model $d_j\left(t,\Theta_j\right)$. Posterior probability distribution for model M_j is given by

$$\pi\left(M_j \mid Y\right)=\frac{L\left(Y \mid d_j\left(t,\Theta_j\right)\right)\pi\left(d_j\left(t,\Theta_j\right)\right)}{\sum_{l=1}^{r} L\left(Y \mid d_l\left(t,\Theta_l\right)\right)\pi\left(d_l\left(t,\Theta_l\right)\right)}, \tag{19}$$

where $p\left(d_j\left(t,\Theta_j\right)\right)$ is prior probability distribution of model $d_j\left(t,\Theta_j\right)$, $p\left(Y \mid d_j\left(t,\Theta_j\right)\right)$ is marginal likelihood conditional on model $d_j\left(t,\Theta_j\right)$.

In the case of non-informative prior distribution, equal discrete probabilities can be assigned for each model $p\left(d_j\left(t,\Theta_j\right)\right)=\frac{1}{r}$, and posterior probability distribution for model $d_j\left(t,\Theta_j\right)$ becomes:

$$p\left(d_j\left(t,\Theta_j\right) \mid Y\right)=\frac{p\left(Y \mid d_j\left(t,\Theta_j\right)\right)\frac{1}{r}}{\sum_{l=1}^{r} p\left(Y \mid d_l\left(t,\Theta_l\right)\right)\frac{1}{r}}=\frac{p\left(Y \mid d_j\left(t,\Theta_j\right)\right)}{\sum_{l=1}^{r} p\left(Y \mid d_l\left(t,\Theta_l\right)\right)}, \forall j=\overline{1,r}. \tag{20}$$

Even though, Bayesian Model Averaging (BMA) seems to have advantages over one-model-fitting, little work has been done in the engineering field to address this for model uncertainty. For example, Alvin *et al.* [4] used BMA to predict the vibration frequencies of a bracket component, Zhang and Mahadevan [139] applied it in fatigue reliability analysis on the butt welds of a steel bridge, and most recent work was done by Inseok Park *et al.* [103]. Those authors analysed uncertainty of four finite element models for laser peening process. However, all of these works used relatively simple models with unsophisticated probabilistic assumptions, and there was no need to adopt advanced probability sampling techniques such as Markov Chain Monte Carlo methods with validation of model selection.

2.2 Modelling of reliability under data heterogeneity

In reliability engineering, applying statistical data analysis, it is a quite common practice to pool statistical information obtained from different sources (for example from different power network overhead lines). At first sight, it seems quite natural to make such decision: if systems are similar and perform the same function, then samples also should be treated as similar. However, such aggregation causes loss of information about specificity of those systems and impact of their environment since assumption that they are identical or at least similar does not hold in every case, e.g., similar components can perform differing ageing behaviour when operating in different environmental and maintenance conditions, even though those components came from the same factory. However, here comes another issue, leading to a decision to pool the data: highly reliable systems, especially if they are time-dependent, do not supply sufficient statistical information for long-term reliability investigation, so that aggregation would strengthen statistical inferences. For aggregated sample one may get estimates with smaller uncertainty bounds, than it would be when between-source (separate samples) variability is considered. Those too optimistic uncertainty bounds will lead to less strict safety margins what itself causes higher risk of safety limits exceedance.

In what follows, the author presents hierarchical Bayesian model, which allows analysis of components, operating in the long term, and prevents from necessity of pooling statistical data. Hierarchical structure enables the analysis of similar "populations" of components without pooling information into one sample, but inferences about each group are strengthened by information shared between separate groups. Application of hierarchical Bayesian modelling for reliability has been used for quite some time (see, for example, Johnson *et al.* [65], Dai *et al.* [35], Younes *et al.* [136], to name a few), with the closest to the present research in its spirit being that by Kelly and Smith [70]. However, there is no information about research for the purpose to investigate behaviour of estimators, based on other than squared error loss function, with nonlinear hierarchical Bayesian model at hand.

As a working-example, count data governed by Poisson distribution with exponential failure rate function was chosen. Poisson distribution is quite common in reliability theory, e.g., Guida et al. [55] performed Bayesian estimation of non-homogeneous Poisson distribution with power-law failure rate for repairable systems, Christiansen and Morris [31] analysed pump failures at a pressurized water reactor nuclear power plant using hierarchical constant failure rate Poisson model, Beiser and Rigdon [19] performed Bayes prediction for the number of failure of a repairable system (they used homogeneous Poisson distribution), Kuo and Ghosh [75] used Bayesian nonparametric inference for real software failure data, Tian et al. [126] considered two real examples from an engine development program and a repairable system with left-truncated power-law Poisson data.

Suppose there are reliability data from N similar sources, which can be units, systems or other entities. Data from each source are indexed by time and possibly by other observable variable. Observed ages of different sources are not necessarily equal. For example, some sources might have started to be observed after several years of operation. Further, some generalization in time variable notation will be made. Suppose that all sources have the same time variable:

$$t = [1, 2, ..., T].$$

If a data sample from i^{th} source is observed, then it will be denoted by $Y^i = \begin{pmatrix} y_1^i \\ ... \\ y_T^i \end{pmatrix}$, so that full data matrix is $Y = (Y^1, Y^2, ..., Y^N) \in R^{T \times N}$.

Denote the probability density function of vector Y^i by parametric expression $f\left(Y^i \mid d\left(t,Z^i;\theta^i\right)\right)$, where $d\left(t,Z;\theta\right)$ is some parametric function embodying relation between time, other variables Z and unobservable variables (parameters) θ.

Examples of function $d\left(t,Z;\theta\right)$ can be:

- Exponential law failure rate trend: $\ln d\left(t,Z;\theta\right) = \ln \lambda\left(t;\theta\right) = \theta_1 + \theta_2 t$;
- Cox's proportional hazard rate: $d\left(t,Z;\theta\right) = \lambda\left(t,Z;\left(\theta_1,\theta_2\right)\right) = e^{Z\theta_1}\lambda_0\left(t;\theta_2\right)$;
- Crack growth rate, which is a solution of Paris-Erdogan differential equation [102] or another law, etc.

Notation of variables Z will be suppressed in further elaboration of concept, because focus is on time-dependency.

According to this notation, likelihood function for the entire sample is given by

$$L\left(Y \mid \theta\right) = \prod_{i=1}^{N} f\left(Y^i \mid d\left(t;\theta^i\right)\right). \tag{21}$$

Now it is necessary to define the first stage of hierarchy, i.e., distributions for unobservable random variables θ^i. Denote $\pi_1\left(\theta^i \mid \xi\right)$ to be a probability distribution of each parameter θ^i, i.e.,

$$\theta^i \mid \xi \sim \pi_1\left(\theta^i \mid \xi\right), i = \overline{1,N}, \tag{22}$$

where distribution index “1” denotes first-stage distribution, and ξ is a vector of hyper-parameters with their own priors in the second stage. For a while, let us denote hyper-prior for ξ as $\pi_2\left(\xi\right)$.

So, full hierarchical Bayesian model can be presented as follows:

$$\begin{aligned} Y^i \mid \theta^i &\sim f\left(Y^i \mid d\left(t;\theta^i\right)\right), \\ \theta^i \mid \xi &\sim \pi_1\left(\theta^i \mid \xi\right), i = \overline{1,N}, \\ \xi &\sim \pi_2\left(\xi\right). \end{aligned} \tag{23}$$

While full posterior distribution of parameters can be expressed (up to normalizing constant) as:

$$\theta,\xi \mid Y \sim L\left(Y \mid \theta\right)\left[\prod_{i=1}^{N}\pi_1\left(\theta^i \mid \xi\right)\right]\cdot \pi_2\left(\xi\right). \tag{24}$$

If function $d\left(t;\theta\right)$ has more than one parameter, i.e., if $\theta^i = \left(\theta_1^i,...,\theta_p^i\right)$ then

$$\prod_{i=1}^{N}\pi_1\left(\theta^i \mid \xi\right) = \prod_{k=1}^{p}\prod_{i=1}^{N}\pi_1\left(\theta_k^i \mid \xi\right). \tag{25}$$

As can be seen from equation (30), whole model likelihood is constituted of data likelihood, the first and the second stage distribution. Graphical representation of hierarchical Bayesian model (and that presented above) can be illustrated by Fig. 4.

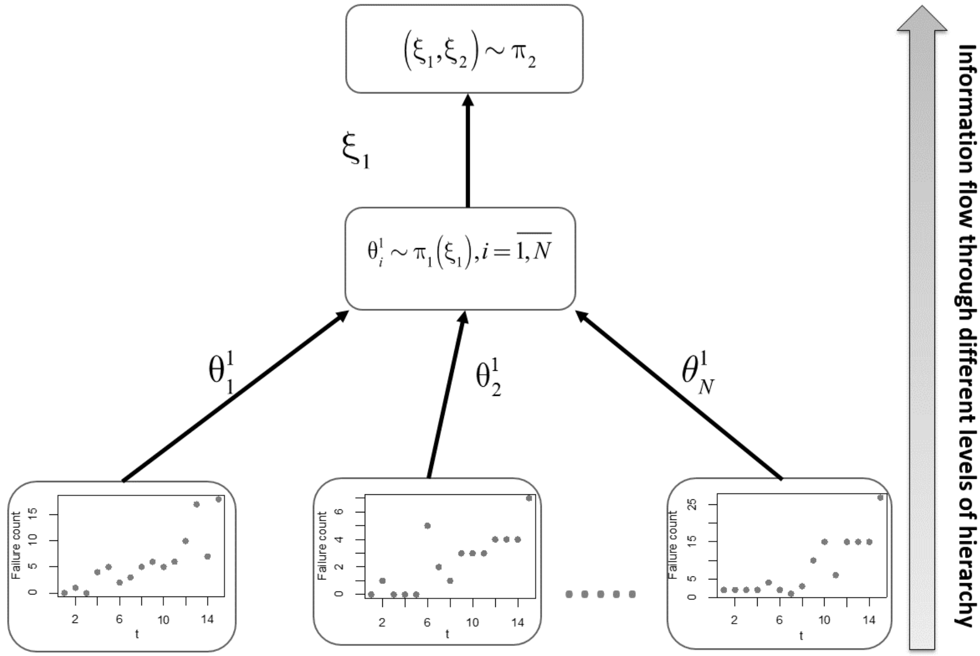

Figure 4. Graphical representation of nonlinear hierarchical Bayesian model with time variable *t*.

The lowest level of the hierarchy represents data and their stochastic behaviour expressed by some parametric distribution. Those data might be collected at different sites or factory places, possibly at various geographical locations or even in different countries. Those data are then partially merged in the second level of hierarchy, where all samples share a part of their information through within-source parameters θ_i^1 and θ_i^2 called unobservable variables. Then this information is used to infer in the third stage, or hyperprior stage, about between-source parameters ϕ. As shown, information goes up to the highest hierarchy level, and its strength deteriorates as it flows deeper and deeper.

Suppose that nonlinear function $d(t,Z;\theta)$ assumes exponential form:

$$d(t,Z;\theta) = d(t;\theta) = \exp(\theta_1 + \theta_2 t). \quad (26)$$

For present purposes, this functional form suffices because of its flexibility and simplicity. For small parameter values, it is approximately linear and under certain parameter transformation can be expressed as a power law.

Now, model (equation 23) becomes

$$y_t^i \mid \theta_1^i, \theta_2^i \sim Poisson\left(d\left(t, \theta_1^i, \theta_2^i\right)\right),\ i = \overline{1,N},$$

$$\theta_j^i \mid \mu_j, \sigma_j \sim N\left(\mu_j, \sigma_j^2\right), \ j = 1, 2, \tag{27}$$
$$\left(\mu_1, \mu_2, \sigma_1, \sigma_2\right) \sim \left(\sigma_1 \sigma_2\right)^{-1},$$

where the normal distribution for θ_j^i is chosen basically due to the fact that parameters are allowed to be positive as well as negative. However, the sensitivity to the choice of second level distribution analysis should be performed in each individual case nonetheless.

In general, the posterior distribution, up to normalization constant, then is

$$\pi\left(\theta_1^1, ..., \theta_1^N, \theta_2^1, ..., \theta_2^N, \mu, \sigma\right) \propto$$
$$\left(\sigma_1 \sigma_2\right)^{-(N+1)} \exp\left\{-\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i\left(t, \theta_1^i, \theta_2^i\right) + \sum_{i=1}^{N}\sum_{t=1}^{T} y_t^i \ln \lambda_i\left(t, \theta_1^i, \theta_2^i\right) - \frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(\theta_1^i - \mu_1\right)^2}{\sigma_1^2} + \frac{\left(\theta_2^i - \mu_2\right)^2}{\sigma_2^2}\right]\right\}.$$

It may be proved (see Appendix) that conditional posterior densities are:

$$\theta_1^1, ..., \theta_1^N, \theta_2^1, ..., \theta_2^N \mid \mu, \sigma, y \sim$$
$$\exp\left\{-\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i\left(t, \theta_1^i, \theta_2^i\right) + \sum_{i=1}^{N}\sum_{t=1}^{T} y_t^i \ln \lambda_i\left(t, \theta_1^i, \theta_2^i\right) - \frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(\theta_1^i - \mu_1\right)^2}{\sigma_1^2} + \frac{\left(\theta_2^i - \mu_2\right)^2}{\sigma_2^2}\right]\right\},$$
$$\sigma_k^2 \mid \mu_k, \theta_{1i}, \theta_{2i}, y \sim Inverse\_Gamma\left(\frac{N}{2} - 1, \frac{1}{2}\sum_{i=1}^{N}\left(\theta_{ki} - \mu_k\right)^2\right), k = 1, 2,$$
$$\mu_k \mid \sigma_k, \theta_{1i}, \theta_{2i}, y \sim N\left(\frac{1}{N}\sum_{i=1}^{N}\theta_{ki}, \frac{1}{N}\sigma_k^2\right), k = 1, 2.$$

2.3 Criteria-Dependent Poison model

In the previous section, a general methodology for handling time-dependent failure data of energy network systems and related uncertainty was set up. Now, it needs to be expanded in order to include some specific issues.

There is a large amount of scientific literature on the pipeline risk assessment, and many ways on how to perform it exist (see a useful Pipeline Risk Manual [90] on this question). For example, the most recent methodologies regarding natural gas network risk and reliability assessment can be found in [64] and [113]. In these methodologies, authors discuss various approaches and aspects of the pipeline risk, e.g., individual and societal risk indexes, gas release rates through cracks, system average interruption frequency index (SAIFI), structural pipeline material aspects and the thermal-hydraulic analysis. Therefore, the information on pipeline risk assessment will not be repeated, and the focus will rather be on the issue of modelling the incident criteria change over time and how it impacts the reliability of the network.

It is not so uncommon to have system reliability data with some discontinuities: criteria, under which the fault is registered as failure, might vary with time. Changing policies of countries operating those systems, increasing or decreasing (surely, the first one is more probable) restrictions on the risk tolerable for society – these and similar aspects result into the change of rules on when the event in the system will be given a failure qualification and put into an appropriate database. For example, such practice is quite common in gas networks (exposition of more specific examples is postponed until later).

In what follows, this section will be confined to the purely statistical analysis without resorting to the questions of cracking process, material strength, etc. Here, the methodology of modelling of incident count data, when the incident definition changes over time, is presented. Although Poissonian data will be considered, the methodology can be extended to other distributions. In addition, at the same time, the OPS database (see Table 13) case will be analysed simultaneously so that more clarity is gained. Suppose there are a number of incidents X_t for pipelines at time t (for simplicity assume equal time lengths of 1), when the cumulative length is L_t (see Fig. 24 for the OPS time-dependent failure frequency data). It is assumed that the incident rate at time t is the function of time and incident registration criterion C_t at that time t, i.e.

$$\lambda = \lambda\left(t; C_t\right), \tag{28}$$

Let C denote the set of all criteria used over the whole observation time period, so that $C_t \subseteq C$. For example, in case of the OPS database (see Table 14), there are three different criteria over the period of 42 years:

$$C = \left\{C_1\left(>5000\$\right), C_2\left(>50000\$\right), C_3\left(>50000\$ \text{ or } >84000\, m^3\right)\right\}.$$

The only problem is that these criteria are not mutually exclusive, i.e., an event with the damage higher than \$50,000 falls under all three criteria for OPS database, while an event of the damage smaller than \$50,000 but larger than \$5,000 falls only under the first criterion. These kinds of incidents would always be recorded in EGIG, UKOPA or NEB databases, but would not be included in the Lithuanian database (before 2004), unless a fire or an explosion. In other words, the following relationships are obtained:

$$C_2 \subset C_3 \subset C_1.$$

A simple, easier, but naïve way to proceed would be to have a base function for failure rate, and for each criterion, to have a different multiplication constant (a parameter to be estimated from the data). Although, this provides no natural interpretation about the meaning of these multiplication constants. In addition, this simplistic approach cannot be easily extended to the case of joining different databases in a single analysis task, unless one is comfortable with an ever-increasing number (and hence uncertainty of the results) of multiplication constants.

Therefore, it is believed that it is reasonable to redefine those criteria in order to obtain mutually exclusive ones. Eventually, multiplication constants for the base failure rate function will be obtained. Yet now, it will have an interpretation of probability of occurrence under the specific criteria. Also, this approach allows decreasing the number of possible parameters.

A word of caution: incidents under those redefined mutually exclusive criteria are not observed. There are only data points each satisfying several mutually exclusive criteria (i.e., a mixture of criteria) at the same time. Therefore, under each (mutually exclusive) criterion, these are not observed directly, only the sum (mixture) of it. The following set of mutually exclusive criteria is easily obtained:

$$C' = \{C_1'(> 5000\$, < 50000\$, < 84000\,\mathrm{m}^3),$$
$$C_2'(> 5000\$, < 50000\$, > 84000\,\mathrm{m}^3),$$
$$C_3'(> 50000\$, < 84000\,\mathrm{m}^3),$$
$$C_4'(> 50000\$, > 84000\,\mathrm{m}^3)\},$$

when the following relations hold:

$$C_1 = C_1' \cup C_2' \cup C_3' \cup C_4',$$
$$C_2 = C_3' \cup C_4',$$
$$C_3 = C_2' \cup C_3' \cup C_4'.$$

Now, assume that a number of incidents at the time t (during a period of one time unit) observed under all of the criteria in C' is $\tilde{Y}_t$. However, only a fraction (say, $\tilde{Y}_{t,i}$) of this number represents the incidents, which occurred under one of the (now mutually exclusive) criterions (say, C_i') and were recorded into a database. For example, 52 incidents were recorded in 1987, when criterion $C_2 = C_3' \cup C_4'$ was used. But the number of incidents falling under all C' was definitely larger with only part of it (i.e., satisfying C_3' or C_4') included into the database.

Let us assume that each incident is a result of an "experiment" or "trial" with outcome falling under the category of one of the criterions, $\tilde{Y}_t$ can be thought of as the sum of multinomial random variables, i.e., $\tilde{Y}_t = \tilde{Y}_{t,1} + \tilde{Y}_{t,2} + \tilde{Y}_{t,3} + \tilde{Y}_{t,4}$. More specifically, if the probability of the incident under criterion C_i' is p_i, then for every t probability density of vector $(Y_{t,1},...,Y_{t,K})$ is a multinomial distribution with parameters $(\bar{p}_1,...,\bar{p}_K)$, where K is the number of all criteria over an entire period. Of course, there is $\sum \bar{p}_i = 1$, while in an analytical form:

$$f(y_{t,1},...,y_{t,K}; Y_t, \bar{p}_1,...,\bar{p}_K) = \frac{Y_t!}{y_{t,1}! \cdot ... \cdot y_{t,K}!} \bar{p}_1^{y_{t,1}} \cdot ... \cdot \bar{p}_K^{y_{t,K}}, \qquad (29)$$

when $\sum y_{t,i} = \tilde{Y}_t$. $K = 4$ in OPS case. Surely, vector $(\tilde{Y}_{t,1},...,\tilde{Y}_{t,K})$ is not observed itself, only the sum of some of its components (i.e., under those criteria which were used at the time t). However, since now the criteria are mutually independent, it may be concluded that each $Y_{t,i}$ is independent and hence:

$$\tilde{Y}_{t,i} \sim Poisson(\lambda_t \bar{p}_i), \tag{30}$$

where λ_t is the intensity function, depending on the time of observation of the pipeline system, i.e., $Y_t = \sum Y_{t,i} \sim Poisson(\lambda_t)$.

So far, a distribution of incidents was obtained for each criterion separately. Now, it has been observed that for some periods of time, not all of the criteria were in action, i.e., the criteria were not used completely "in blind". Therefore, for each t, there is a following result, Criteria-Dependent Poisson model:

$$\tilde{Y}_t \sim Poisson\left(\lambda_t \sum_{C_t'} \bar{p}_i\right), \tag{31}$$

where the summation is over those probabilities, of which the corresponding criteria were in action at that time, and $\tilde{Y}_t$ is the actual number of incident recorder in the database at time t. As it will be seen, sometimes C_t coincides with an entire set C. In the case of OPS there are the following multiplication factors for the criteria set $C = \{C_1, C_2, C_3\}$:

$$(1, \bar{p}_3 + \bar{p}_4, \bar{p}_2 + \bar{p}_3 + \bar{p}_4) = (1, 1 - \bar{p}_1 - \bar{p}_2, 1 - \bar{p}_1). \tag{32}$$

It is obvious that there indeed are multiplication constants, but now the interpretation is clear: it is the probability of incidents under each criterion. In addition, due to the condition that all probabilities sum to 1, one probability parameter can be expressed in terms of other probabilities, i.e., if there is a set of two criteria, then there is no need to have two multiplication factors, only one suffices.

This model enables an analyst to use all available data in order to evaluate and predict pipeline system reliability. Such possibility leads to more accurate inferences, since there is no need to discard part of statistical information. In addition, it was possible to estimate the fractions of data falling under each criterion separately, which gave additional level of information. The general framework can be schematically represented as in flow-chart (Fig. 5).

It should be noted that since the fault occurrence, process Y_t follows non-homogeneous Poisson distribution; probability distribution of time until the first fault is equal to the probability of zero occurrences over some period of time and is expressed as follows:

$$\mathbb{P}[T > t] = \mathbb{P}[Y_t = 0] = \exp\left[-\int_0^t E_\tau \lambda(\tau)\, d\tau\right]. \tag{33}$$

When predictions are made for some period of time of the reliability function we will have in mind the above equation and the Bayesian model attached to the failure rate $E_t \lambda(t)$ will be bared in mind.

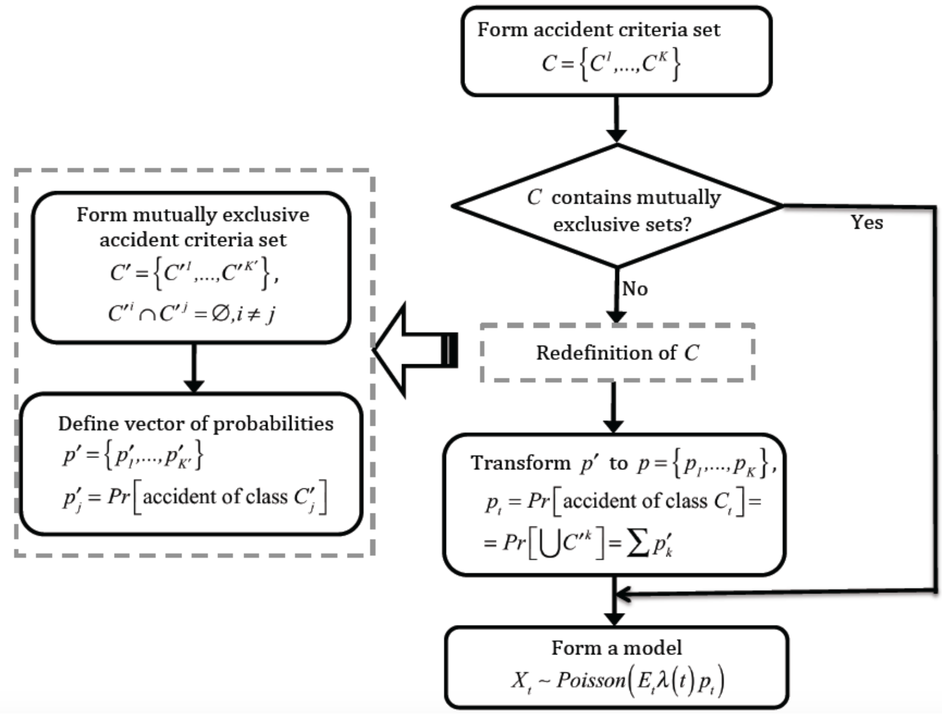

Figure 5. Methodology of pipeline network reliability model construction under different incident criteria.

2.4 Hierarchical Criteria-Dependent Poisson model

The next addition to the methodology of network reliability analysis, when incident criteria change over time, is the application of hierarchical Bayesian modelling approach for the CDP model (Hierarchical CDP, or HCDP). To do this, it is necessary to decide which parameters will be treated as hierarchically dependent, and which should be estimated independently of the others. The most general case would be to put hierarchies on all parameters in model; however, this might be impossible to do for probability vectors $p = (p_1, ..., p_k)$, since different operators or databases may employ some totally different criteria.

The hierarchical construction opens a possibility to not just investigate international incident databases jointly and assess the variability of energy network reliability, but also to investigate networks not covered by any of databases: if few data samples are available, hierarchical structure will strengthen the inferences; if no statistical information is at hand (newly deployed transmission network), more realistic uncertainty boundaries can be obtained from HCDP rather than just using one database information.

2.5 Hierarchical Borrel-Tanner model

In this section, the attention is given to one particular aspect of power transmission network reliability – cascading outages. This part of the methodology cannot be extended directly to the gas transmission network, as it does not experience the outages similar to those of the power network.

The statistical framework to be used is Bayesian as well. At present knowledge, there is no previous work dealing with cascading outages in Bayesian fashion; thus, no similar attempts to account for uncertainty of parameters of models were used. Here the possibilities and implications of hierarchical modelling applied to cascading outages will also be explored.

As a baseline for current investigation, the thought of Dobson [40, 39] is followed, and it is assumed that a branching process with Poisson offspring distribution can approximate cascading phenomena. The total number of individuals, starting from the M_0 parents, is distributed according to Borel-Tanner distribution:

$$\mathbb{P}\left[R=r\mid\theta,M_0\right]=M_0\theta\left(r\theta\right)^{r-M_0-1}\frac{e^{-r\theta}}{\left(r-M_0\right)!},\ r\geq M_0,\ 0<\theta<1; \qquad (34)$$

where parameter θ is the cascading outage propagation rate.

When the initial number of outages is assumed to be of Poissonian nature, Borel-Tanner distribution leads to unconditional distribution of the following form [39]:

$$\mathbb{P}\left[X=r\mid\lambda,\theta\right]=\frac{\lambda\left(\lambda+\left(r-1\right)\theta\right)^{r-2}}{\left(r-1\right)!}\exp\left[-\lambda-\left(r-1\right)\theta\right],\ r=1,2,3,...; \qquad (35)$$

where parameter λ is the intensity of initial number of outages, and this distribution is known as a generalized Poisson distribution (GPD).

By applying this distribution to the transmission network outages it is assumed that each cascade is similar to another, i.e., initiation occurs at the same intensity λ and propagates at the same rate θ no matter at which part of the network it occurred. But is it an appropriate assumption? May the knowledge about the behaviour of cascading failures be improved by a more proper management of data uncertainty?

It turns out one can extend the generalized Poisson model to fit the present case by incorporating source-to-source or more cascade-to-cascade variability. This gives the model capable of incorporating, reflecting and propagating information about occurrence of cascading outages as similar, but non identical processes.

Suppose that the total number of outages in m^{th} cascade is generated by a GPD with parameters λ_m and θ_m.. The next assumption is that these parameters λ_m and θ_m are themselves generated from lognormal and normal distributions with parameters μ_i and σ_i, $i\in\{1,2\}$. Besides, as in previous sections, the prior on hyper-parameters is left aside, and uniform distribution is employed. These assumptions lead to full Bayesian hierarchical model for the cascades in the considered electrical power transmission network:

$$r_m \mid \lambda_m, \theta_m \sim GPD(\lambda_m, \theta_m), m = \overline{1, M},$$
$$\lambda_m \mid \mu_1, \sigma_1 \sim LN(\mu_1, \sigma_1^2),$$
$$\theta_m \mid \mu_2, \sigma_2 \sim N(\mu_2, \sigma_2^2),$$
$$\pi(\mu_1, \sigma_1, \mu_2, \sigma_2) \propto 1, \quad (36)$$

where *LN* denotes the log-normal distribution, while *N* denotes normal (Gaussian) distribution.

At the end, a general model is obtained, which also captures the variability or degree of differences between separate cascades. Although the state and environment of a network, whether it is a different network loading, weather severity, inhomogeneous maintenance policy at the cascading event, are not encoded into the present model, it proves to be superior over simple GPD model. This is because a certain degree of dissimilarity (through first stage distribution) has been allowed, and at the same time, some information (through second and third level) has been borrowed from each cascade to make inference.

2.6 Maintenance with time-dependent uncertainty modelling

Having developed the methodology of time-dependent reliability assessment, a natural extension would be to consider the possibility to apply the results in maintainability, inspectability or availability. Therefore, in this section, maintenance task under uncertainty will be considered.

Maintenance concept, especially when related to the context of power network, is of great importance, as while time is passing, the reliability requirements for systems are possibly increasing, while the maintenance cost is not getting any cheaper [45]. However, the common practice is to use a certain time-independent mathematical model for maintenance and parameter values are just point estimates obtained from the historical data. Moreover, it does not really matter, what specific problem might be considered. Whether it is a network maintenance/inspection optimization task, or reliability predictions under the influence of maintenance/inspection rates, the "tradition" is the same: to obtain point estimates and not to bother about the data any more, as they are treated as a priority. In addition, uncertainty of data is almost never taken into consideration, or if it is, then not as the central part of the whole picture. In this section, it is demonstrated that Bayesian framework is applicable to propagation of data uncertainty through all the maintenance analysis aspects.

Surely, with Bayesian methods penetrating into every engineering problem, there is already some work in maintenance modelling field, like application of Bayesian networks [72]. However, it is basically for modelling of the maintenance decision processes under data uncertainty and the time-dependent maintenance specificity of power network components was never (at least the author is not aware of it) widely considered. On the other hand, mathematical modelling literature on maintenance of power network components is not so vast in general.

In this section, the problem of power transformers and Markov process as a reference model of their maintenance will be investigated. A complete picture of how transition rates between different states of transformer degradation can be estimated are provided, and a view on how to do it sequentially is presented, i.e., updating parameters once new data arrive without recalculating everything starting with old data. It is also shown how such updating makes life easier when optimization of the maintenance rates is taken into consideration.

Since no specific case investigation is sought for, a general model of maintenance will be analysed (its cost will not be considered), which is a slight modification of the one considered in [63, 18]. The general scheme of the Markov model is presented in Fig. 6. For example, using it, three states D_i may be assumed, which approximate the deterioration of a power transformer, while the last state F represents a complete failure and inability to carry through electricity. F state is considered absorbing. The states after deterioration can be repaired at the rates μ_1, μ_2 and μ_3.

It is assumed that if the transformer fails (in state F), lines going through it cannot transmit electricity, i.e., failure of the transformer is equivalent to the outage of connected lines.

In this model, the maintenance rates μ_i are assumed to be fixed. As its effect on the reliability of the network will be investigated, the estimation of these rates is out of the scope of this investigation.

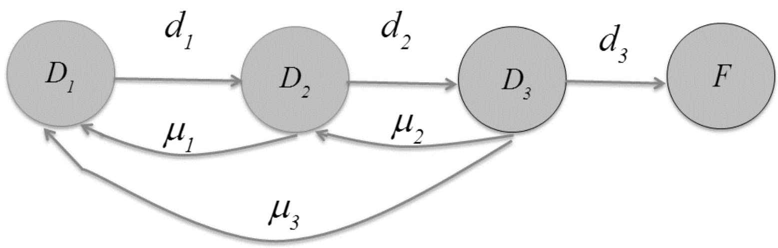

Figure 6. Markov model for maintenance.

A note about the deterioration is required here. Any transition rate d_i between the deterioration states does not reflect correctly the ageing process. Ageing during long time period generally weakens the entire system: components of power transformers fractures, cracks, etc. Thus, as time goes by, more often transitions between D_1, D_2, D_3 and F states will occur, and transition rates may increase. In order to be able to model this effect, time-dependent transition rates should be employed, i.e., time-inhomogeneous Markov processes should be considered. However, this kind of modelling will not be used here.

One of the purposes of this section is to stress the importance of properly handling uncertainty in the maintenance-inspection modelling and the influence on overall network reliability. The prevailing practice in maintenance modelling is to plug-in point estimates of the parameters (in this case, there are six unknown transition rates) and go along with it. However, estimates are uncertain, i.e., there is no infinite amount of information in order to be able to have absolute certainty about its value. Thus, by blindly using point estimates without any consideration of uncertainty bounds it is possible to make inspection or maintenance decisions, which are not very well informed. For example, there is a significant difference between the 95% confidence intervals [400; 700] and [200; 1300] of the MTTF (Mean Time To Failure), even though the expectations of MTTF may be equal in both cases. Quantified uncertainty allows having a more informed insight on possible variations of parameters and their impact on considered result, e.g., cost.

Bayesian framework is perfectly suited for such purposes. In this section, Bayesian inference for Markov chain parameters will be briefly presented. More in depth and more general considerations can be found in references [21, 23, 61], on which the following model formulation is based.

For the ease of general presentation, transition rate matrix is denoted by $Q=(q_{ij})$. Then the likelihood function can be expressed as follows:

$$L_t(Q)=\prod_{i=1}^{n}\prod_{j\neq i} q_{ij}^{N_{ij}} \exp\left\{-q_{ij}M_i(t)\right\}, \qquad (37)$$

where n is a number of states, N_{ij} is the number of transitions from state i to state j in the time interval [0,t], whereas $M_i(t)$ is the time that is spent in state i before time t.

It is sought to estimate the transition rates, so according to Bayesian framework, they have to have a prior distribution. For each unknown parameter, a uniform distribution is placed, so that the posterior distribution is proportional to the likelihood.

Since no prior dependencies between the transition rates are assumed, there is a very nice simplification. Namely, the likelihood function (1) can be decomposed into likelihood functions for each separate transition rate q_{ij}, and the posterior distribution for each transition rate given data X is proportional to the following expression:

$$q_{ij}^{N_{ij}} \sim \exp\left\{-q_{ij}M_i(t)\right\}, \qquad (38)$$

which is (after renormalization) a gamma distribution:

$$q_{ij} \mid X \sim Gamma\left(N_{ij}+1, M_i\right). \qquad (39)$$

Having posterior distribution for the unknown transition rates, now inspection schedule analysis and its influence on the reliability can be performed. Suppose it has been decided upon some optimal value of the inspection rates. As time goes by, additional data will become available, and it would be reasonable to adjust optimal inspection and maintenance plans. Fortunately, Bayesian approach allows performing such updating in a very natural way.

If the new data are X' and if previous posterior distributions $\pi\left(q_{ij} \mid X\right)$ are considered as prior distribution, an updated posterior distribution is obtained, which is expressed again as a gamma distribution:

$$q_{ij} \mid X \sim Gamma\left(N_{ij}+N'_{ij}+1, M_i+M'_i\right), \qquad (40)$$

where N'_{ij} and M' are calculated for the new data.

Since parameters might change considerably (especially in case of initially small samples), inspection rate can be reconsidered and another optimum can be found. Thus, it can be proposed that Bayesian framework in addition to being the most natural way of propagating uncertainty in model parameters allows a kind of sequential updating of not just parameters but optimal inspection plans as well.

Because changes of parameters, as supported by the new data, are allowed, the sequential parameter updating has another useful feature; if due to ageing effect the transition rates do in fact change, this will partially be reflected in updated posterior distributions. Even though expectations of parameters might not change very much, uncertainty bounds will react to the additional information. Hence, this is another important reason for the uncertainty consideration.

2.7 Power consumption estimation in systems

In order to be able to assess how system reliability impacts the performance in terms of energy consumption, the best way to do that is to be able to imitate\simulate the work or processes of that particular system. This is because the physical processes are the main factors that drive the energy consumption of the system.

The energy simulation is first of all a calculation procedure, both manual and automated (computerized), giving quantification of energy volume handled within the industrial manufacturing process or in provision of any service for industrial process. Thus the present concept of energy simulation is represented as follows (Fig. 7):

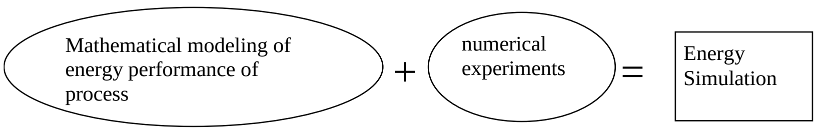

Figure 7. Structure of energy simulation concept.

Energy simulation is neither a common term in engineering and academic worlds nor is it clearly defined. It is not differentiated from energy modelling.

From the definition as that in Figure 7, the following definition of *energy simulation tool* is derived. It is a software tool (program module, program, code, package) based on effective mathematical model of energy analysis of a specific industrial process, designated for multiple calculations of energy parameters of that process. Similarly, this concept is shortly represented as (see Fig. 8):

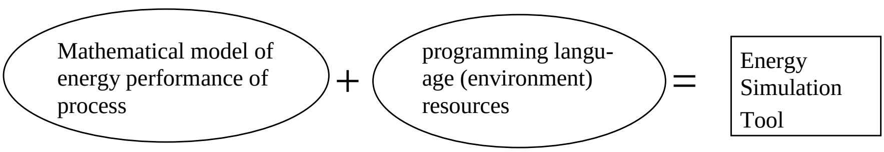

Figure 8. Structure of concept of energy simulation tool.

Thus a mathematical energy model *represents* a base of an energy simulation tool and *presents* a major input into development of a tool.

There is a number of ways to classify the modern energy simulation tools. The rationale for classification should be their application properties. The following set of relevant categorization criteria may be taken into account:

1. Process area – it points to the area of industrial enterprise, where the energy related industrial process takes place;

2. Output results – indicate what results a simulation tool presents – energy use data, energy generation data or other energy related process parameters (e.g., geometrical layout, pressure, flow rate);

3. Stage of the process ("flexibility" of process) – distinguishes between the changeability of process *in situ* after application of energy simulation tool:

- the existing (commissioned) process is "stiff" and cannot be reengineered; hence, the tool is applied practically for evaluation and benchmarking, with small possibilities to improve energy effectiveness by improved operation of process;
- the existing (commissioned) process is flexible to a certain extent, i.e., it can be slightly reengineered; hence, the tool is applied to improve the process;
- the process is in a planning stage and thus is "very flexible"; hence, the tool is applied in design stage with significant opportunities to improve energy effectiveness as compared to analogical/prototype process;

4. Linkage with process design tool – identifies whether energy simulation tool is embedded into the process design tool;

5. Uniqueness of the process – specifies whether process is single, i.e., applied by one industrial user, or typical, i.e., suitable for a lot of users.

In order to simulate energy use for an existing process or any what-if scenario of the innovative process, a simulation tool should be available.

Simplistically, software program should be a set of instructions to the computer as how to calculate energy use. The energy use calculation needs formulas/models, i.e., each case should be described by mathematical models. A "library" of mathematical models for energy use calculations should be developed for the components of each case, while a specific what-if-scenario should be represented as a combination of the models in the "library". Each component used in the scenario needs to be represented by the appropriate model (e.g., a compressed air installation may be represented by a combination of models for each of the installed compressors, filters, receivers and dryers). The architecture of the Energy Simulator component architecture is understood as its structure, which could be represented by subcomponents and their ties (relations). The subcomponents hereafter are referred to as blocks. The architecture of the component is presupposed as follows:

- Data Input block;
- Energy-use mathematical Model block;

- Energy-Use Calculation/Simulation Process block;
- Data Output block. This block will collect simulation results from Simulation Process block.

Physical model of the process is defined here as a comprehensive description of the manufacturing/technology process. The descriptive model should provide:

- Set-up of the process (list of sub-processes, their links and sequence in time);
- List of equipment used and its geometrical layouts (lengths, widths, heights, diameters, distances);
- List of physical parameters of the process and their ranges;
- Boundaries of the process;
- Interdependencies of physical parameters.

The descriptive model has to be detailed as much as sufficient to get information about energy use in the process. If appropriate, it may include mathematical description of process configuration and other characteristics.

The Energy Simulator software was developed under the project DEMI "Product and Process Design for AmI Supported Energy Efficient Manufacturing Installations". The concept of the Energy Simulator was applied for three cases: Compressed Air Systems, Plastic Parts Moulding, Steal Annealing.

The Energy Simulator concept was improved and extended to include the characteristics of reliability and then was implemented in the case study of gas network reliability influence on power consumption in compressor stations. Therefore, in what follows, there is a description of methodology applied for this particular case.

It is assumed that the network is composed of pipelines, compressors and load points. Valve modelling is of no interest here. The topology of the network is not necessarily tree-like, closed loops are allowed. Suppose that at each load point, the consumption follows some stochastic time series of flow rates, like the one in Figure 32. Time is measured in days and the point in time series is an average flow rate over one day. This time granulation is required, because there is no intention to model transient flow, instead a steady state flow is assumed with all the physics that follows from this assumption. In addition, this allows modelling a crack growth dynamics with more continuity rather than just abrupt jumps in case of time measured in years.

A crack is assumed as pseudo-consumption point (the flow rate calculations will be presented in the following sections). Hence, at different time slices, there will be different number of consumption points. This leads to the time-dependent topology of the network and different number of equations and variables to solve. As the cracking process will be modelled as being stochastic, the size of the system of equations is also stochastic and hence cannot be pre-programmed *a priori*.

In the following section, these questions will be tackled:

- How to model the network with time-dependent topology?

- How to model the development of and leaking from the crack processes?
- What is the stochastic nature of cracking occurrences?

Entire network imitation program can be divided into two major parts: one for cracking dynamics imitation and another for solution of flows, pressures and power consumption (see Fig. 9 and Fig. 10, respectively).

In order to obtain results as close to reality as possible, parameters presented and validated in various literature and established as representative were used. In addition, crack occurrence rates were obtained by applying methodology presented in this dissertation, where several different databases are used in order to estimate the rates as accurately as possible.

Cracking-inspection-repair dynamics

1. Generate time series of flow rates for each load of consumption;
2. Repeat for each day for entire simulation period:
   - 2.1. Predict number *N* of new cracks in the network by the data-based cracking rate estimates;
   - 2.2. Generate *N* initial crack depths;
   - 2.3. Generate *N* initial crack lengths;
   - 2.4. Generate *N* growth rates.
3. Repeat for each new i^{th} crack:
   - 3.1. Use POD curve to predict length *L* at which i^{th} crack is detected and repaired;
   - 3.2. Grow that crack until the length *L* and terminate it after $\lceil L / \text{Growth rate} \rceil$ days.

Figure 9. Cracking dynamics imitation program.

As it has already been mentioned, due to the assumption of a crack/hole in a pipeline being a randomly occurring gas consumption point, the systems of equations (i.e., number of equations, number of unknown variables) representing flows and pressures will be stochastic and time-dependent. With each occurrence and repair of the cracks, the system of equations has to be altered by adding or deleting corresponding variables. The location and time of crack occurrence are random. This unforeseeable nature prohibits one from pre-programming a system of equations and reusing it at each time series point. At a random moment in time, the system of equations has to be changed. That is why an algorithm for automatic construction of the system of equations and its solution is needed.

Network flow and energy consumption simulation

Repeat for each day:

1. Alter the adjacency matrix, which represents new/repaired leakages, by adding/destroying nodes;
2. Estimate leak sizes for each new crack depending on its dimensions and consider it as a consumption point with nonzero flow rate;
3. Automatically construct new system of nonlinear equations and solve network flows and pressures for a steady state;
4. Estimate compressor station power consumption.

Figure 10. Gas pipeline network flow, pressure and energy consumption simulation program.

Now, the way the flow problem in the network was solved earlier will be briefly defined. Each new topology of the network is handled by the same solution algorithm. The algorithm proposed in [109] is adopted, and some basic ideas are briefly presented.

In order to solve the steady state condition of the network, one has to use the nonlinear system of equations described by the potential equations (i.e., the loop equations), the flow equations (i.e., the continuity equations) and the resistance equations (i.e., pressure loss equations). The resulting system of equations is therefore:

$$\begin{cases} KQ = 0 & : \text{ n-1 cut-seet equations,} \\ CP = 0 & : \text{ p-n+1 circuit equations,} \\ P(Q) = S(Q) & : \text{ p resistance equations,} \end{cases} \quad (41)$$

where K is the cut-set matrix (cut-set is a minimal set of links, which disconnects some nodes from the others), Q – the flow vector, P – pressure loss vector, C – circuit matrix, S – is the resistance coefficient, n – number of nodes, p – number of pipe-links.

However, authors applied some graph theoretical manipulations and obtained a representation which can be solved with less demanding calculations:

$$\begin{cases} F(Q_T) = P(Q_T) + C_T P(Q_T) = 0, \\ \text{with } Q_T = C_T^t Q_{\bar{T}} + S_{TD}, \end{cases} \quad (42)$$

where C_T – is the matrix obtained from the partition $C = \begin{pmatrix} I & 0 & C_D \\ 0 & I & C_T \end{pmatrix}$, $Q_{\bar{T}}$ are the discharges for co-tree chords as obtained from the initial network graph, S_{TD} is the discharge vector along the tree branches assuming that the co-tree chords are discarded.

In addition, compressor performance equation [130] is as follows:

$$\left(\frac{P_d}{P_s}\right)^m = K\left(A + BQ + CQ^2 + DQ^3\right) + 1, \tag{43}$$

where *K* is the constant depending on the properties of gas, *A*=6.35e-05, *B*=-7.08e-05, *C*=2.54e-05, and *D*=-2.92e-06 are compressor constants, and *Q* is the output flow. The power of the compressor needed to create discharge pressure P_d is as follows [87]:

$$Power = 4.0639 \cdot \frac{\gamma}{\gamma - 1} \cdot Q \cdot T \cdot \frac{Z_s + Z_d}{2} \cdot \frac{1}{\eta} \cdot \left[\left(\frac{P_d}{P_s}\right)^{\frac{\gamma-1}{\gamma}} - 1\right], \tag{44}$$

where γ is ratio of specific heats, *Q* is the inlet flow rate, *T* is the inlet gas temperature, Z_s, and Z_d are compressibility of inlet and discharge gas, η is compressor adiabatic efficiency.

All information that is needed to solve the equations is adjacency matrix of the corresponding graph, flow rate consumption at load points, physical parameter of the pipelines (like length, diameter) and working parameters of compressor in the network.

Each time the crack/hole occurs or is repaired, i.e., each time a new consumption point is created or destroyed, the adjacency matrix has to be automatically altered. Since other parameters like pipeline length and diameter stay unchanged, all one needs to do to enable random changes of network topology is to alter appropriately adjacency matrix.

From computer implementation perspective, it is necessary to have a procedure that without human interruption, program by itself would create a system of equations as presented above and solve it just by using adjacency matrix information.

There are two processes that need to be taken into account: the occurrence of crack and leaking rate through it. To address modelling of the first process, a probabilistic approach is used: how often cracks occur in the pipeline network, what the initial sizes and growth rates are; all these are assumed to be of random nature.

In order to simulate the occurrence of the crack pipeline incident, databases were used, and the occurrence rate was estimated from them. This allowed a more realistic assessment of energy/fuel consumption in the network. More in depth discussion of these estimates will be presented in the following section.

In order to simulate initial size (see Fig. 11) and the growth rate of the crack, information provided in the references [58, 77] was used. The generating model is a lognormal distribution.

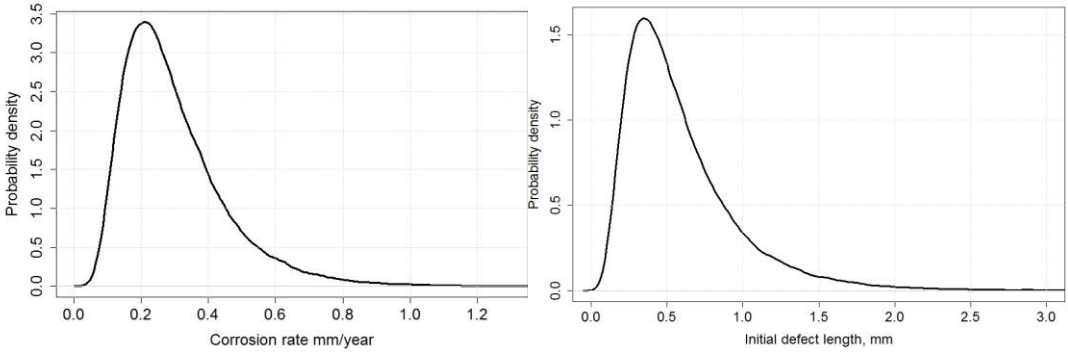

Figure 11. Crack growth rate and initial length distributions (lognormal rate distribution: location '-1.33', scale 0.5; length distribution: location '-0.7', scale 0.6).

Also, there is one more issue: how long does the crack grow until it is noticed and repaired? This question will be discussed in the application part, where different POD curves will be used to imitate the maintenance efficiency and the influence on the energy consumption.

Since a steady-state approach was taken for calculating the network flow, leaking process will be assumed being of the similar nature. In fact, since the leaking is into the atmosphere, and the pressure in the pipeline at the leak point is much higher, a sonic flow through the orifice with the rate equal to [137] is obtained:

$$Q = A_{cr} P \sqrt{\frac{M}{ZRT} k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}, \quad (45)$$

where A_{cr} – is area of crack, P – pressure at the crack point, M – molecular mass of natural gas, Z – compressibility factor of gas, R – specific gas constant, k – ration of specific heats. Used values are $M = 19.5 kg / \mathrm{k} mol$, $R = 9.314 \mathrm{Pa} \cdot \mathrm{m}^3 / (mol \cdot K)$, $Z = 0.9$, $k = 1.27$. Hence the calculated flow rate is then further used as a consumption point and added to the network topology as a separate node with an appropriate alteration of the adjacency matrix.

The question of how many new cracks occur during a certain period of time is quite uneasy to answer due to a two-fold uncertainty issue: the cracking process is stochastic, and hence, statistical estimation techniques have to be applied. On the other hand, in order to perform statistical inference, sample data has to be collected. However, the sad nature of the issue is that cracks cannot be detected with complete certainty, i.e., with probability being equal to 1.

To account for the uncertainty due to stochastic nature of cracking, Bayesian inference machinery is used, and estimates are obtained by joint analysis of data collected from various pipeline incident databases like EGIG, UKOPA, EBA, OPS.

Each database has its own data collection/registration criteria, which might change over time (e.g., OPS database incident criteria have changed three times since 1970s). Thus, one has to have a model to take into account those different criteria. A

Criteria-Dependent Poisson model (CDP) was opted for in order to take into account data collected under different criteria. Another problem was to join information from different databases for one analysis and not to lose information about the between-database uncertainty, i.e., to include crack occurrence rate variability as caused by varying soil, environment, maintenance, etc. conditions. Hierarchical Bayesian method proved to be of great use in this issue.

The dynamics of failure rate was estimated by using Hierarchical CDP model, and the resulting trend as well as uncertainty bounds are as in the figure.

Resulting average number of cracks and related 95 % uncertainty bounds are presented in Fig. 12.

Unfortunately, these estimates are slightly off, and it is not known how much, because some cracks were not detected. This means that the estimated curve is an underestimation of the real number of cracks in the pipeline network. To evaluate how much this affects the overall leaked gas amount, a sensitivity analysis will be performed by slightly varying the estimated number of cracks.

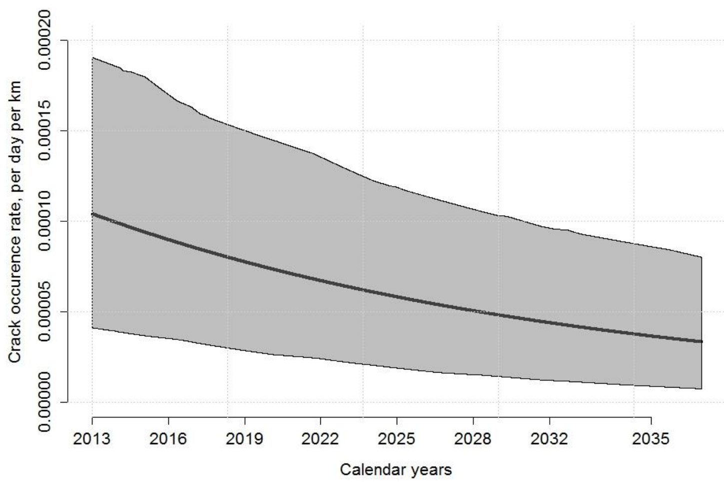

Figure 12. Prediction of cracking rate dynamics for 24 years as obtained by Bayesian analysis of CDP model.

2.8 General approach to network reliability

Consider a network of any kind. As it has already been presented, reliability of network can be represented by the following expression (see also equations (9) and (10)):

$$R(t)=\varphi\left(p_1(t), p_2(t),..., p_n(t)\right),$$

where $\varphi(\cdot)$ is a structure function representing the state of network (failed or not), dependent on the states of each its structural parts (e.g., pipelines, overhead lines, or roads). Basically, $\varphi(\cdot)$ represents minimal path required to pass through the network from one node to another and $p_i(t)$ is the probability that i[th] element is available at the moment t. More specifically,

$$p_i(t) = 1 - \exp\left(-\int_0^t \lambda_i(\tau;\theta)\,d\tau\right).$$

One can observe that the only unknowns are parameters θ of failure rate function. But here comes in the methods presented in previous chapters, and the steps of network reliability estimation can be expressed as follows:

1. Define a time-dependent reliability model for network edges and nodes (Section 2.1)
$$\lambda(t;\theta)$$
2. Discretize the model (see equation (12)):
$$\lambda(t;\theta) = \sum_{i=1}^{N-1} \mathbf{1}_{\{t_i < t < t_{i+1}\}} \lambda(t_i;\theta).$$
3. Obtain Bayesian representation of the model (see equations (14) and (15)).
4. Perform goodness-of-fit techniques to validate the fit of the model to data (see equations (16) and (17)).
5. Extend the model to account for data heterogeneity (see section 2.2 and equation (23)).
6. Obtain posterior distributions of unknown parameters θ (see equations (24) and (25)).
7. Construct structure function of the network $\varphi(\cdot)$ and obtain network reliability function
$$R(t) = \varphi(p_1(t), p_2(t), ..., p_n(t)).$$

The above steps are general and can be done for any network. If, however, there are more specific networks, additional methods can be used. Namely, for natural gas transmission network Criteria-Dependent Poisson model (section 2.3) and its hierarchical extension (section 2.4) applies. These models should be used in the above step No. 1 when the change of data collection criterion occurs. For electricity transmission network, hierarchical Borrel-Tanner model (see section 2.5) applies, which allows assessing the phenomena of cascading outages.

One can go even further, once the above steps are carried out and the level of network reliability is assessed, it can be later reused in other tasks, e.g. assessment of how the level of gas transmission network reliability affects power consumption in gas compressor stations (section 2.7). Or, in case of power network, it can be turned to the problem of power transformer optimization, which depends on time and the reliability of the network.

2.9 Results of the section

In this section methodology for the assessment of reliability of networks when taking into consideration uncertain and time-dependent data was presented. It was showed how time-dependency as well as data heterogeneity should be included in the overall model of network reliability. The construction of time-dependent model, inclusion of data heterogeneity and formation of network reliability model are general steps applicable for any type of network. Borrel – Tanner model was extended to hierarchical and Criteria-Dependent Poisson model was developed having in mind the cascading phenomena in power network and changes of data registration criteria in pipeline failure databases. In addition, main guidelines were presented on how to infer about the dependency of energy consumption in the system on the level of reliability of that system.

3 ANALYSIS AND DEMONSTRATION OF THE METHODOLOGY

3.1 Investigation of the methodology and developed tools

The general time-dependent reliability assessment methodology presented in Section 2.1 has to be analysed in order to assess its strengths and weaknesses as well as the range of application. Therefore, what follows is the application of the methodology to the real data sets. In order to show the applicability of the methodology to the general case of networks and systems, a wide range of data samples were considered. First, the case is considered where only time-dependency (in this particular case data is time-dependent) is present in the failure frequency observations, and the steps of time-dependent model construction and its validation by goodness-of-fit measures (section 3.1.1) are described. Then (in Section 3.1.2) failure data are assumed to be not only time-dependent but also heterogeneous and with very small samples – here only pseudo data, generated from *a prior* known model, were considered and small sample behaviour of Bayesian estimation procedure were investigated. Sections 3.1.3 and 3.1.4 are devoted to the analysis of methodology through examples of North America electricity and natural gas transmission networks. A small detour is taken in Section 3.1.5, where reliability of power transformers and their maintenance is considered when outage data are time-dependent and uncertain. The last Section 3.1.6 of this chapter is devoted to the application of methodology in order to infer the relationship between the level of gas network reliability and energy consumption in gas compressor stations.

3.1.1 Time-dependent reliability assessment of electronic control components

It is important to realize that significant part of the power network reliability comes from the reliability of generating facilities like nuclear power plants, wind farms, etc. The location of power generating facility maybe viewed as a location of a node in the graph of the network with several edges (high voltage power lines) coming out of it. Here an example of nuclear power plant and its Instrumentation and Control system is taken.

The instrumentation and control (I&C) system architecture, together with plant operations personnel, serves as the “central nervous system” of a nuclear power plant (NPP). Through its various constituent elements (e.g., equipment, modules, sensors, transmitters, redundancies, actuators, etc.), the plant I&C system senses basic physical parameters, monitors performance, integrates information, and makes automatic adjustments to plant operations as necessary. It also responds to failures and off-normal events, thus ensuring goals of efficient power production and safety. Essentially, the purpose of the I&C system architecture at an NPP is to enable and ensure safe and reliable power generation. Therefore, it is important to have a model for the assessment of reliability of I&C components. This model can be incorporated into a general Probabilistic Safety Assessment framework of power plant in order to obtain the reliability of the facility to supply electricity to the network.

In what follows, data at hand represents the failure and replacement dates of electrical instrumentation and control (I&C) components. The considered data are quite similar to the real operating experience data collected in French or German nuclear power plants (data were encoded, and exact places where they were collected could not be identified). In particular, it is a large sample that represents one technological group of continuously operating components. The data set contains records from type “T” reactors, which are operated by a single utility with a single management philosophy. The components and composition of them in all reactors are similar (design, manufacturer, technology, etc.). In all type “T” reactors, the components of type “A” are subjected for ageing effect during their operation in the environment with more stressful pressure and temperature. The scope of maintenance is the same for all components.

All data were collected during eleven years, from January 1, 1990 through December 31, 2000. The components in the sample do not all have same date of being put into service, and as a consequence do not have the same ages at the beginning and the end of observation. This reason caused the expansion of age scale from 11 years to 15 years, i.e., at the beginning of the observation, some units had already been operating for several years, and as a consequence, they were older than 11 years. The failure counts were taken from a review of the maintenance data, so any reported date of failure is actually the date of the periodic test. A “critical” failure is one that causes the component to lose its safety function modelled for PRA.

There were 20 reactor units of type “T”, each with 20 components of type “A”. So, each year, there would be 400 component-years except for the fact that some of the reactor units were commissioned before and after the start of the data collection (see Table 1). This caused differing cumulative operating times.

Failure rates presented in Table 4 give the first impression about failure behaviour over time: failure rate increases in time showing system ageing effect. Also, several statistical tests were performed for the ageing effect confirmation and were presented in the report of JRC Institute for Energy [116].

In this analysis, failure rates are considered as constant values in each year, but in every year, this value jumps at the value which can be calculated from linear, Weibull or other model.

Consider as the model for the failure rate $\{\lambda(t); t \geq 0\}$ a jump process structure described above:

$$\lambda(t) = \sum_{i=1}^{N} 1_{\{t_i < t \leq t_{i+1}\}} \lambda_i. \tag{46}$$

In each year, period failures occur as homogeneous Poisson process but with different failure rate parameters $\lambda_i, i = 1, 2, ..., 15.$ At every time period (which in this case is equal to one year), equipment was in operation for τ_i time (operating time). Denote number of failure that occurred in one year as N_i. Probability of N_i failure can be expressed as:

$$\mathbb{P}\left(N_i = k\right) = \frac{e^{-\lambda_i \tau_i} \left(\tau_i \lambda_i\right)^k}{k!}. \tag{47}$$

Table 4. Failure data of I&C components under consideration.

Age, Years	Number of failures	Cumulative operating time, Years	Failure rate, 1/Year
1	1	126.60	0.0079
2	1	171.62	0.0058
3	3	231.36	0.0130
4	1	314.80	0.0032
5	10	396.60	0.0252
6	8	400.00	0.0200
7	16	396.76	0.0403
8	11	380.00	0.0289
9	12	363.34	0.0330
10	8	336.73	0.0238
11	16	281.68	0.0568
12	9	273.42	0.0329
13	10	288.44	0.0347
14	16	168.58	0.0949
15	15	85.16	0.1761

Likelihood function that contains all information obtained from data is:

$$L\left(Y \mid \Theta\right) = \prod_{i=1}^{n} \exp\left\{-\lambda\left(t_i, \Theta\right)\tau_i\right\} \frac{\left(\lambda\left(t_i, \Theta\right)\tau_i\right)^{N_i}}{N_i!}. \tag{48}$$

Since in data source (116), there is no available information about which particular I&C components were under observation, diffuse distribution is chosen as prior distribution for parameters of failure trend function.

As it has already been mentioned in Section 2.1, to express diffuse knowledge about model parameters, one can choose to use uniform, gamma, normal distributions, etc. Gamma distribution with small parameters (which is the usual way to obtain diffuse prior) can lead to incorrect estimates. This occurs due to the nature of gamma distribution – all its mass is concentrated close to zero (Fig. 13).

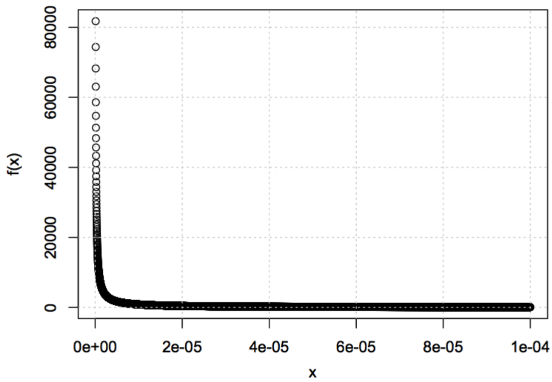

Figure 13. Gamma distribution with parameters $\alpha = \beta = 0.001$.

Due to this high concentration, prior gamma distribution sometimes can pull parameter estimates towards zero. This effect misrepresents the real underlying failure trend and causes making overly optimistic decisions.

For the sake of an example, assume linear model $\lambda(t) = a + bt$ and in the first case, all priors are gamma with parameters $\alpha = \beta = 0.001$, while in the second case, all priors are uniform distributions on interval $[0;100]$. Resulting posterior distribution for parameter a are highly biased towards zero under gamma prior distribution (Fig. 14).

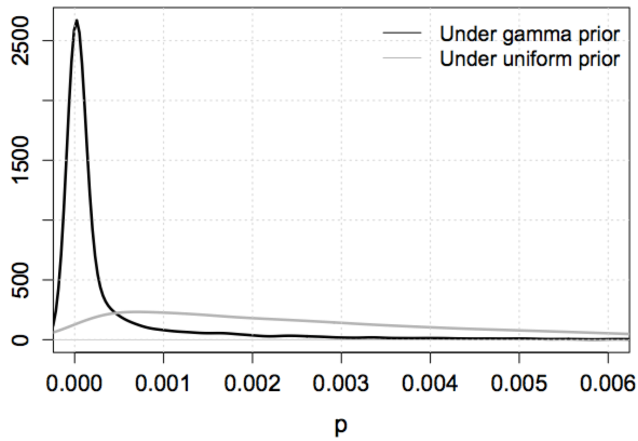

Figure 14. Influence of prior distributions for posterior inference.

Due to this observation, uniform prior distributions for all models and all parameters are confined with. Having this, prior distribution can be generally expressed as follows:

$$\pi(\Theta)=\prod_{i=1}^{m}\frac{1}{range(D_i)},$$

where range is a length of the interval D_i with i[th] parameter is defined.

If the parameter is defined on positive part of real axis, it is not necessary to define prior in the same range, i.e., on an infinite interval. It is usually sufficient to choose “big enough” real value instead of infinity. Of course, posterior distribution has to be inspected and if at least one distribution is found to be truncated, one needs to extend the interval of prior distribution. Numerical experiments and constraints of parameters led to the following intervals of uniform prior distributions:

Table 5. Prior distributions of parameter definition ranges.

Failure rate model	Parameters and their ranges for prior distributions
Linear	$(\theta_1,\theta_2)\in[0,100]\times[0,100]$
Log-linear	$(\theta_1,\theta_2)\in[-100,100]\times[-100,100]$
Power-law	$(\theta_1,\theta_2)\in[0,100]\times[-100,100]$
Xie & Lai	$(\theta_1,\theta_2,\theta_3,\theta_4)\in[0,100]\times[0,1]\times[0,100]\times[1,100]$
Generalized Makeham	$(\theta_1,\theta_2,\theta_3,\theta_4)\in[0,100]\times[-100,100]\times[0,100]\times[0,100]$

Having guaranteed convergence of the Markov chain of MCMC algorithm[2], one has to decide (since no rigorous calculations can be done) on how long to run chains in order to obtain posterior summaries. These summaries can be obtained very easily without any additional burden. Since the present work is not directed to the analysis of various estimators, the research is confined to the posterior expectation as an appropriate posterior summary.

As mentioned in Section 2.1, five trend models (Fig. 15) of failure rate were considered. Linear, exponential and power models represent class of trends, which is common in ageing analysis, and Makeham and Xie & Lai models represent a more flexible bathtub trend class. A constant failure rate model was excluded, because ageing effect of considered data has already been validated in another analysis [116].

As can be seen from posterior p-values based on discrepancy measure $D_2(Y,\theta)$ (see equation (16)) presented in Table 6, in this case, none of the proposed trend models of failure rate give good enough fit, and all models should be rejected. However, p-values $p(D_1)$ show satisfactory discrimination abilities: linear, generalized Makeham and exponential trend models can be interpreted as a better fit than Xie & Lai and power low failure rate trend models. Validity of p-values $p(D_1)$ is

[2] Due to the complexity of Bayesian posterior distribution, the approximation schemes should be used. In this thesis, all calculations were carried out by using Markov Chain Monte Carlo algorithms (see Appendix).

supported by a visual inspection of replicated and observed number of failures (Fig. 16). The sudden drop of the number of observed and replicated numbers of failures after the age of thirteen years is explained by the fact that many components were taken out of operation (due to complete degradation or other unknown reasons). This resulted in the drop of the cumulative number of failures, even though the failure rate increased continuously.

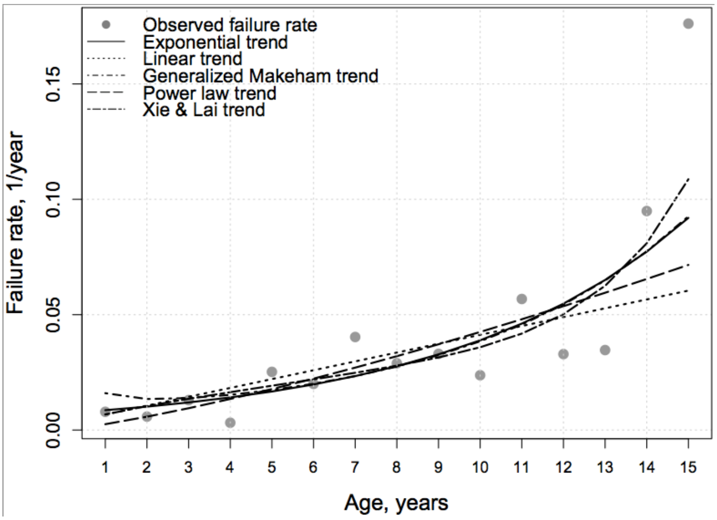

Figure 15. Comparative representations of fitted trend models.

Estimated posterior p-values for different failure rate models are given in Table 6:

Table 6. Posterior p-values for different failure rate models.

	Linear	Log-linear	Power law	Gen. Makeham	Xie & Lai
$p(D_1)$	0.5458	0.6333	0.7356	0.6178	0.7006
$p(D_2)$	0.0042	0.0278	0.0084	0.0306	0.0110

It is worth taking notice of the inability of chi-square discrepancy measures $D_2(Y,\theta)$ to assess model's goodness-of-fit, even if graphical investigation shows a quite tolerable fitness. Even though standard deviation measure seems to work, it might be that applied to another data sample it fails, as is the case with chi-square measure in this circumstance. This leads to the conclusion that discrepancy measures (and as a consequence, posterior predictive *p* values) do not provide an automatic model assessment tool for practitioners.

It is well known that more complex curves will fit data more precisely, but fitness of very complex models can lead to over-fitting (e.g., perfect fitness can be achieved by splines, but this apparently leads to nonsensical inference).

Nevertheless, this obscurity can be solved by using DIC (see equation (17)). This criterion naturally adopts Occam's razor principle, because it incorporates penalty, the effective number of parameters: more complex models will be penalized more severely. DIC values for all models under consideration are presented in Table 7.

Figure 16. Replicated number of failures from exponential model compared to observed data.

Table 7. Values of Deviance Information Criterion.

Model	Linear	Exponential	Power law	Generalized Makeham	Xie & Lai
DIC	91.39	86.48	88.42	94	88

As can be seen from DIC values, exponential model shows best fit. Also, Xie & Lai and power law model can be accepted.

Two measures of fitness – discrepancy measure and DIC – show different results, and an unambiguous answer cannot be given. Preference to one model over another can lead to too pessimistic or optimistic predictions of ageing phenomena behaviour. Such uncertainty related to the selection of model for further use has to be quantified to make sure that applications of model will not be influenced on incorrect choice of trend. Such quantification will be demonstrated in further analysis, where Bayesian model averaging (BMA) will be applied.

As was concluded previously, discrepancy measure and DIC gave quite ambiguous results; subsets of models, selected by these criteria are not exactly the same. In practice, the usual decision is to adopt just one model, but as it has already been mentioned in the theoretical part, this could lead to overoptimistic results if model uncertainty is not incorporated into modelling process.

Next, the application of Bayesian model averaging to analyse time-dependent failures will be demonstrated. The averaging procedure will be performed for all models that were considered in this thesis. To be able to average over the set M of models, probabilities of each model have to be obtained by calculating marginal likelihoods. Calculation of marginal likelihoods is not a trivial task. However, Friel and Pettitt [50] proposed a method when marginal likelihood can be estimated via power posteriors, defined as follows:

$$\pi_s(\Theta \mid Y, t) \propto L^s(Y \mid d(t,\Theta))\pi(\Theta), \ s \in [0,1].$$

It can be proved that:

$$\log L(Y \mid d(t,\Theta)) = \int_0^1 \mathbb{E}_{\Theta|Y,s}\left[\log L(Y \mid d(t,\Theta))\right] ds,$$

where expectation is taken over the power posterior. Then this integral can be approximated by a trapezoidal rule like this:

$$\log L(Y \mid d(t,\Theta)) \approx \frac{1}{2}\sum_{i=0}^{n-1}(s_{i+1}-s_i)\left(\mathbb{E}_{\Theta|Y,s_{i+1}}\left[\log L(Y \mid d(t,\Theta))\right] + \mathbb{E}_{\Theta|Y,s_i}\left[\log L(Y \mid d(t,\Theta))\right]\right).$$

To be able to obtain power posteriors, one needs to define new sampling distribution with additional power parameter s . Then calculate expectations of log-likelihood for original model with regard to power posterior:

$$\mathbb{E}_{\Theta|Y,s_i}\left[\log L(Y \mid d(t,\Theta))\right]..$$

Suppose for a moment that rates for fourteen years are available and need to be predicted for the next one. Obtained probabilities for each model are presented in Table 8.

Table 8. List of probabilities of analysed models.

	Linear	Exponential	Power law	Gen. Makeham	Xie & Lai
$p(d_j(t,\Theta_j) \mid Y)$	0.107	0.422	0.108	0.216	0.147

The interpretation of the above probabilities is as follows: given data Y and a set of models $d(t,\Theta) = \{d_1(t,\Theta_1), d_2(t,\Theta_2), d_3(t,\Theta_3), d_4(t,\Theta_4), d_5(t,\Theta_5)\}$, probability that the model, which generated data, was $d_j(t,\Theta_j)$, is equal to $p(d_j(t,\Theta_j) \mid Y)$.

Calculated probabilities partially justify assessments made by DIC and do not confirm conclusions based on discrepancy measures. Predictions together with 95% confidence regions are presented in Fig. 17.

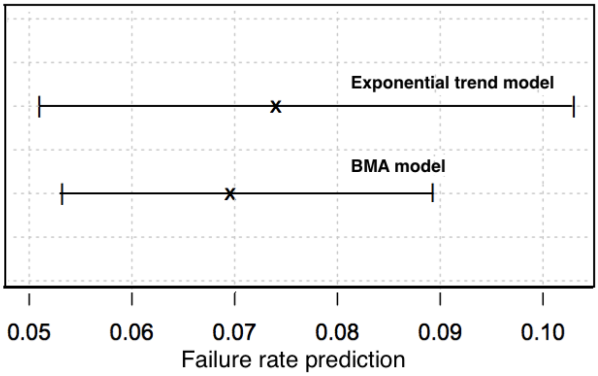

Figure 17. Bayesian predictive confidence intervals under exponential and BMA models.

Differences in confidence intervals suggest that the statistical information about future values is not the same for exponential and BMA models. The smaller confidence interval more information is carried by the model from which it is obtained. The manifestation of shrinkage effect in BMA case is due to aggregation of information over the set of different models. It is seen that BMA is superior to a single model (exponential in this case) in reliability prediction task.

Having this, it was concluded that Bayesian averaging procedure could be a good alternative to various goodness-of-fit approaches since it prevented decision-maker of exclusion of models, which had a good fit and could lead to reasonable posterior inferences. Also, the information aggregation inherited by BMA approach can be advantageous over the single parametric model in component reliability prediction.

In this section, application of methodology of Bayesian method application for time-dependent analysis was presented. It was shown that the methodology is able to deal with disperse and small data amount along with multiple parameter set (Makeham and Xie & Lai trend models). As an illustrative example, the proposed methodology was applied for ageing analysis of electrical I&C components. This application was carried in terms of piecewise homogeneous Poisson model with several failure trends.

When fitting and screening various trend models, it was noticed that none of model selection approaches could give an unambiguous answer. P-values can be quite misleading and can either show no discriminatory abilities (as in case of chi-square p-value) or can suggest more than one model as having a good fit (as in case of standard deviation p-value).

Deviance information criteria can also suggest more than one model (and not necessarily the same one as p-value criteria). That is why there is a high chance to omit model, which can also lead to satisfactory results. Thus, model selection should be performed very carefully. It is worth mentioning that other model selection and validation criteria (such as Bayesian information criteria, Bayesian factors, etc. not described or used in this thesis) can also suffer from such shortcomings.

To evade the drawbacks of model selection and validation tools, Bayesian posterior model averaging procedure was performed for the entire set of models, which were analysed in this section. Such averaging over set of selected trends finally results in better predictive performance, because, due to shrinkage effect inherited by BMA approach, averaged future failure rates will not be underestimated in terms of their uncertainty. Notwithstanding all the advantages of Bayesian model averaging, this approach also undergoes some problems: BMA cannot deal with an infinite set of models, and when one chooses a finite set of them, the best one may not be included in this set; it also fails to "emit the alert signal" when all models fit data very poorly, and so averaging will not result in better performance.

3.1.2 Small sample behaviour of hierarchical Bayesian reliability assessment

While the previous section was devoted to time-dependent reliability assessment validation (application of deterministic jump process to simulate the trend, analysis of several goodness-of-fit tools and Bayesian averaging), in this section, one particular aspect will be analysed in depth. Namely, the behaviour of Bayesian estimates, when sample sizes are very small, and how they are influenced by the choice of an estimator. It will be demonstrated that Bayesian inference produces estimates of time-dependent failure rate parameters quite close to true values of parameters. In addition, the sensitivity to the estimator will be demonstrated. This analysis serves as a supporting argument to the previous claim that Bayesian inference is a suitable framework for the small sample data analysis.

In order to get some insight about the behaviour of hierarchical Bayesian model estimators under small samples, thousands of samples of different sizes and with alternating parameters were simulated. In addition, for each parameter vector (see Table 9), which is considered a true parameter vector, number of data sources was $N = 10$, and each data source had data in each year until the time since the start of operation $\mathrm{T} \in \{5, 7, 9, 11\}$ years.

Table 9. Parameter vectors $(\mu_1, \mu_2, \sigma_1, \sigma_2)$ used for synthetic data simulations.

	Values			
μ_1	-0.5	-0.5	0.2	0.2
μ_2	0.2	0.2	0.2	0.2
σ_1	0.1	0.01	0.1	0.01
σ_2	0.1	0.1	0.1	0.1

Note that first two cases $\mu_1 = -0.5$ reflect smaller initial failure rates (and higher initial reliability); components might be bought from the same factory with highly controlled manufacturing process, while $\mu_1 = 0.2$ indicates less strict production control. Also, variability measure $\sigma_1 = 0.01$ might mean that components came from the same factory, while interpretation of the case $\sigma_1 = 0.1$ might be that components (though similar) were produced by different factories.

For each composition of $(\mu_1, \mu_2, \sigma_1, \sigma_2, N, T)$, 2000 samples were simulated and estimates of vector $(\theta_1^1, ..., \theta_1^N, \theta_2^1, ..., \theta_2^N, \mu_1, \mu_2, \sigma_1, \sigma_2)$ obtained under squared errors and LINEX (as defined in Section 1.2.5) loss functions. Due to the huge total amount of samples and the computational intensity, computations were performed in parallel using Altix ICE Grid supercomputer, owned by Lithuanian Energy Institute.

Further, simulation results will be presented for separate parameters and in the following order: μ_1, μ_2, (σ_1, σ_2). Two loss functions under different sample sizes, their drawbacks and strong sides will be compared. Also, influence of one parameter (specifically μ_1) on the estimation of another parameter (μ_2) will be assessed.

Parameter μ_1 together with σ_1 represents initial level and distribution of failure rates of components/systems under consideration. The more negative μ_1 value is, the smaller initial failure rates it indicates (look at failure rate expression (3)) and value σ_1 sheds light on degree of similarity of components at the beginning of operation.

In case of first parameter μ_1, it is initially observed (Fig. 18) that the expected value of estimator is closer to the true parameter value when $\mu_1 = 0.2$. This might hint that in order to obtain estimates of small failure rates, one needs to gather more data than it would be needed to obtain equally accurate estimates of larger failure rates.

Another aspect that can be inferred is that when $LINEX(1)$ loss function is used, on average, estimator performs better, i.e., bias is smaller than in estimator, based on squared error loss function, case. This is also true for small (*T*=5) to moderate (*T*=10) within-source samples. This effect occurs due to the fact that this estimator "prefers" higher estimates instead of the smaller ones. The question is whether estimator based on squared error loss function always underestimates (at least in Poisson distribution)? However, this cannot be easily answered, and another extensive study is needed.

Another parameter of interest is μ_2. This parameter controls the degree by which component failure rate process is affected by different environments, which is characterized by parameter σ_2. In this case, differences (in terms of bias) between two estimators are not so pronounced, and both show quite a satisfactorily performance. This observation needs more attention; in previous, μ_1, case, quite a severe bias was observed (towards more negative values), but now, with the same size of samples, estimators perform almost equally well and are closer to the true value. So it seems that the information, carried in sample, has different effect on parameters.

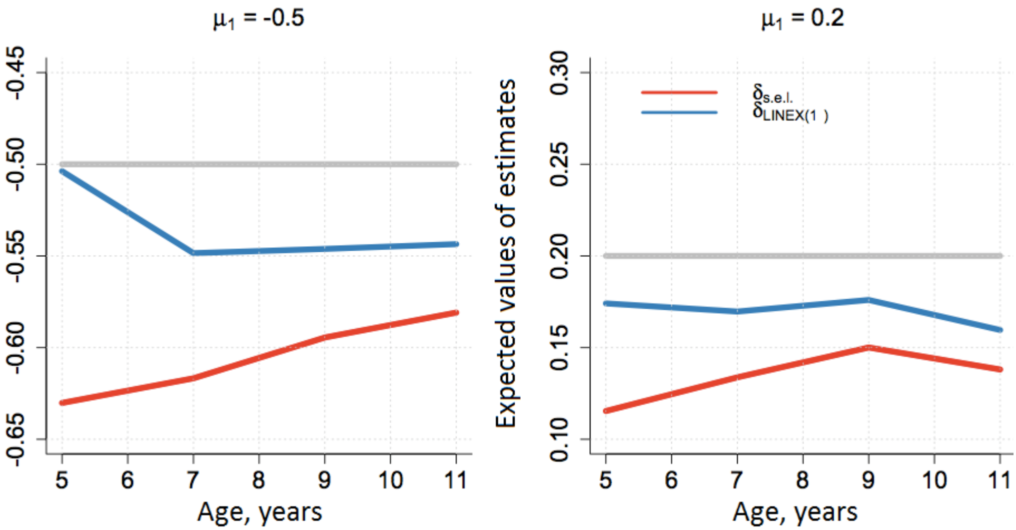

Figure 18. Averaged behaviour of estimators based on squared error ($\delta_{s.e.l.}$ equation (10)) and $\delta_{LINEX(1)}$ (equation (11)) loss functions. Variances of estimates based on both loss functions are about the same and give no additional information, so they are not presented here; grey line represents true parameter value.

In addition, simulations showed differences in variances of distributions of estimators. When comparing different μ_1 values and their influence on estimation of μ_2 , estimators based on both loss functions give less dispersed estimates when $\mu_1 = 0.2$ (Fig. 19), which confirms the conjecture that smaller failure rate values affect estimation procedures in a negative way.

In order to analyse simulation results for parameters σ_1 and σ_2, at first, it is necessary to answer a question about what does it mean to overestimate and underestimate these parameters? If estimate $\hat{\sigma}_i, i = 1,2$ is greater than the true parameter $\sigma_i, i = 1,2$, then from its value, one will falsely conclude higher variation of initial failure rates and ageing acceleration than it really was. Although overestimation of initial failure rate variance does not lead to severe consequences, this cannot be said about overestimation of second parameter, because in a decision-making phase, it will lead to much wider safety margins than they should be. It is still preferable to have a bit higher variation estimates though. This calls for estimator with the ability to produce slightly biased estimates.

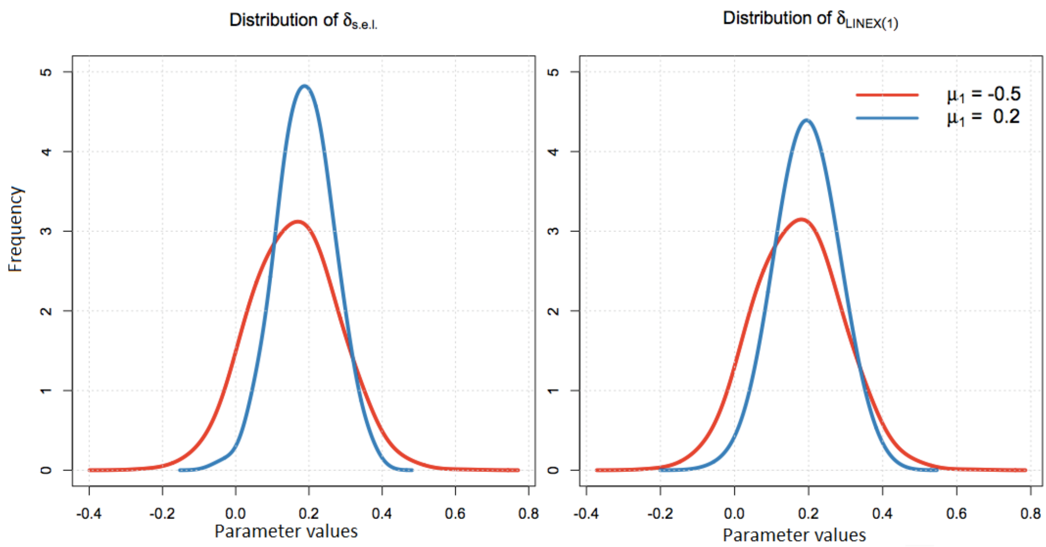

Figure 19. Distribution of estimates for two estimators when $T = 5$.

Simulation results show that expectation of estimator based on squared error loss function is very far from the true parameter values, and in $\mathrm{LINEX}(1)$ case, estimates are even farther from the true values. The use of $\mathrm{LINEX}(1)$ in this case will result in extremely wide safety margins. However, estimator based on $\mathrm{LINEX}(-1)$ loss function, which causes underestimation for parameters μ_1 and μ_2 thus less accurate than estimates under $\mathrm{LINEX}(1)$, still on average overestimates $\sigma_i, i = 1, 2$, but expectation is closer to the true values than in squared error and $\mathrm{LINEX}(1)$ cases.

Given the above analysis, it might be concluded that given properly chosen (and justified) estimator, small sample problem can be at least partially overcome. Even when the hierarchical structure is present, values of estimator groups are close to the true value. This justifies the use of Bayesian approach to the Poissonian data under considered hierarchical structure. However, this is by no means the actual prove. Analysis only provides evidence for our considered model structure. It remains interesting conjecture that observations presented above extends to larger class of models, e.g. non-Poissonian and non-Gaussian cases.

3.1.3 Heterogeneity of North America power network reliability

For the purpose of validation, time-dependent Bayesian reliability assessment methodology will be applied to the outage data of North America power transmission network (500 kV part).

The transmission network will be analysed in two aspects. The first one is the estimation of the reliability of transmission network lines. It will be discussed how uncertain data and estimates of reliability of separate transmission lines can be propagated through the Bayesian inference framework to obtain reliability of arbitrarily chosen subset of North American electric transmission network. The second aspect that will be considered is Bayesian analysis of cascading outages and what are possible ways to account for differences and uncertainty present in cascading phenomena.

The data that are analysed in subsequent sections were obtained from Bonneville Power Administration (BPA) public database [26] that contains information about the outages, timings and their causes. The considered transmission network spans over the areas of California, Oregon, Idaho, Montana and Washington.

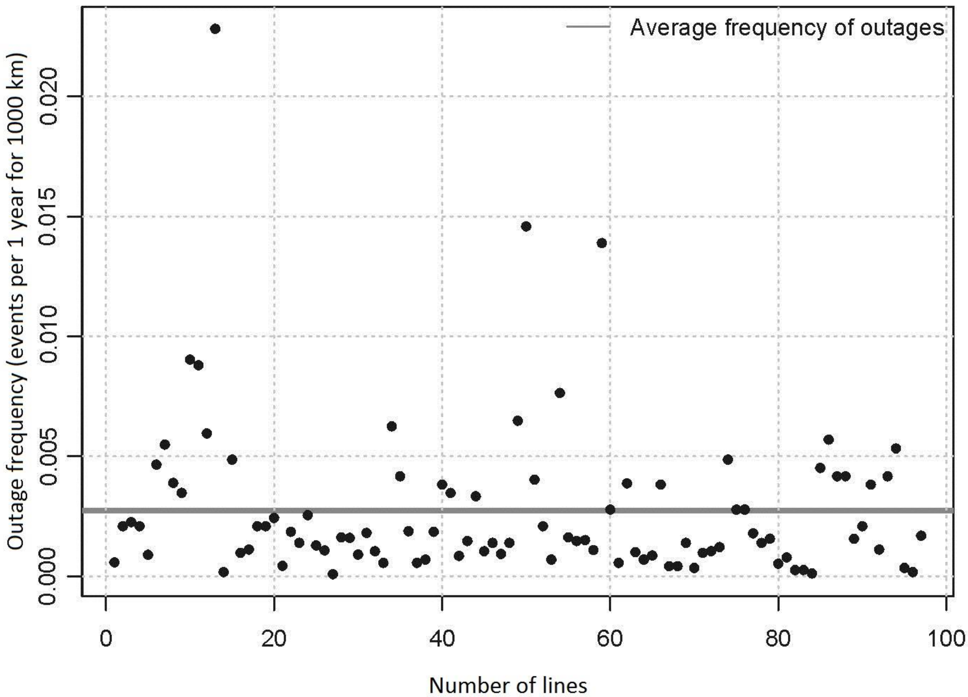

Figure 20. Outage frequencies for each transmission line compared to overall average frequency.

Since this research does not seek for full investigation of this particular power network, it has been confined to electrical power transmission network of 500 kV lines. Time span of outage events is eleven years, and it involves 3179 events (non-planed outages) produced by 97 transmission lines (distribution of frequencies of outages for each line is presented in Fig. 20). At this stage of the research, the consideration of causes of the outages has been discarded in order to lay grounds for more robust Bayesian treatment of the outage phenomena. These events will be modelled as coming from Poisson distribution.

In order to investigate the cascades of the outages, the data was grouped into different cascades and into different stages of each cascade according to the rules used for cascading events in electric power transmission systems [40]; successive outages separated by more than an hour belong to different cascade, and outages in a given cascade separated by time more than a minute belong to different stages. There were 1799 cascades, 327 out of which had more than one stage (Table 10).

Table 10. Summary of cascades statistic.

Stage number	1	2	3	4	5	6	7	8
Number of lines	2572	411	117	47	19	10	2	1

The number of outages can be modelled by the Poisson distribution. In this section, analysis and calibration of the various modifications of simple Poissonian model will be performed.

Suppose that the data are described by the triplet $\left(X_i, L_i, t_i\right)_{i=\overline{1,N}}$, where X_i is the number of outages collected over the period t_i for the i^{th} line with length L_i when the number of lines is N. According to these notations, the model can be expressed as follows:

$$X_i \mid \lambda \sim Poisson\left(\Delta t_i L_i \lambda\right), i = \overline{1, N}, \tag{49}$$

where λ denotes the intensity of outage events of transmission network.

In order to have a full Bayesian model, initially an improper prior distribution might be chosen for intensity:

$$\pi\left(\lambda\right) \propto 1_{(0,+\infty)}. \tag{50}$$

Resulting posterior distribution is a gamma distribution:

$$\lambda \mid X \sim Gamma\left(\sum X_i + 1, \sum L_i \Delta t_i\right). \tag{51}$$

This distribution summarizes all the information about parameter λ necessary to make inference. Expected value and standard deviation the above gamma distribution are 0.00215 and $4.73e-05$, accordingly. No normality assumptions are required (as in maximum likelihood estimation) in order to obtain confidence interval, which in this case is

$$\mathbb{P}\left[0.00205 \le \lambda \le 0.00224\right] = 0.95.$$

This model, although is very simple and straightforward to analyse, as it assumes that all lines produce outages with the same intensity. However, this assumption could be misleading, since the transmission network is established over a wide geographical area, and its different parts experience different weather loads (which lead to various degree of lines deterioration) and different power loads (which impact how hidden failures are exposed [88]). Due to inability to account for heterogeneity in intensity of outages, simple Poisson model fails to incorporate all uncertainty appropriately. Hence, this assessment may produce inadequate estimates.

In order to account for source-to-source (or line-to-line) variability, hierarchical model is used, which enables borrowing the strength of intensity over all samples and at the same time models each outage data sample separately. This type of model can be thought of as intermediate case between complete data pooling and no pooling. Bayesian hierarchical model for Poissonian data is obtained as follows:

$$X_i \mid \lambda_i \sim Poisson\left(\Delta t_i L_i \lambda_i\right),\ i = \overline{1,N},$$
$$\lambda_i \mid \theta \sim N\left(\mu, \sigma^2\right), \qquad (52)$$
$$\pi\left(\theta\right) \propto 1.$$

By stating this model, several assumptions were made. The first is that distribution of the (so called) unobservables $\{\lambda_i\}_{i=\overline{1,N}}$ is Gaussian. The second assumption is regarding uniform hyperprior distribution applied for mean and variance parameters. It might be equally assumed for parameters $\exp\left(\lambda_i\right)$ lognormal distribution or gamma or other distribution defined on positive real axis. To select and to compare them with the homogeneous Poisson distribution, the Deviance Information Criterion (equation (17)) was employed.

Table 11 summarizes the results of fitness of different probability models for outage statistics. The smallest DIC is of hierarchical model under lognormal second stage distribution. It also has the smallest p_D value of all hierarchical models considered.

Table 11. Network outage model selection criterions for different types of distributions

Distribution	Nonhierarchical	Normal	Gamma	Lognormal	Weibull
DIC	29497.24	28219.73	27158.41	26986	27160.01
pD	1.01	97.21	89.53	82	89.37

Replication of outages from non-hierarchical and from hierarchical (with lognormal second stage distribution) models confirms the superiority of hierarchical structure. The hierarchical model properly incorporates within- and between- sources of uncertainty and, as a consequence, has better prediction properties – in case of individual as well as overall rate of outages.

Fitness of hierarchical structure implies that intensity by which outages occur varies across different transmission lines. Some lines are more vulnerable than others and this gives the information which parts of the network should be targeted when maintenance plan is being set. A drawback of this model is in its inability to differentiate between various causes of outages – it just shows that there is significant variability or difference between separate transmission lines. However, it can be easily extended to include information about different causes of outages by including, e.g., a regression part into Poisson rate.

The difference of two models analysed above can be seen in long-term outage number predictions. Simple Poisson model, as compared to the hierarchical Poisson model under lognormal population distribution, favours smaller expected number of outages over all (500 kV) transmission network: the difference of predicted number of outages for 2 years is 20 and as the prediction interval increases this difference increases as well. Estimation was performed for the data collected from years 2001-2012, and data from 2013 to 2014 were used for the comparison of the predictions. Poisson model predicted overall number of outages to be equal to 344 (with 95 % credibility interval being [306; 356]), while hierarchical model predicted that 360 (with 95 % credibility interval being [309; 402]) outages should be expected. True number of outages during those two years was 391. While true number of outages is still quite far from the predicted numbers, it is well within the credibility interval bounds in case of hierarchical model. Non-hierarchical Poisson model fails even to cover the true value of number of outages. This implies that the maintenance planning as well as prediction of loss of load probability might be underestimated when simple Poisson model is employed.

The following few paragraphs are devoted to the demonstration of the reliability assessment of a network in terms of Bayesian framework. Still the results presented in the previous chapter will be used: hierarchical Poisson model structure for transmission lines together with various posterior distributions of outage rates. In addition, several assumptions will be used:

1. Transmission line outages are independent;
2. Transmission lines, once outaged, stay in this state for some time period of time $[0;T]$;
3. No failures occur in network nodes, i.e., nodes are perfect in terms of reliability;
4. Transmission line can be in two states: not outaged (state 1) and outaged (state 0).

It is hard to tell at what degree the first assumption (i.e., independence of outages) holds, but in general, it is not true, e.g., in cascading failures, outages are at least dependent on the previous cascade stage. However, due to lack of knowledge addressing these dependency issues, at this stage no assumptions about dependencies between lines are made. In addition, about 20 % of outages were involved in cascades longer than one stage; therefore, the independency assumption (1) should be a close approximation.

In addition, the second assumption (i.e., the fixed time period of outage) is not realistic in general; all transmission lines sooner or later get repaired. However, if short enough time periods are considered, the network can be partly treated as a non-repairable system. When the raw outage data were analysed, cascades were constructed by assuming that failures separated by more than one hour belonged to different cascades. Hence, a one-hour window could be thought of as a short one enough, so that lines could be regarded as non-repairable components.

With this in mind, the probability that a considered part of the entire network is connected in terms of ability to deliver electricity between the opposite nodes in time period $[0;T]$, e.g., between Monroe and Keeler (Fig. 21) is investigated.

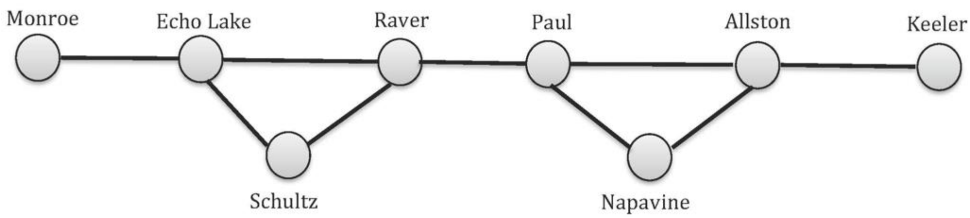

Figure 21. Structural representation of network over 8 nodes.

If the state of k^th^ transmission line by $Y_k(t)$, the structure function $\phi(Y_1(t),...,Y_8(t))$ for the considered example represents the state of the network and posterior probability of failure [80], which are expressed as:

$$R(t) = \mathbb{E}\left[\phi(Y_1(t),...,Y_8(t)) \mid X\right] = \int ... \int \phi(p_1(t),..., p_8(t))\pi(p_1(t),..., p_8(t))dp_1(t),...,dp_8(t) \tag{53}$$

where $p_k(t) = \mathrm{P}\left[Y_k(t) = 1\right]$.

While hierarchical Poisson distribution is assumed over outage statistic, there is

$$p_k(t) = e^{-tL_k\lambda_k}. \tag{54}$$

Since in the previous section samples from posterior distributions of Poisson rate parameters have already been considered, it is straightforward to obtain the posterior distribution of reliability of the network part by feeding structural function with $p_k^i(t) = e^{-tL_k\lambda_k^i}$, where λ_k^i are random draws from posterior λ_k distribution. In case of the present network, a distribution of reliability is as presented in Fig. 22.

Comparison of Poisson intensity parameter estimate from the last hierarchical model with maximum likelihood (ML) estimate shows the impact of the hierarchical structure; the expected value of posterior probability of the network failure (within one-hour window) is smaller than the estimate obtained by ML method. By including additional information regarding variability, the reliability assessment is achieved, which is closer to reality.

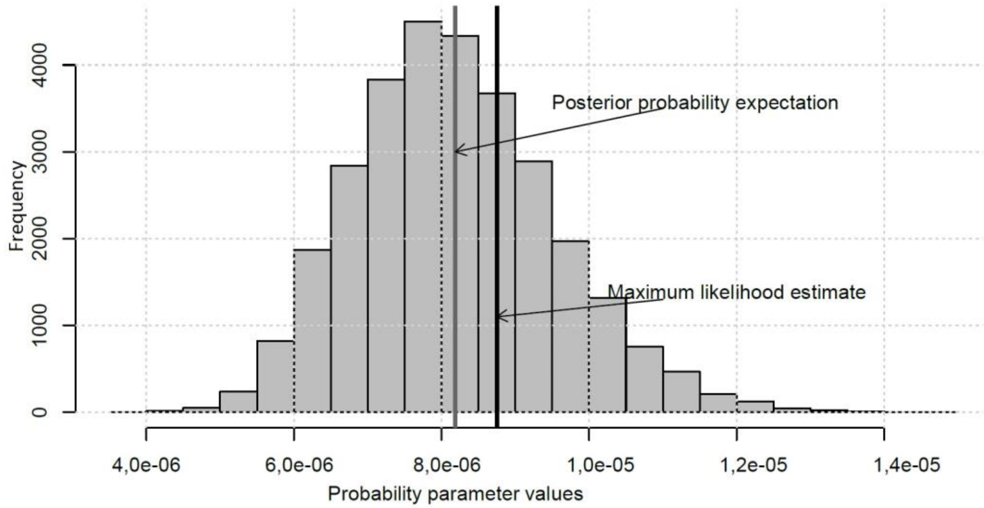

Figure 22. Posterior network failure probability distribution.

Also, one could go further and obtain, once again very easily, posterior distributions of other network reliability characteristics.

It is important to note that although the most general model was stated (see Section 2.5), it does not mean that it will fit better than intermediate cases: λ varies across the cascades, and θ is constant, and vice versa. Thus, for comparison purposes, four models will be considered (or variations of the general models) for cascading outages in 500 kV transmission network. Again, DIC criterion allows discriminating between them in terms of their predictive power.

Table 12. Cascading outage model selection criteria.

Model	Nonhierarchical	Hierarchy on λ	Hierarchy on θ	Full hierarchical
DIC	2642.7	-1106.1	6042	-636.4
pD	1.92	1499	1810	1041.8

Results (Table 12) show high superiority of λ - hierarchical Bayesian model over non-hierarchical – DIC dropped by almost 4000 points, and effective number of parameters is approximately only 1500, while factual number of parameters is 1802. This shows the strength of hierarchical model by explaining the variability of cascading failure data.

Concluding, it may be seen that hierarchical model for initial number of cascades outages is a natural consequence of the hierarchical Poisson model over transmission lines failures. Since each line generates outages according to its unique (i.e., different) rate, it is reasonable to expect that when it comes to cascading phenomena, the initial number of outages are also generated by hierarchical model.

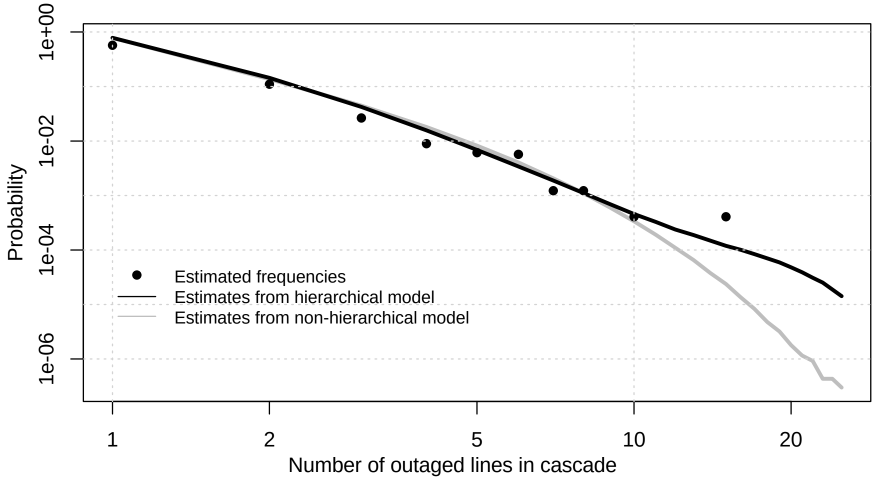

Figure 23. Comparison of empirical distribution and distributions obtained from simple GPD and λ-hierarchical models.

Inference from simple and λ-hierarchical GPD models is almost identical except for higher number (more than 10) of outaged lines in cascade (see Fig. 23). Under λ-hierarchical-GPD, the occurrence of more severe (i.e., longer) cascades has higher probability compared to simple GPD. Probability estimate (obtained by the use of λ-hierarchical GPD) for the largest (spanning over 11 lines) considered cascading event is $4 \cdot 10^{-3}$ while according to the assessment based on the non-hierarchical model it equals $1 \cdot 10^{-5}$ and a more accurate estimate $1 \cdot 10^{-4}$ were obtained by means of hierarchically generalized model This result is of great importance, since the influence of severe cascading events on network risk assessment in a more realistic case will be non-negligible. Large blackouts (like in North America in 2003 [20] that affected roughly 50 million people with 63 GW of load interrupted, or blackout in Italia [33] in the same year that left the system with 6400 MW shortage) in past several decades have already demonstrated their significant effect of occurrence, and they get even more severe as the old network infrastructure operates closer and closer to its limits.

Thorough analysis of electrical power transmission, network failure data was presented in this section. Random outages of network lines were dealt with, as well as with more complex phenomena, cascading outages. Both cases are a part of the network reliability, and both are important aspects of overall network risk assessment, so proper probabilistic attention is of high importance. Bayesian methods and, in particular, hierarchical methods served as a base line for all assessment.

It has been shown that employing Bayesian methods could enhance transmission network risk analysis; this is due to ability to deal with uncertain data and parameters as well as to tackle complex hierarchical structures imposed on that data. Bayesian machinery in line outage case permitted to obtain evidence about the dissimilarities of outage processes generated by transmission lines. Results of hierarchical outage analysis were directly transferred to assess reliability of configuration of representative network part. Assuming that all the minimal cut-sets of the network could be found, it is straightforward to obtain reliability of any complex configuration. In addition, this shows how easily random samples from posterior distributions can be reused in further analysis of the same phenomena.

Analysis of cascading outage phenomena revealed dissimilarities between different cascades as well. Hierarchical structure of the model resulted in higher probability of severe cascading events, which is contrary to the results of a simple Galton-Watson branching process.

Although the analysis was performed just for North Americas' electrical power transmission network, and general conclusion about hierarchical nature of outages cannot be drawn, results provide evidence of the presence of the uncertainty issue phenomena. This evidence should be thought of as an alert for other researchers tackling the problem of power network or any other complex network reliability.

In addition to more realistic proposed estimation of general level of transmission network reliability, various other application possibilities are seen, for example, investigation of network connectivity and vulnerability, where results from hierarchical Poisson model can be used to achieve more truthful connectivity measures, such as line failure probability dependent of the geographical line locations. On the other hand, for transmission network expansion or modification planning, present results can be used as well, as they enable to investigate areas more prone to failures. However, the results of this research do not have a direct application in network reliability assessment part, where such measures as SAIDI, CAIDI, etc. are evaluated, as it involves estimates of not supplied energy, customer interruption times and so on. Although, if distribution network and customer data are included in calculations, then the results can be used to assess reliability from customer point of view as well.

3.1.4 Time-dependent dynamics of North American gas transmission network

As it has already been explained, there are several databases that reflect different experience in gas pipeline networks in various countries or geographical regions. Mostly used and cited international pipeline incident databases are as follows:

- OPS (data from 1970 to 1990 in [54], from 1991 to 2011 in [105]);
- EGIG [46];
- UKOPA [83];
- NEB [48].

However, these databases are not identical; they differ in covered time periods, incident criteria, geographical location of pipeline networks, record types, etc. Main differences and similarities are summarized in the following table.

As it can be seen, the data collection criteria highly differ for OPS database; the entire time series is divided into three regions (Fig. 24), which cannot be analysed as one sample without an appropriate model. Incident criteria can be though of as the basis for screening out insignificant events or applying censoring procedure.

It is important to realize that the data represented by considered databases do not cover every country, and their usage for analysing samples from other, not covered by database, gas transmission networks might be questionable. On the other hand, samples from small countries, like the one from a Lithuanian gas pipeline network, are not representative enough, and then there is no other choice but to use the international experience.

Table 13. International pipeline incident databases: similarities and differences.

Name (Location)	**Incident registration criteria**	**Database record types**
OPS PHMSA (USA)	1970-1983: damage higher than \$5,000; 1984-2002: damage higher than \$50,000; 2003-2012: damage higher than \$50,000; leaks above 84,000 m^3.	Incident frequencies and causes; detailed information for each incident as independent study; In all periods, fatalities and injuries were recorded, explosions and fires as well.
EGIG (Europe)	All detectable unintentional gas releases.	Number of incidents, causes, distribution by detection methods, pipeline diameter, diameter, wall thickness, age, cover type; ignition frequency grouped by hole size and pipe diameter; injuries.
UKOPA (Great Britain)	All detectable unintentional gas releases.	Incidents frequency and causes; leakage volume distribution grouped by detection methods, pipeline diameter, wall thickness, soil type, age, type of cover.
NEB (Canada)	All detectable unintentional gas releases; death or a serious injury of a person.	Incident frequencies.

In this section, the application of CDP model (see section 2.3) for OPS gas transmission incident database sample will be demonstrated. Until 1983, incidents were recorded to OPS if the damage was above \$5,000, or there were events with injuries or deaths. Then in 1984, the criterion increased to \$50,000, and finally, the additional criterion was introduced in 2002, leakage above 84000 m^3. Accidents with deaths and injuries were always recorded, so that they could be discarded from this set of criteria, so that three not mutually exclusive criteria are left. Hence, the set is formed:

$$C = \left\{ C_1\left(>5000\ \$\right), C_2\left(>50000\ \$\right), C_3\left(>50000\ \$ \text{ or } >84000\ \text{m}^3\right) \right\}.$$

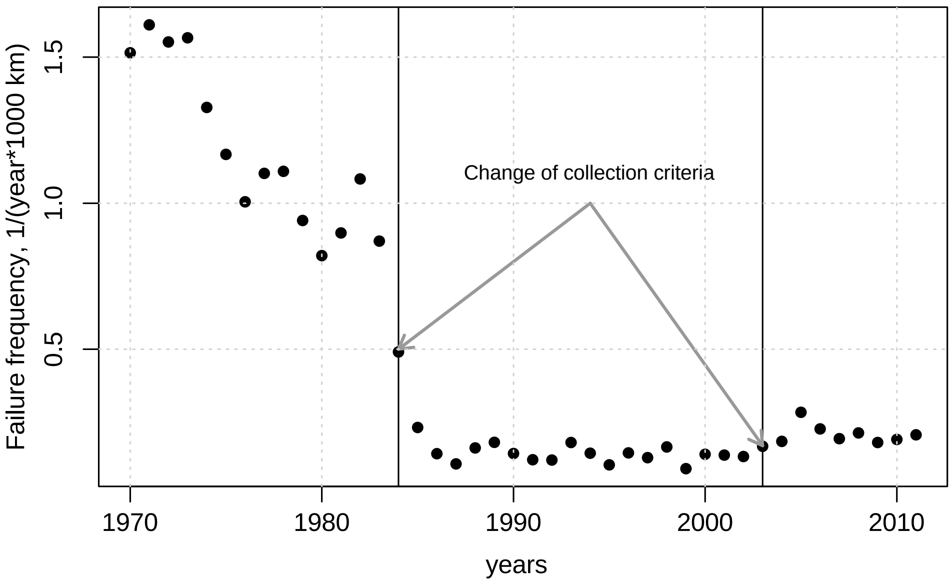

Figure 24. Influence of collection criteria to incidents frequency (OPS database).

None of the criteria are mutually exclusive. and following relations hold:

$$C_2 \subset C_3, C_3 \subset C_1, C_2 \subset C_3,$$

hence, there is a situation when redefinition of C is needed. However, it is not so trivial with C_3, since the incident in this group might be the damage greater than \$50,000, but with the leakage less than 84,000 m^3 or vice versa. It may also happen that an incident falls in both categories. This leads to the following redefinition of the set of incident criteria:

$$C' = \left\{ C_1' \left(> 5000\$, < 50000\$, < 84000 m^3 \right), \right.$$
$$C_2' \left(> 5000\$, < 50000\$, > 84000 m^3 \right),$$
$$C_3' \left(> 50000\$, < 84000 m^3 \right),$$
$$\left. C_4' \left(> 50000\$, > 84000 m^3 \right) \right],$$

where all criteria are mutually exclusive, and the following expressions holds:

$$C_1 = C_1' \cup C_2' \cup C_3' \cup C_4',$$
$$C_2 = C^{3\prime} \cup C_4',$$
$$C_3 = C_2' \cup C_3' \cup C_4'.$$

If a probability vector for C' is denoted by $p' = \left(p_1', p_2', p_3', p_4' \right)$, then it is obvious that probability vector for C will be $p = \left(1, p_3' + p_4', p_2' + p_3' + p_4' \right)$ or (since $\sum p_k' = 1$) $p = \left(1, 1 - p_1' - p_2', 1 - p_1'\right)$. It should be mentioned that the last expression of probability vector should be used to avoid identification problems. Having this, the final expression of the model is as follows:

$$X_t \mid \lambda(t), p_1', p_2' \sim \begin{cases} Poisson\left(E_t \lambda(t)\right), t = \overline{1,14} \\ Poisson\left(E_t \lambda(t)\left(1 - p^{1\prime} - p^{2\prime}\right)\right), t = \overline{15,33} \\ Poisson\left(E_t \lambda(t)\left(1 - p^{1\prime}\right)\right), t = \overline{34,52} \end{cases} \quad (55)$$

To enable Bayesian analysis, uniform distributions for probabilities have been used as an expression of prior beliefs about the proportion of data in each category. Such prior can be interpreted as a non-informative one, since no value is given a priority. By performing goodness-of-fit checking procedures, the power law trend $\lambda(t) = at^b$ has been validated as having the best fit. The goodness-of-fit analysis is not presented in this thesis, as the purpose of this research is to present new models rather than repeat classical procedures. The posterior distribution is then proportional to:

$$\pi\left(a, b, p_1', p_2' \mid X_1, ..., X_{42}\right) \propto \left(\prod_{t=1}^{14} e^{-at^b E_t} \left(at^b\right)^{X_t}\right)\left(\prod_{t=15}^{33} e^{-at^b E_t\left(1 - p_1' - p_2'\right)} \left(at^b\left(1 - p_1' - p_2'\right)\right)^{X_t}\right) \cdot \left(\prod_{t=34}^{42} e^{-at^b E_t\left(1 - p_1'\right)} \left(at^b\left(1 - p_1'\right)\right)^{X_t}\right) \quad (56)$$

All uncertainty bounds, trends and forecasts were estimated by first sampling from the above posterior distribution. The main measures of parameter estimates are presented in Table 14. As can be seen from these posterior estimates, parameters are well identified with quite narrow uncertainty bounds.

Table 14. Main posterior measures for unknown parameters.

Parameter	Posterior expectation	95 % Bayesian credibility interval
a	0.93	[0.925; 0.930]
b	0.00144	[0.0014; 0.0015]
p_1'	0.004	[5E-06; 0.02]
p_2'	0.48	[0.44; 0.52]

Replicated values from the model were compared to those from a classical Poisson model, which account just of incidents that occurred after the last change of criterion in regulatory documents, i.e., since 2003. In short, CDP model is able to incorporate all the data since 1970, while classical Poisson can make inferences just based on data collected after 2003. Surprisingly, 95 % credibility intervals (measures of uncertainty boundaries) in the inference regions (see Fig. 25) are almost identical for both cases, contrary to the expected wider intervals for partial sample. Hence, this leads to the conclusion that in this particular OPS case, it does not matter whether all available data (since 1970) are used or just a part of the sample (since 2003) in the simple model. The differences should become more obvious in case of even smaller data samples. However, when the prediction region is examined, superiority of using all available data becomes clear; a smaller sample leads to rapidly increasing credibility bounds, while CDP model under full data sample provides a less uncertain forecast. This is especially important in analysis of long-term reliability.

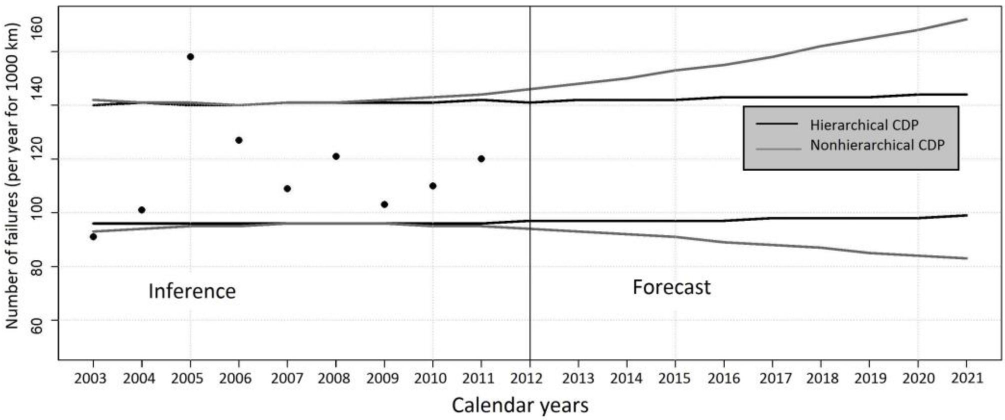

Figure 25. 95 % credibility intervals for replicated and predicted data dynamics (see equation 7 for the used model).

The probabilities of incidents to fall in one or another category (see Fig. 26) are somewhat a redundant result; although it allows efficient inclusion of all available pipeline reliability data, it could be left without any further consideration if the modification of incident criterion is not planned. However, it might be of use to analyse how the change of criteria would influence reliability predictions, and how it would affect general risk level (i.e., level of total incident number) expressing frequency or severity for corresponding incidents. It might turn out to be beneficial to go back to some previous criterion. In addition, these probabilities actually represent fraction of data falling into category of particular criterion, and in this way, it represents a kind of relationship between different categories. The correlation of these probabilities can be easily obtained from posterior distributions of those probabilities; however, it would not contain any useful information, and it is refrained from further correlation analysis.

Regarding the prediction of failure, when the dynamic state is not well understood, it has to be stressed here that the research in not concerned with physical modelling of the pipelines. Physical modelling (based for example on fracture mechanics) is more of a local analysis, while this analysis is based on putting a global data generating stochastic model. The global state of the reliability of the network is represented by the failure rate function, which is estimated by the methods that were proposed here. In addition, physics-based and data-governed modelling techniques are two valid paradigms, aiming at slightly different goal. Statistical data modelling results in the prediction of expected number of failure events for the whole network, while knowledge of the dynamic state of particular pipeline will predict failures locally, i.e., for that particular section of the pipeline.

However, if one would be able to have a perfect knowledge on the dynamic state of the whole network, then probabilistic fracture mechanics might lead to a much more accurate prediction of overall number of failures.

This section is concluded with the stress upon the validity of proposed model and its superiority due to ability to account for data collected under varying incidence criteria.

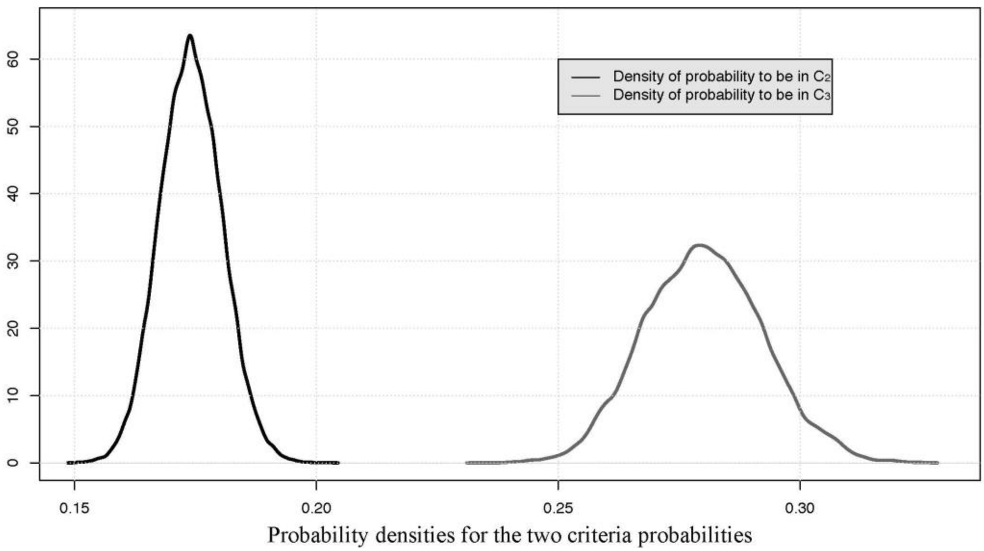

Figure 26. Densities of probabilities of incidents in pipeline network to fall in categories C_2 and C_3 (these posterior distributions were obtained from distribution defined by equation 56).

In this section, the model has been validated by comparing inferences obtained from CDP model against what would be seen if one would use a classical Poisson model. The strength and usefulness of CDP is especially obvious when it comes to future reliability forecasting. If one accepts the proposition that larger data sample will produce more certain future values, then the present CDP model will always dominate over classical Poisson model in terms of wideness of forecast uncertainty bounds.

3.1.5 Power transformer reliability under time-dependent uncertainty

Let us assume that a certain data sample has been observed over the time period of 10 years. The sample is such that data for posterior distributions of transition rates (except the maintenance intensity, as those will be assumed to be known) are as presented in the table below.

Table 15. Data for gamma distributions of uncertain parameters.

Parameter	N_{ij}	M_i	Expectation
d_1	83	177	0.47
d_2	6	20	0.30
d_3	1	3	0.33

Table 27 represents 100 realisations of failure probability (i.e., unreliability) curves within 95% credibility belt, as it is possibly considered in Bayesian framework. These curves represent the uncertainty level as it is encoded in the gamma distributions of transition rates. Example clearly shows how uncertain one's decisions might be should one just use some point estimates and reflect the uncertainty related to the data.

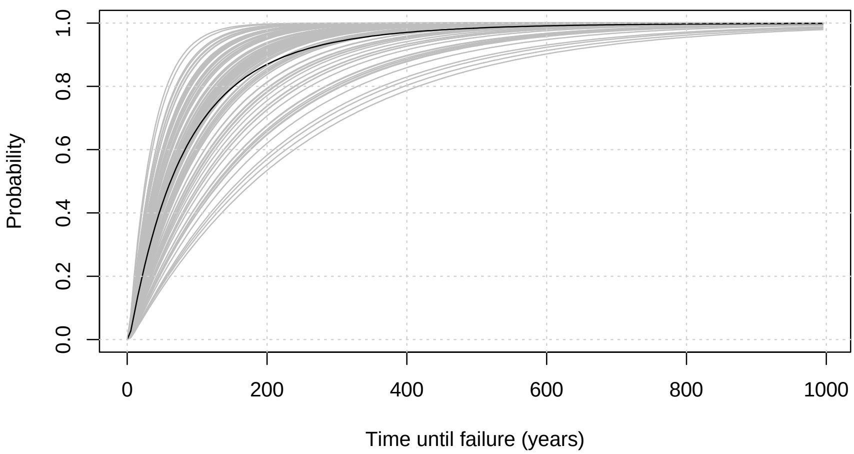

Figure 27. Uncertainty of unreliability function

The uncertainty directly can be transferred to other measures of interest, like Mean Time To Failure (MTTF). For example, using the collection of the parameters as above, the 95 % credibility interval of MTTF is [47; 192] years. This interval is very wide and it is not encouraging that there is a 0.95 probability that the MTTF lies somewhere in that range. So, for better decision-making, say in optimal maintenance scheduling, or in optimal cost analysis, it may be suggested to take into account this uncertainty.

Let us fix μ_1 and μ_2 parameters (see Fig. 6) to the values of 0.5 and 0.7, accordingly, and vary μ_3 over the interval [0, 20]. The MTTF and its upper/lower bound for 95% credibility interval changes as represented in Fig. 28.

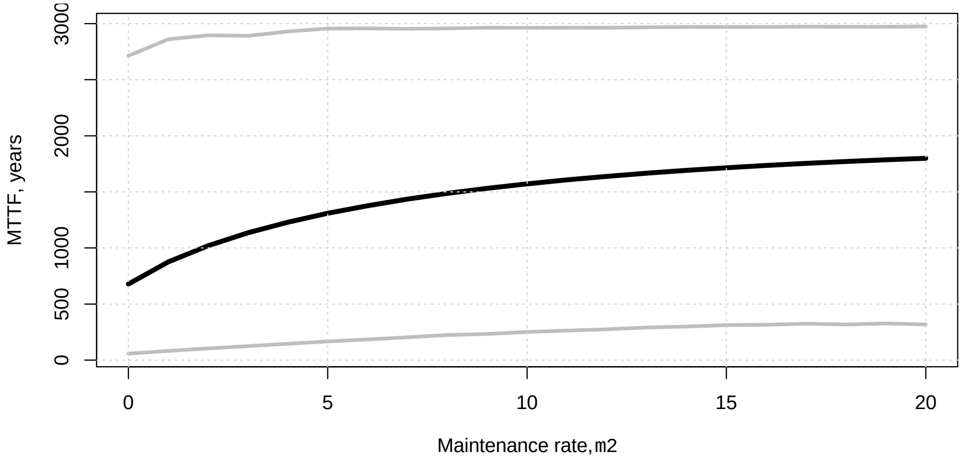

Figure 28. MTTF and its 95% confidence interval dependence on the maintenance rate.

MTTF dependence on the maintenance rate can be used in the maintenance optimisation. However, the same problem with uncertainty as in reliability case remains; uncertainty bounds show that the application of point estimates may not be the best choice. Yet once again, uncertainty bounds can be used to make better-informed decisions; maintenance cost evaluation when uncertainty is taken into account should serve as a better money allocation tool. From the plot, it can be seen how wide MTTF uncertainty bounds can be, and making decisions purely by means of expected values might lead to under- or overestimation.

Let us assume in further investigation that maintenance optimization is equivalent to choosing maintenance rate μ_2 such that the upper bound of power transformer failure in the next four years probability (i.e., transition rate d_3) is not higher than 0.07, i.e., it is desirable to choose such maintenance rate, so that in the next four years, the probability of transformer outage would not be higher than the value of 0.07. The choice of time period and probability threshold is purely arbitrary and is only for the purposes of possible strategy demonstration rather than to imply some existing practice. Figure 30 illustrates the dependence on the maintenance μ_2 of 95% bound for the probability of failure in the next four years.

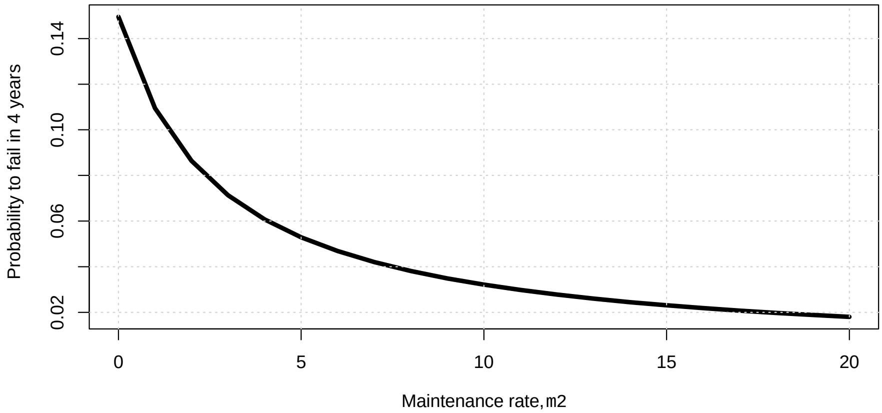

Figure 29. Upper 95% confidence bound of probability to fail in the next four years.

Now, let us assume one wants to update the optimal maintenance every four years (i.e., reflect changes of data in the adjustable maintenance rate μ_2). Due to the convenience of Bayesian updating procedure and the resulting gamma posterior distribution, such maintenance corrections can be done without any recalculations of the previous analysis. All one needs to do is to update posterior gamma distribution with new data and then find another maintenance rate so that the probability in another four years is kept lower than 0.07.

Markov chains generate time-dependent probability distribution of transitions (even though it will eventually approach some steady state value), and optimal maintenance μ_2 or MTTF (Mean Time to Failure) is shown to change each time transition rates are updated. The following figure demonstrates how this optimal maintenance rate changes when the model parameters are updated (i.e., deterioration rates updated) every four years.

The initial maintenance rate was 0.7. Since the upper bound of probability to fail in the next four years was lower than the 0.07 value, there was no need to increase the maintenance rate. That is the reason for going fourteen years with the same maintenance rate. Then, since the probability to fail increases with time, maintenance with optimal schedules has to be updated. As transition rates are updated every four years, the optimal maintenance is updated as well. However, after certain age, the corrections of only one maintenance rate will not be enough, since keeping the upper bound of probability to fail in the next four years lower than 0.07 value, the maintenance rate will have to be extremely high (at the age of 30, it has to equal approximately 58 in order to keep the probability below 0.07).

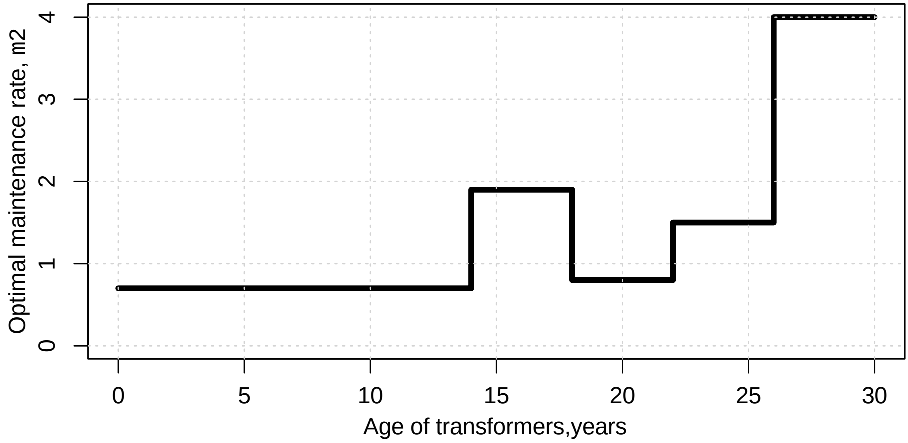

Figure 30. Change of optimal maintenance rate after updating every four years.

If time-dependent transition rates were considered (i.e., time inhomogeneous Markov chains), the easy transition rate updating procedure by gamma posterior distribution would not work, as the posterior distribution would be much more complicated, and the analytical expression would not be known. Thus, in order to obtain posterior estimates, one would have to employ numerical methods, like Markov Chain Monte Carlo methods (see Appendix).

In this section, it was demonstrated how uncertainty in the data can be easily incorporated into the problem of maintenance optimization and how sequential updating of parameters using Bayesian approach can be applied when maintenance is adjusted according to the available information and possible uncertainty.

Bayesian updating of model parameters allows straightforwardly dealing with uncertainty in data, which is directly transferred to the decision about the optimal maintenance schedule. In addition, a sequential nature of the problem enables (at least at some degree) to deal with uncertainty in transition between transformer degradation states.

The aspect of power transformer reliability influence on overall power network reliability has been left unexplored. It is a very common approach to consider power network nodes as perfectly reliable and just proceed further with failing electricity transmission lines. However, even though the MTTFs are much longer for power transformers as compared to transmission lines, transformers have much more significant influence on the connectivity of the network and hence on the overall reliability. As it was shown, reliability bounds for transformer can be very wide (especially when data sample is small); taking just point estimates of failure probability in a power network, reliability analysis would lead to high under- or overestimation degrees.

The work presented in Section 3.1.3 has provided some evidence that power lines do not follow identical distribution. Heterogeneity due to geographical locations, different weather conditions, etc. results in heterogeneity in failure rates of transmission lines. Since power transformers also function in different geographical areas, the same heterogeneity can be expected to be present in their reliability as well. This brings into picture an additional level of uncertainty, the so-called between-source uncertainty.

3.1.6 Power consumption estimation for gas network

The calculations were performed on a theoretical pipeline network. However, it was intended to have a topology as representative of the real world as possible. In addition, the results and calculations were meant to demonstrate qualitative investigations rather than quantitative, i.e., one wants to obtain insights on how large is the effect of network leakages due to cracking processes on the consumption of energy in compressor stations.

Assume the network topology as in Fig. 31: one gas compressor (or compressor station), each line of equal length (100 km) and diameter (1000 mm). Since the calculations are of a theoretical nature, they are confined to the simple network topology with few lines, although complex enough for the present purposes.

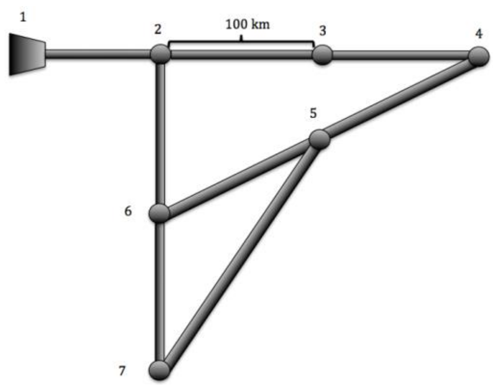

Figure 31. Pipeline network used for calculations.

The adjacency matrix is as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

Adjacency matrix is the only information necessary in order to automatically create a set of equations for the network flow. When a crack appears in one of the lines, an additional row and column are added to the adjacency matrix, and this new matrix is then used to automatically generate the set of equations. Such automation is very handy since in order to establish valid estimates, thousands of iterations need to be handled, and it would not be possible in practise to alter the code by hand.

Next, it will be assumed that there is a positive consumption at each point. The consumption time series will be generated from lognormal distribution: each point has its unique time series. Lognormal distribution is chosen for no particular reason except that it is defined for positive values only. Gamma distribution would have been acceptable as well. Parameters were chosen such that expectation and variance would be close to realistic values of consumption. This will allow estimating energy consumption in the compressor. Example of flow time series is presented in Fig. 32.

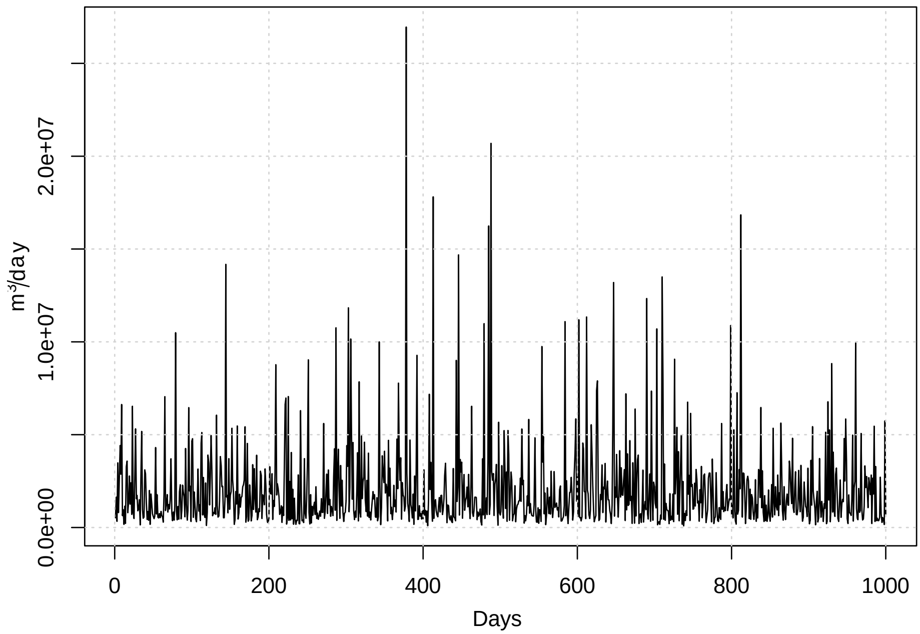

Figure 32. Flow consumption time series at one of the points.

The average consumption rates are representative of the real network. However, seasonality is not included in the consumption time series.

At first, simulations were run as if the network were perfectly reliable without any cracks occurring in its entire body. Distribution of the average power consumption per day is visualised in Fig. 33.

Perfect network simulation results will serve as a baseline for comparison with cases when failure as well as occurrence of holes have nonzero probability.

500 simulations were run each time, taking a random sample from the posterior distributions of pipeline cracking rates (see Fig. 12). Each such sample contained around 12,000 points, as the simulation was run for the period of 25 years. Thus, each simulation resulted into average power consumption point. Since each simulation out of those 500 was run under different crack occurrence rates (as sampled from posterior

distributions), a random sample of average consumptions was obtained. The distribution of the sample and its comparison with consumption in ideal network is presented in Fig. 34.

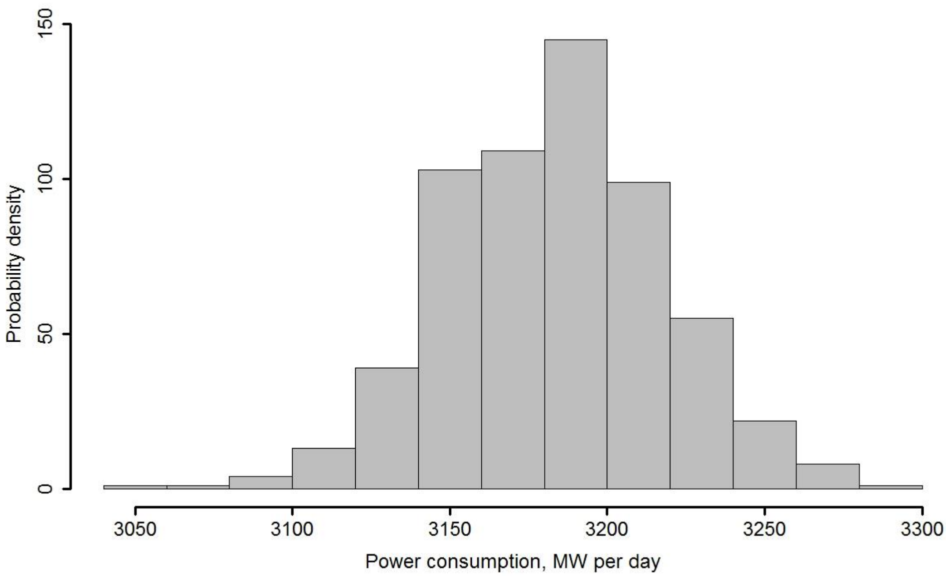

Figure 33. Power consumption distribution for ideal network.

Results demonstrate how significant the influence of leakages through crack-damaged pipelines is. The difference between compressor station power consumption expected values for ideal and imperfect network cases is approximately 8%. This difference would result in 8% increase of daily costs. Over a year or several years, this would result into a large amount of money, which would result in the increase of gas price. Thus, the necessity to include reliability considerations into overall gas network planning is clearly seen.

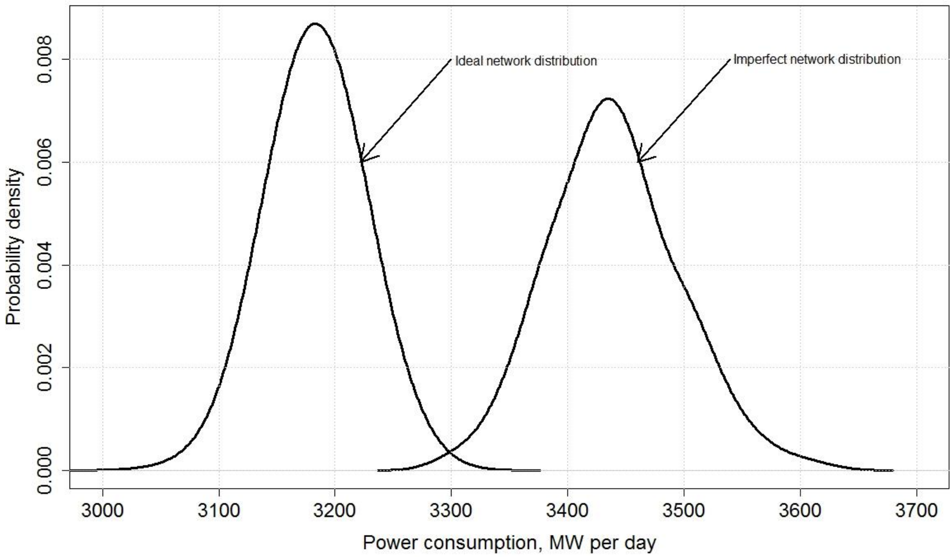

Figure 34. Distribution of average power consumption over 25 years.

Therefore, the evidence that the influence of reliability parameters on overall network performance can be significant was established. Next, since crack occurrence rate as discussed previously is definitely an underestimation of the real rate (due to a fraction of leakages not observed by any inspection instrument), a sensitivity analysis to the variation of the crack occurrence rate (variation range [0.2, 0.7] events per day per 1000 km) was performed. In addition, it was taken into consideration that the true ability to detect cracks in the pipelines is unknown; hence, three different POD (Probability Of Detection) curves were used.

It was assumed that probability to detect follows gamma distribution with scale parameter equal to 0.007 and shape parameters equal to 20, 30 and 40. POD are plotted in Fig. 35.

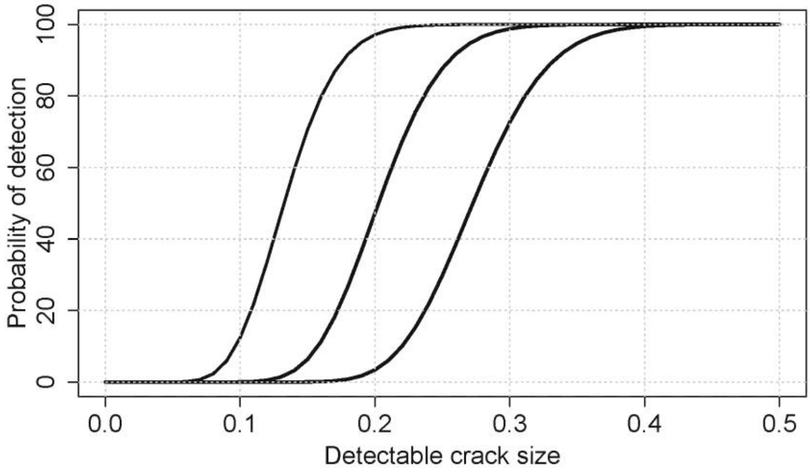

Figure 35. POD curves used for gas network inspection imitation. Curves with shape parameters 20, 30, 40 are from left to right, correspondingly.

Results from sensitivity analysis are presented in Fig. 36. As can be seen from simulation results, the average power consumption increases linearly with crack occurrence rate, i.e. additional power consumed in compressor station due to wasted gas is proportionally dependent on the failure rate of network pipelines. This implies that true difference between average power consumption of compressor station in ideal network and imperfect network will differ by more than 8 %, as estimated in the above analysis. That is because true crack occurrence rate is slightly higher than is estimated from the data.

The speed, at which average power consumption increases (by increasing cracking rate), depends on the quality of leakage detection.

In what follows, the author explored the possibility to include reliability characteristics of gas pipeline network into power consumption estimation at the compressor station. Main guidelines on how to perform such an inclusion and how to implement it were drawn.

Even though some significant assumptions and simplifications, like steady state flow instead of transient flow, were made, as much realistic data as currently available were included: crack growth parameters, time-dependent crack occurrence rate. Average gas consumption rates were also taken to be representative to the real network. A validated orifice gas consumption model was used as well. Therefore, it is highly probable that the results may be taken as qualitatively valid.

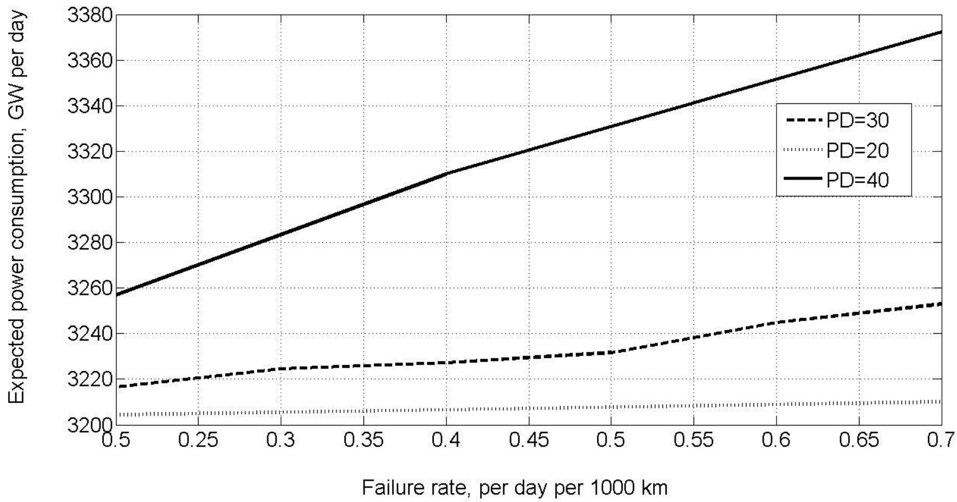

Figure 36. Power consumption dependence on cracking rate and probability of detection.

The main conclusions of the present work are that forecast of costs due to compressor station work may be quite significantly different if network reliability characteristics are included as compared to the case of ideal (in terms of reliability) network. Power consumption and thus cost of running compressors are tightly related to the level of network reliability. Roughly 8 % over a long run will be added to the network operating cost due to the leaking pipelines.

It was also concluded that the maintenance level and its efficiency have a significant impact on the average power consumption. The better maintenance, the more leakages will be traced down and resolved, leading to reduced power consumption. However, since the maintenance strategies cannot be improved indefinitely, eventually degradation will "outperform" the maintenance and leaks might start to increase. Unfortunately, this question – maintenance improvement and degradation process balance – cannot be taken into account at this stage due to a complete lack of maintenance technology dynamics modelling approaches.

In addition, there is another dimension to the problem of significant leaks from the pipelines, that is, the environmental aspect. 8% of power consumption increase means that a significant amount of gas is wasted to the atmosphere increasing amount of greenhouse gas.

Even though it was possible to make inferences about the cracking process influence on overall power consumption in compressor stations, it is not the only network reliability aspect that will eventually lead to additional spending. Risk was not considered here due to fires or explosions. The research was confined to the aspect of power consumption due to additional load caused by wasting gas to the atmosphere.

3.2 Demonstration of methodology in the case of Lithuanian energy networks

The methodology developed and analyzed in previous chapters was applied to the Lithuanian context. Namely, time-dependent reliability of the 330 kV power transmission network and gas transmission network was assessed. Following paragraphs will be devoted to the main characteristics of these systems.

There are 6687 km of high voltage (110-330 kV) airlines with 51 km of underground cables.

Figure 37. Lithuanian power transmission system (Courtesy of Litgrid).

The 330-110 kV Lithuanian power transmission network includes 234 transformer substations and switchyards as well as 6687 km of power transmission lines. The installed capacity of the 330 kV transformers totals 3900 MW, and that of the 110 kV transformers totals 92.6 MW. The Lithuanian transmission network is well connected with some of the neighbouring power systems: by four 330 kV lines and three 110 kV lines with Latvia, five 330 kV lines and seven 110 kV lines with Belarus, and three 330 kV and three 110 kV lines with the Kaliningrad Region.

Every year, Litgrid (the Lithuanian electricity transmission system operator) staff or contractors carry out inspections of the high-voltage overhead power transmission lines. An inspection involves checking that the line and its equipment (supports, insulators, etc.) are intact, not outdated, and not overgrown with trees and bushes.

In terms of connectivity, a 330 kV network can be represented as in the following figure (note: the distances do not correspond to the real ones).

Red dashed lines represent plans for new lines. Those new lines will provide additional reliability to the overall network, as new paths for electricity to flow will be available. The effect of those new lines will be analysed as well.

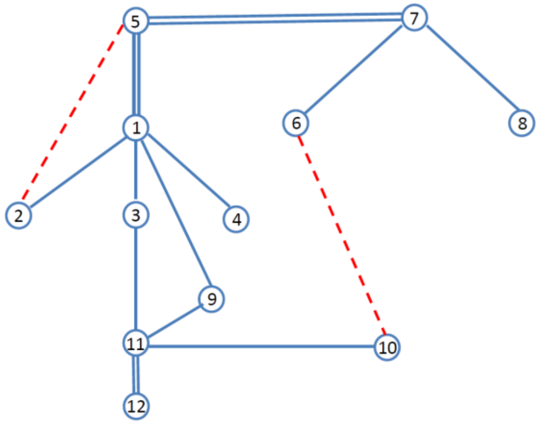

Figure 38. Power transmission system of Lithuania.

The transmission system, operated by AB Amber Grid is interconnected with the natural gas transmission systems of the Republic of Latvia, Republic of Belarus, Kaliningrad Region of the Russian Federation, Klaipeda Liquefied Natural Gas Terminal and distribution systems, operated by the distribution system operators within Lithuania.

The pipelines that have been in operation for the longest time were constructed back in 1961. The diameter of the largest pipeline is 1,220 mm. The design pressure of the largest part of the gas transmission system is 54 bar.

Lithuania’s natural gas transmission system is interconnected with the natural gas transmission systems of Belarus, Latvia and Russia. The largest volumes of natural gas are imported via the gas transmission pipeline from Belarus and are transported to customers of Lithuania and in transit to customers of the Kaliningrad Region, Russian Federation. Gas transportation via the Lithuania-Latvia cross-border gas interconnector is bi-directional.

Figure 39. Lithuanian natural gas transmission network (Courtesy of AmberGrid).

3.2.1 Time-dependent reliability estimation of 330 kV power network

Due to data availability, the analysis will be confined to the 330 kV power transmission network part. Time-dependent analysis of network reliability requires that specific alignment of the data, since different lines were put into operation at different time periods therefore resulting at differing ages at any calendar year. For further analysis, 13 overhead lines were considered, and data for it graphically represented in Fig. 40 (even though the visualization is very chaotic due to large number of zeroes, the overall decreasing trend is clear).

The data are quite scattered; several lines had just one or two outages over the entire observation period, while others were much more prone to faults. Therefore, the only way to use classical statistical tools is in a case where all data would be pooled into one sample. This is because for lines, where just one or two points of data are available, no reasonable inference could be performed.

First of all, it is necessary to screen for a model with best fit to the data. According to the methodology developed in earlier chapters, the goodness of fit will be analysed through DIC measure. The governing model was chosen to be Poisson as there are count data. The intensity function for the Poisson distribution was selected to be power law, exponential, linear and constant (i.e., time-independent) trend. In addition, hierarchical extensions were checked as well. The evidence (obtained in validation part) of heterogeneity in the outage data forces one to expect that the same phenomena will be present in other power networks deployed over the wide area.

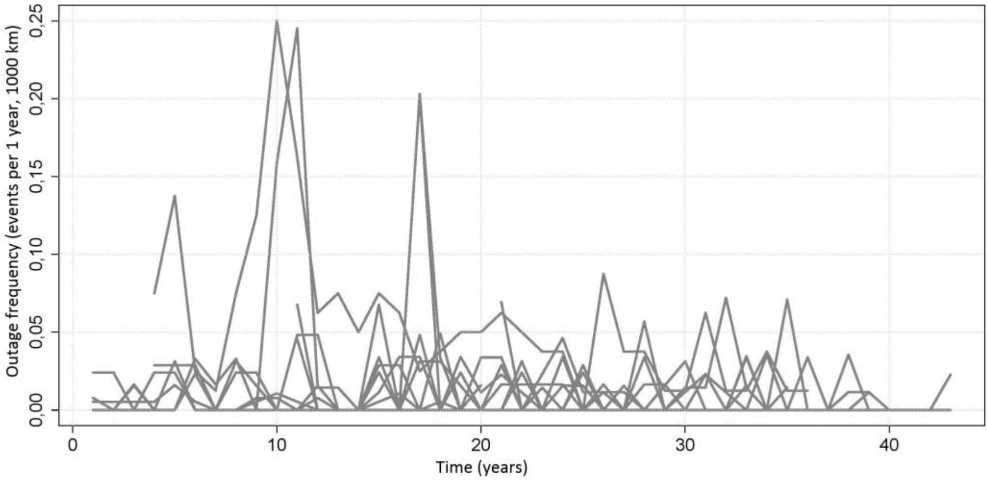

Figure 40. Plot of outage frequency data for all 13 lines.

The DIC values for all the models are presented in the following table (N – non-hierarchical, H – hierarchical, C – constant, P – power low, R – exponential, L – linear models).

Table 16. DIC values for considered models.

	NC	NP	NE	NL	HC	HP	HE	HL
DIC	3795.2	3794.4	3794.2	3773.0	3953.5	3399.3	3489.028	5079.80
pD	1	1.89	1.6	1.93	120.77	21.42	20.79	136.1

As the DIC measure shows, there is a significant increase in the predictor power if hierarchical model extensions with power law is considered. In other words, outage rate is currently decreasing according to the power law. A rather strange behaviour is seen in hierarchical linear model, as DIC values show that it is the worst of all considered models. Therefore, the Bayesian hierarchical extension of power law is chosen for further inference on network reliability. Full model mathematically is expressed as follows:

$$\begin{aligned} &X_{i,t} \mid \theta_{i,1}, \theta_{i,2} \sim Poisson\left(L_i \lambda\left(\theta_{i,1}, \theta_{i,2}, t\right)\right), t = \overline{1,43}, \ i = \overline{1,13}, \\ &\theta_{i,\mathrm{k}} \mid \mu_k, \sigma_k \sim N\left(\mu_k, \sigma_k^2\right), \ k = 1,2, \\ &\pi\left(\mu_1, \sigma_1, \mu_2, \sigma_2\right) \propto 1, \end{aligned} \tag{57}$$

where $\lambda\left(\theta_{i,1}, \theta_{i,2}, t\right) = \exp\left[\theta_{i,1} + \theta_{i,2} \ln(t)\right]$, L_i is the length of ith overhead line, $\pi\left(\mu_1, \sigma_1, \mu_2, \sigma_2\right)$ is the prior distribution which is chosen to be flat. The first line of the model expresses the uncertainty of the data for each line separately. The second line puts a model describing the uncertainty arising due to the heterogeneity of operating conditions. Without the second level model, outages of each line would be modelled separately, as if the data for other lines does not exist. Such choice would lead to the large uncertainty levels and in some cases (e.g., where there are just one or two data points) to invalid inference. The second level can be thought of as being a bridge for information: information in the data for one network line is transferred partially to all other lines through this second level model. In this way, each line model is strengthened by the information carried in other lines' data. In addition, the second level model expresses the global character of the uncertainty (usually called between-source uncertainty).

Estimates of model parameters show strong evidence of decreasing outage rate trend - the most part of θ_2 posterior distributions for each line lies below the zero value. Since this particular parameter describes the direction of the trend, it is concluded that overall failure rate of 330 kV network lines is decreasing. However, this conclusion cannot be extended to the rest of the network, e.g., to the distribution part. This is because the distribution part suffers very much from the age-related causes of outages (see an excellent book on ageing power delivery infrastructures by Willis *et al*. [129]); therefore, it may be conjectured that the outage rate would be increasing. Transmission part of the network usually receives much more attention from the maintenance personnel. In addition to that, components used are often more immune to the degradation processes.

It is interesting to observe how estimated parameters are "distributed" over the entire network. Fig. 41 represents the posterior expectations of each line trend parameters θ_1 and θ_2.

As for parameter θ_1, which represents initial failure rate at the beginning of the operation, it looks like some dependencies on connectivity might be traced, i.e. lines connected to the same node have similar values of parameter. On the other hand, parameter θ_2 describing the speed of the trend, demonstrates more homogeneity (values are distributed around -0.5) and there are few lines with significantly different parameter values. The homogeneity of the trend parameter suggests a conclusion (or more likely a conjecture), that the speed at which outage rate of the 330 kV lines decrease is independent of the geographical location and is mainly influenced by the inspection and maintenance efficiency.

The time-dependency also transfers directly to the overall network reliability. Consider a flow of power from node 11 to node 1. Due to the structure of the network, there are two paths. Namely, paths 11-3-1 and 11-10-9-1, which will be denoted by P_1 and P_2, respectively. Suppose one is interested in the possibility that for one day, power flow between nodes 11 and 1 will be discontinued due to the outages of lines. Therefore, it is necessary to calculate the reliability of the network part between nodes 1, 9, 10, 11 and 3.

As the failure rate is time-dependent, it is also desired to trace the behaviour of network reliability over a long time. Therefore, a prediction for 10 years was made under two cases of models: hierarchical time-dependent and non-hierarchical time-independent models (Fig. 43). There is a significant difference between the predictions, showing sever underestimation of reliability in case when constant failure rate was assumed.

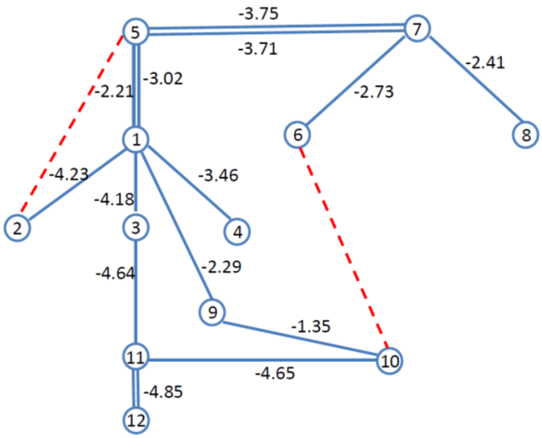

Figure 41. Posterior expectations for parameter group θ_1

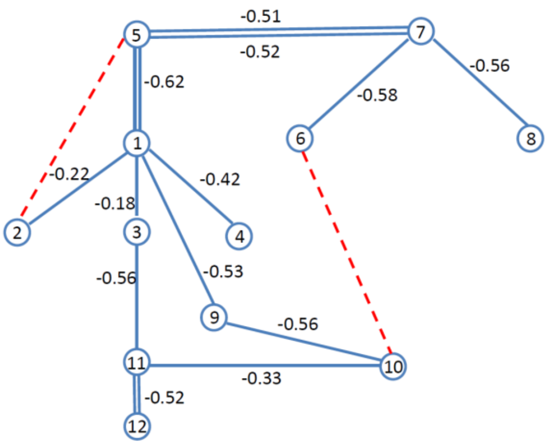

Figure 42. Posterior expectations for parameter group θ_2

It has already been stated, at the stage of time-dependent Bayesian inference tools analysis, uncertainty bounds allow making better-informed decisions. In addition, the time-dependency uncovered by this analysis enables one to plan network reliability for a long term and with better accuracy. This is justified by the comparison with reliability level, when the assumption of constant failure rate is considered (Fig. 43). Evidently, the constant failure rate assumptions significantly overestimate the future outage rate, which is decreasing.

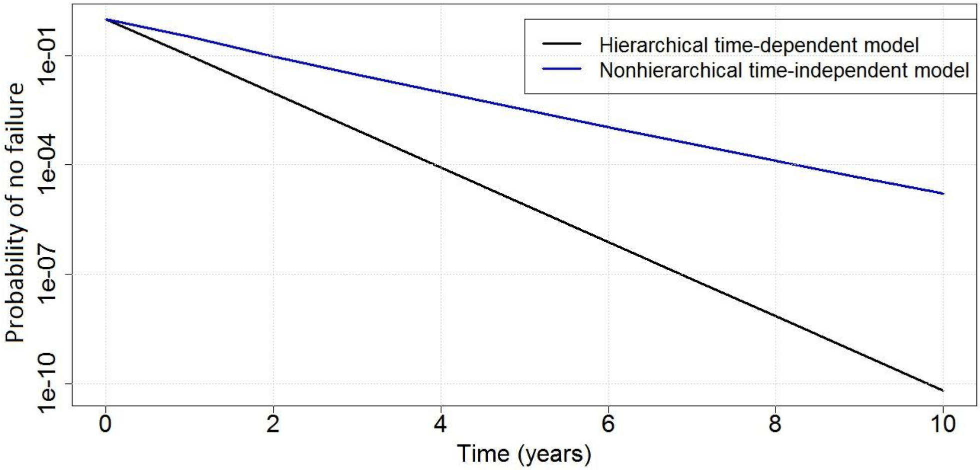

Figure 43 Probability of event that now electricity flow is available from node 11 to node 1 (Logarithms of probabilities were taken for the more clear view).

Now consider that two additional lines are put into operation. Their age would be 0, i.e. time since the start of operation is 0 years. There is a question of how to define the outage rate of new lines, since there are no data to make any inference about it. However, one has to look at the hierarchical model that was presented above. The first stage prior distribution (i.e., Gaussian distribution for parameters θ_1 and θ_1) is a distribution of the entire population of lines, i.e., it is an assumption that parameter values are some unobservable data and come from those two distributions. Hence, it is justifiable to use these distributions to define values for the outage rates for the new lines. It is done as follows:

$$\begin{aligned} \hat{\theta}_{1,k} &\sim N\left(E\left[\mu_1\right], \mathrm{E}\left[\sigma_1^2\right]\right),\ k=1,2, \\ \hat{\theta}_{2,k} &\sim N\left(E\left[\mu_2\right], \mathrm{E}\left[\sigma_2^2\right]\right),\ k=1,2, \end{aligned} \tag{58}$$

where $\hat{\theta}$ are trend parameters for the new lines, k - is the index of the new line, expectations are taken with respect to posterior distribution of second stage parameters. Clearly, the addition of a line between points 5 and 2 does not change reliability of the flow between nodes 11 and 1. Therefore, first, a line is added between nodes 10 and 6, and after that, another line is added. The results are shown below.

Results (Fig. 44) demonstrate that the addition of one line from point 10 to 6 reduces the probability of the event “there will be no flow from 11 to 1” by approximately 4 orders, while the second additional line reduces the probability by additional 2 orders. In addition, the time-dependent analysis enables to assess the dynamics of influence of additional lines in terms of the time variable. The usefulness of having such additional information is that maintenance and inspection planning can be made more efficient, as periodic adjustments may be made depending on the probability (failure rate) value.

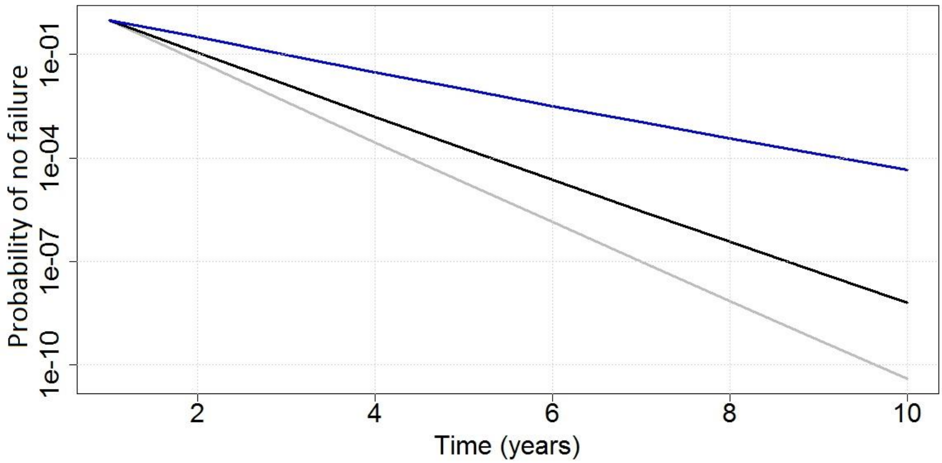

Figure 44. The effect to the transmission reliability of additional lines.

3.2.2 Reliability of time-dependent Lithuanian gas transmission network

The application of proposed model is continued to a set of available different databases or data samples. As presented previously, there are samples from OPS, EGIG, UKOPA, NEB and Lithuania. Hence, overall there are five samples, which traditionally would not be analysed jointly, but the current methodology resolves this issue. Differences in each data samples have already been presented: EGIG, UKOPA and NEB could be regarded as very similar data samples, since incident criteria are almost identical; OPS used three criteria and a sample from Lithuania is related to two criteria. Therefore, in order to apply the proposed hierarchical model, it is necessary to construct a set C of criteria. Failure frequencies for all considered databases are presented in Fig. 45.

Comparing all data samples one can get a clear impression of failure frequency similarity in the decade from year 2000 until 2010. However, OPS database use different criterion for data inclusion. While other countries makes a record of all the leakages, OPS contains data with damage greater than $50,000 or with leakage more than 84000 m^3. Hence, if the same criterion as in other databases would be used in OPS database, overall failure frequency for North America gas transmission system would be considerably higher. This is due to the fact that more extreme weather conditions dominate in North America as compared to Europe, Canada, United Kingdom or Lithuania as well. Hence, OPS data could be considered as kind of an outlier. The convergence of failure frequencies of other databases (including Lithuanian) to almost the same level is particularly interesting. While at the beginning of operation of different networks the failure frequencies are very different (this represents the difference in initial history of different gas transmission networks), at the end they become quite similar. This is most certainly due to the homogenization of the practices of inspection and maintenance.

Because the sample of failures for the Lithuanian gas transmission system is very small, the degree of uncertainty in the estimates is high. The use of only this sample may provide very misleading estimates. However, there is information from other natural gas transmission systems, namely, from OPS, NEB, UK, EGIG and OPS databases.

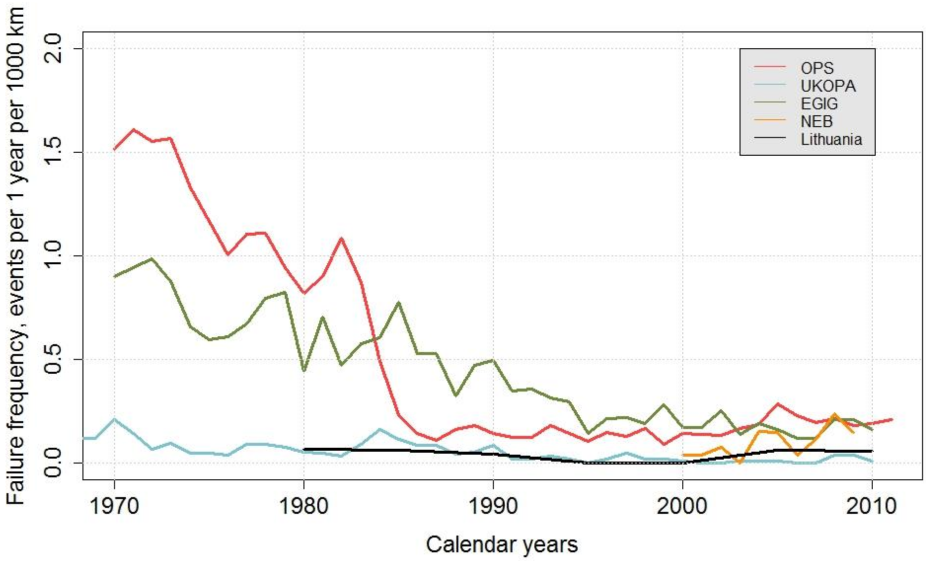

Figure 45. Comparison of failure frequencies of all databases used in the analysis

Due to the reasons discussed above, OPS database should be included with great care. Therefore results with and without OPS database included will be presented (it is a sort of sensitivity analysis). Therefore, there are five data samples which will be used to strengthen inference about the reliability of Lithuanian gas transmission system.

Since 2004, Lithuania collected information about incidence with gas leakage (denote it C_2), so until this moment, previously used criterion (gas explosion) forms a subset of C_2 . Denote the first criterion C_1, then there is a relation $C_1 \subseteq C_2$. $C = (C_1, C_2)$ transformed as follows:

$$C' = \{C_1'(\text{Explosion}), C_2'(\text{Gas leakage without explosion})\}.$$

Evidently, criteria C_1' and C_2' are mutually exclusive, and $C_2 = C_1' \cup C_2'$, $C_1 = C_1'$. This leads to probability vector $p = (p_1', 1)$ for C_1.

It is difficult to relate probability vectors for different databases through hierarchical structure; the number of components differs, and the criteria are not identical. Hence, a hierarchical structure is applied just to parameters of trend $\lambda(t) = \lambda(t;\theta)$.

Mathematical representation of hierarchically structured whole data is as follows:

$$\text{Level I}\begin{cases} X_t^1 \mid \theta^t \sim \begin{cases} Poisson\left(E_t^1\lambda\left(t;\theta^1\right)\right), t=\overline{1,14} \\ Poisson\left(E_t^1\lambda\left(t;\theta^1\right)\left(1-p_1'-p_2'\right)\right), t=\overline{15,33} \\ Poisson\left(E_t^1\lambda\left(t;\theta^1\right)\left(1-p_1'\right)\right), t=\overline{34,42} \end{cases} \text{ for OPS case} \\ X_t^2 \mid \theta^t \sim \begin{cases} Poisson\left(E_t^2\lambda\left(t;\theta^2\right)p_1'\right), t=\overline{19,34} \\ Poisson\left(E_t^2\lambda\left(t;\theta^2\right)\right), t=\overline{35,44} \end{cases} \text{ for Lithuania case} \\ X_t^2 \mid \theta^t \sim Poisson\left(E_t^k\lambda\left(t;\theta^k\right)\right), \mathrm{t}=\overline{3,5} \text{ for EGIG, UKOPA, NEB cases} \end{cases}$$

$$Level\ II \begin{cases} \theta_1^k \sim LN\left(\mu_1,\sigma_1^2\right) \\ \theta_1^k \sim LN\left(\mu_1,\sigma_1^2\right) \end{cases}, \mathrm{t}=\overline{1,5}$$

$$Level\ III \quad \pi\left(\mu_1,\mu_2,\sigma_1,\sigma_2\right)=\pi\left(\mu_1,\sigma_1\right)\pi\left(\mu_2,\sigma_2\right)$$

Here, an assumption is made that $\lambda\left(t;\theta\right)$ has two unknown parameters $\theta^k=\left(\theta_1^k,\theta_2^k\right)$, the first of which is positive, hence has lognormal distribution, and the second one is normally distributed. Normality and log-normality are assumptions (could be changed by another), and their sensitivity has not been investigated here. This is the shortcoming of this research, but at this stage, while presenting application of the methodology, this issue is not the focus.

There is a particular interest in how well the hierarchical model covers all observed data, and how much, including the international experience on the incidents in pipeline networks, it strengthens inferences on Lithuanian gas transmission network.

The first issue, performance of the model with regard to all data samples, is not easy to address. Since the number of data samples is small, parameters θ^k form the so-called unobservable data sample, which is small. Due to this reason, the uncertainty in the second level hierarchy parameter estimates will be high, resulting in very broad credibility intervals. Hence, the influence of the hyper-prior (Level III) distribution has to be investigated. Three prior distributions of the following forms will be considered [49]:

- $\pi\left(\mu,\sigma\right)\propto 1$ - uniform distribution;
- $\pi\left(\mu,\sigma\right)\propto \sigma^{-2}$ - Jeffrey’s prior;

- $\pi(\mu,\sigma) \propto \sigma^{-1}$ - invariant Haar measure.

For the sake of consistency, failure rate trend $\lambda(t;\theta)$ will be considered alone, i.e., when all incidents are observed. The influence of the prior distributions on the trend line is negligible; no significant differences were observed, hence enabling to conclude that even under small sample of unobservable parameters, the parameter estimates are very robust. However, the impact is much more pronounced when the estimation of credibility intervals is examined, as Fig. 44 reveals.

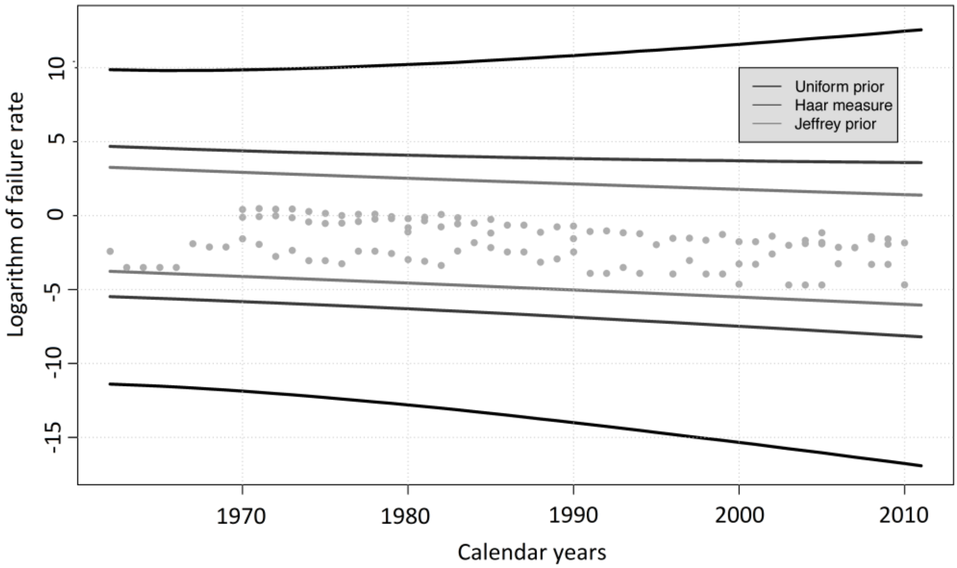

Figure 46. Comparison of the influence of hyper-prior distribution on Bayesian credibility intervals.

This prior is not the only model that heavily influences posterior results, though. Due to very small sample in Lithuanian gas network case, posterior expectation of probability for incident to occur under one of criterion applications is highly sensitive to prior distribution, which was a beta distribution with shape and scale parameters *a* and *b*. To gain insight in the level of sensitivity, each parameter (*a* and *b*) was varied over the range from 0 to 500. The 3D plot (Fig. 47) shows the final results.

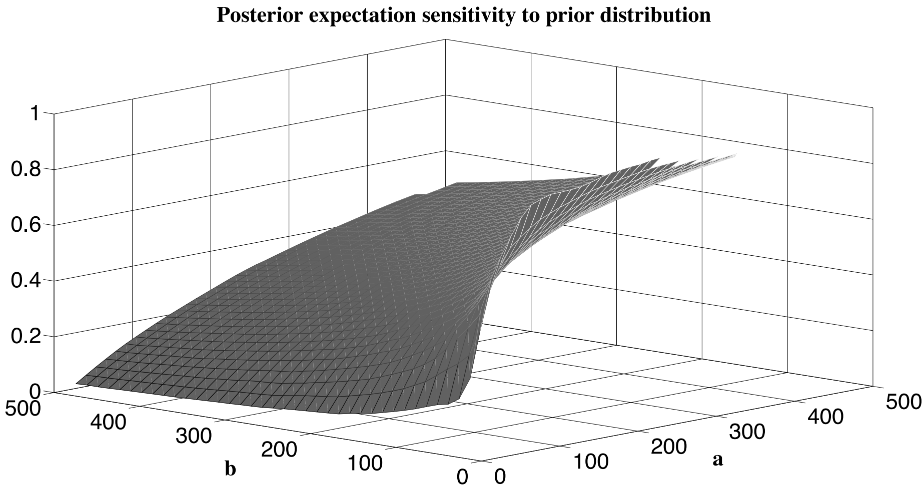

Figure 47. Analysis of prior distribution influence on probability of criterion application.

The steepest changes are for parameter values close to zero; then, posterior expectation does not react to the prior distribution influence so extremely. This is because for small Beta distribution parameter values, the data, no matter how small the sample is, still provide some information. While for high parameter values, the prior completely overshadows the information provided by statistical sample. This sensitivity analysis shows how important in this case the prior distribution selection is, but it does not quite answer the question of what parameters of beta distribution should be selected. It is recommended using parameters (1, 1), or in other words, using uniform distribution over the interval [0; 1], as uniform distribution does not give a priority to any value in that interval. In fact, the information about the parameter in question is evenly spread over the interval and each value "attracts" the posterior distribution with equal strength. Hence, the dominating influence cancels out.

The widest intervals are under the uniform hyper-prior distribution; hence, it is the least informative distribution. However, the bounds in this case are extremely wide (the figure is in log-scale) and clearly unreasonable: any experienced expert would discard such bounds as carrying no useful information. Hence, it can be said that by using such subjective knowledge, uniform hyper-prior distribution is discarded from the further analysis, and more informative ones are turned to. Upper bounds for Haar measure and Jeffrey's hyper-prior are still very high, resulting to 107 and 26 incidents for 1000 km of pipelines, accordingly. A subjective judgement is used again, and the most informative prior out of three considered, that is Jeffrey's prior, is selected for this analysis. This distribution represents the most realistic situation of failure frequencies in pipeline networks, because the highest observed frequency is 1.61 incidence for 1000 km at OPS database; thus, 107 would be clearly too high even for the credibility bounds. In Jeffrey's case, a good coverage of the incident frequencies plotted from all data samples is also observed. Hence, the validity of this prior model is supported as well.

Now, the inference for Lithuanian pipeline network will be discussed. It is interesting that inclusion of OPS database does not provide anything significant (see Fig. 48) – posterior estimate of failure rate trend function is almost unaltered if additional data from OPS database are included. Except for the uncertainty estimate – small difference can be observed in the predictions for these two cases. This insensitivity to the data from OPS database shows a very useful feature of Bayesian procedure – the robustness to the outliers (it was also observed in the case of North American power network reliability estimation).

Therefore, one is left with a decision – to use OPS data sample, or not to use it. Due to the concerns expressed about the data of this database (i.e., that they are outliers as compared to the rest of the data samples), it will be excluded from the analysis that follows.

Due to the small size of the network and the incident criterion used until 2004, the data are quite scattered, and the inferences based on them alone would be questionable. But until now, nothing was to be done: at best, those few data could be translated into failure frequency estimate and qualitatively compared to international experience to validate that the situation in network reliability was not worse or better than on the general level.

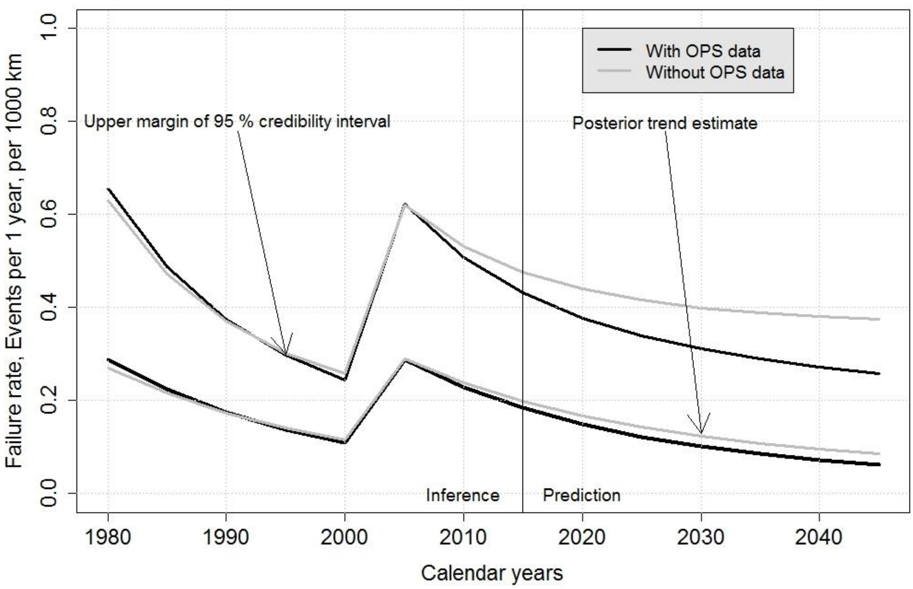

Figure 48. Comparison of estimates with and without OPS database sample

When comparing the estimated trend (see Fig. 49, inferred zone), the hierarchical model generally provides wider credibility bounds than those obtained from a non-hierarchical variant of CDP model (only considering Lithuanian case).

This is due to the fact that hierarchical structure of the model allows incorporating and quantifying additional level of uncertainty, i.e., variation between different databases is now accounted as well. In addition, two data points that were collected under new incident criterion (since 2003) are less underestimated by hierarchical model.

On the other hand, when failure frequency was predicted for the next 40 years (see Fig. 49, predicted zone), uncertainty about the future failure frequency increased for CDP. While in case of hierarchical model for the same prediction period, the uncertainty bounds increase relatively slowly. This is due to the fact that this additional information from various databases, the same information that gave rise to wider uncertainty bounds in inference part, is now making future predictions more certain (as it involves clearly revealed decreasing trend). In other words, one is more informed about the future state of network reliability than one would be if just a small Lithuanian pipeline network data sample were used.

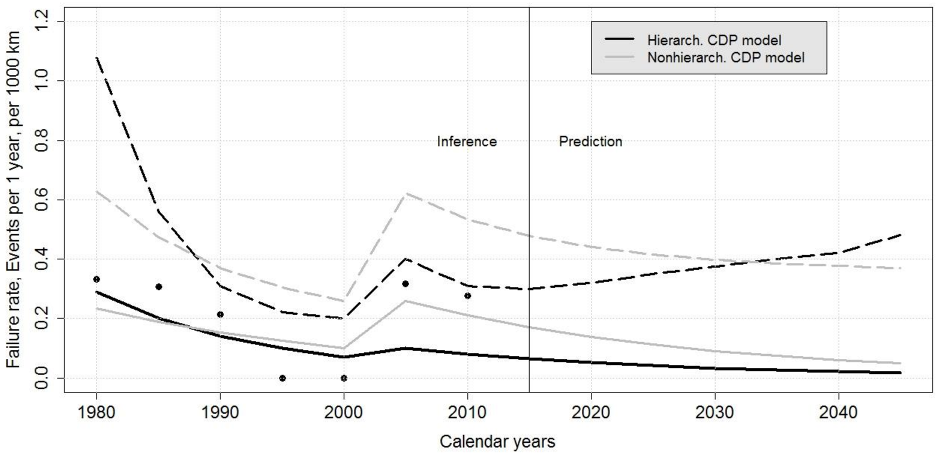

Figure 49. Inferred and predicted failure frequencies for simple CDP and their hierarchical modification for Lithuanian gas transmission network (sudden change of the failure rate is due to change of incident registration criterion).

Reliability function was also predicted for the next 20 years of 1000 km of Lithuanian pipeline network (see Fig. 50) – probability that a failure will occur in next 20 years is approximately 0.4 with credibility interval [0.15; 0.8]. The differences between hierarchical and non-hierarchical variants of CDP are remarkable. Failure probability in next 20 years according to the non-hierarchical CDP model is 0.1 with uncertainty estimate [0; 0.85]. The consequences of using just simple Poisson model (without incorporating information about the criteria) or regular CDP model to predict reliability for Lithuanian gas transmission network would be underestimation of failure frequency or overestimation of the true level of reliability.

A short review of international natural gas pipeline network incident databases, their differences and difficulties arising in inferences about data contained in there was provided.

It has been demonstrated that Criteria-Dependent Poisson model provides a coherent way to handle changes of data collection criterion and to use this information to obtain more correct inferences on the current state of gas network as well as more certain predictions.

Nothing has yet been said about the dynamics of probability vectors. It was treated as time-independent. Theoretically, there is no difficulty to impose such functional form on it with dependence on time variable. However, it is not clear what forms might be assumed, and whether they are needed at all. Hence, further testing and probably some theoretical in-depth analysis would be necessary to answer this.

In addition to the presentation of CDP model, it was extended to hierarchical framework, which allows using information from various databases. This enables to quantify an additional level of uncertainty, arising due to the variation of different pipeline operating conditions as well as maintenance programs. Through hierarchical model, HCDP information from one database is supported by information contained in other databases. However, such information sharing does not overshadow probabilistic evidence already contained in the single database.

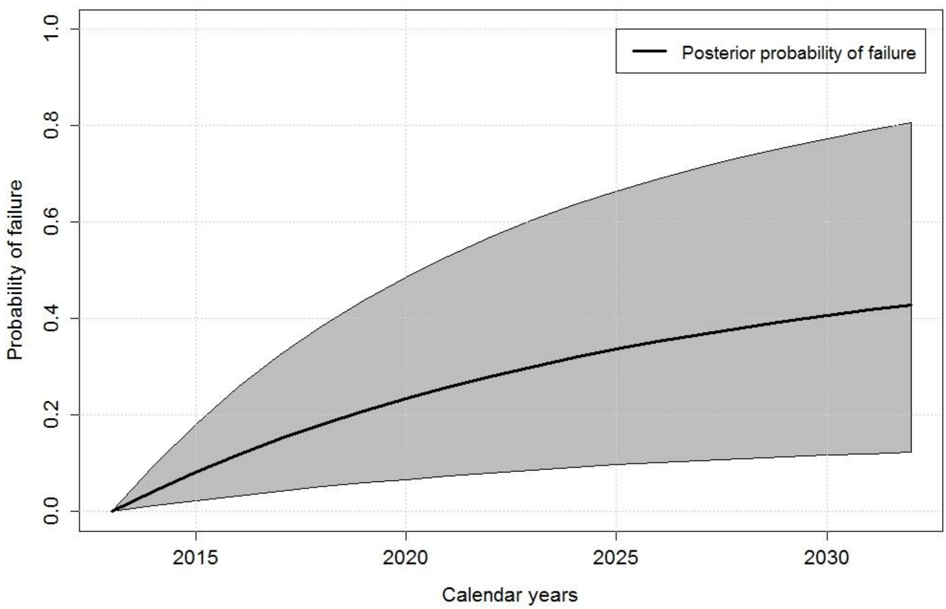

Figure 50. Posterior expectation of probability of failure in the Lithuanian natural gas transmission network

CDP model for OPS database sample was demonstrated, while hierarchical extension HCDP was applied to assess the level of Lithuanian pipeline network reliability. In case of OPS data sample, CDP enabled to use all collected information about the incidents in North American pipeline network. Curiously, this increased amount of information did not result in more informed inferences (significantly lower uncertainty) about current state of OPS gas network reliability, but when a forecast was made for next 40 years, uncertainty about future value differed significantly. Hence, due to ability to incorporate information collected under various incident criteria, the current model enabled to predict future reliability more certainly. As a result of this more certain future, it provides a way to make more informed decisions, e.g., when planning maintenance.

Hierarchical extension of CDP was used for Lithuanian pipeline network due to a small size statistical sample. In addition, as it has been already discussed, there is one time point when incident criterion was changed. Under new criterion, there are just two data points as abnormalities to the previous ones, and no valuable inferences could be made. Hence, CDP allows incorporating information collected under the older criterion, while HCDP allows supporting information from Lithuanian database sample with information contained in other international databases. It was demonstrated that reliability of Lithuanian pipeline network could now be predicted with more certainty as well. Information borrowed through hierarchical model structure enables more informed future predictions as compared to the case when just local country-specific but rare information and CDP were used. In addition, hierarchical CDP model showed a better fit for the data points observed after the change of criteria and hence, provides a more realistic way of failure frequency prediction.

Implementation of hierarchical CDP model presented some difficulties with regards to prior distribution selection. It was demonstrated how sensitive posterior results can be to the prior distribution of between-database variation parameters as well as to the prior distribution of probability vectors. However, practical advises were given on how to proceed in these ways, and how to obtain posterior results of higher certainty.

3.3 Result of the section

This chapter was divided into two major subsections. The first one is devoted for the analysis of the methodology and application to various data samples. The second section is for the application of methodology to the cases of Lithuanian energy networks.

In this section, analysis of the methodology from various aspects was performed. This analysis was carried out by employing various case studies. The case when failure data are only time-dependent provided insight into how the methodology works when no additional assumptions about data heterogeneity are made. Deviance information criterion showed evidence of superiority over the posterior p-values in its discriminatory power between various models as well as interpretability.

Investigation of small sample problem lead to a conjecture that Bayesian procedure provides sufficiently robust results – even with very small sample, on average, estimates concentrates near the true parameter values. Hence, there is evidence that methodology can safely be used in cases of heterogeneous data with small samples.

Next, analysis of North American power transmission and natural gas transmission systems was carried out. This shed some light on the applicability of methodology for the energy networks. More specifically, hierarchical extension of Borrel-Tanner model resulted in the higher probabilities of larger power network cascade. This is particularly important, as non-hierarchical version of the model underestimates these probabilities, which are in disagreement with data observed in reality. Application of the developed Criteria-Dependent Poisson model resulted in the reduced uncertainties in the forecast of the reliability of North American gas transmission network.

Finally, Lithuanian power transmission and natural gas transmission networks were considered, and reliability estimation as well as prediction were performed.

CONCLUSIONS

Development and demonstration of the methodology, which enables a more comprehensive analysis and assessment of time-dependent reliability and its uncertainty for heterogeneous energy networks, resulted into following conclusions:

1. Power transmission network cascading outage probability estimate, obtained by application of hierarchical generalization of Borrel-Tanner model, is closer to the estimate based on the observed data, as compared to the results of the non-hierarchical model. This estimate for the largest (spanning over 11 lines) considered cascading failure is $4 \cdot 10^{-3}$ while according to the assessment based on the non-hierarchical model it equals $1 \cdot 10^{-5}$ and a more accurate estimate were obtained by means of hierarchically generalized model;
2. Failure rate of natural gas transmission network was modelled in greater detail with developed data registration criteria-dependent Poison model by taking into account time-dependent dynamics of uncertainty. For this reason, even if there is a significant change in data collection criteria and failure data, the uncertainty level of predictions does not change significantly;
3. Application of the methodology to the gas network with compressor station led to the determination of 8 % difference between energy consumption in ideal (i.e., without failures) and real networks. It was demonstrated that energy consumption in compressor station is directly proportional to the failure rate of network pipelines;
4. Outage rate of Lithuanian electricity transmission network lines can be modelled more comprehensively by applying the developed methodology and taking into account data and uncertainty dependence on time. It was demonstrated that outage rate is currently decreasing according to the power law;
5. Failure rate of Lithuanian gas transmission network pipelines can be modelled more comprehensively by applying the developed methodology and taking into account data dependence on time and on data registration criteria. Estimate of probability of failure over the next 20 years for 1000 km pipeline section is 0.4 (as obtained by hierarchical model), while uncertainty interval is smaller as compared to the results of the non-hierarchical model (probability is 0.1).

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APPENDIX

Bayesian posterior distribution approximation

In all cases of Bayesian inference presented in this thesis, posterior distribution approximations were needed due to the complex nature of the models. The simplicity and usefulness of the so-called Markov Chain Monte Carlo (MCMC) methods led to the choice of this particular group without any exceptions. Therefore, it is reasonable to briefly sketch here the main ideas underlying MCMC methods.

Suppose a random sequence is generated $\{X_0, X_1, X_2, ...\}$ in such a way that at every moment $t \geq 0$ the next state X_{t+1} is sampled from distribution $P(X_{t+1} | X_t)$. This distribution depends only on the current state and not the previous history. This sequence is called a *Markov Chain,* and distribution $P(\cdot|\cdot)$ is called a *transition kernel* of that chain. Under some particular regularity conditions, the chain eventually will "forget" its initial state X_0 and distributions $P^t(\cdot | X_0)$ will converge to a stationary distribution, which is the same as posterior distribution $\pi(\cdot)$ that one wishes to approximate, i.e., $X_t \sim \pi(\cdot)$.

One of the main ways to describe the construction of transition kernel $P(\cdot|\cdot)$ is the Metropolis-Hastings algorithm[3,4]. At each moment t, the next state X_{t+1} is chosen by first sampling a point Y from proposal distribution $q(\cdot | \mathrm{X}_t)$. For example, $q(\cdot | \mathrm{X})$ may be a multilinear normal distribution with expectation X and fixed covariance matrix. Point Y is accepted with probability $\alpha(X_t, Y)$, expressed as follows:

$$\alpha(X,Y) = \min\left(1, \frac{\pi(Y)q(X|Y)}{\pi(X)q(\mathrm{Y}|\mathrm{X})}\right).$$

If point Y is accepted, then $X_{t+1} = Y$, if rejected, the chain stays at the same state, i.e., $X_{t+1} = X_t$.

Proposal distribution $q(\cdot|\cdot)$ may have any form, and the produced Markov chain still converges to the necessary posterior distribution. This is because Metropolis-Hastings method generates distribution, which satisfies the so-called detailed balance equation.

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[3] Hastings, W.K. Monte Carlo Sampling Methods Using Markov Chains and Their Applications. Biometrika 57 (1): 97–109, 1970;

[4] Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., Teller, E. Equations of State Calculations by Fast Computing Machines. Journal of Chemical Physics 21 (6): 1087-1092, 1953.

However, Metropolis-Hastings was inefficient for reliability models applied to Lithuanian energy networks. The generated Markov chain in these cases could hardly achieve the stationary distribution. For this reason, modification of Metropolis-Hastings method was employed – so-called Adaptive[5] Metropolis-Hastings method.

The main idea behind this algorithm is not to fix the covariance matrix *a priori*, but to adapt it according to the previous states of the Markov chain. At every moment, covariance matrix may be calculated as follows:

$$C_t = \begin{cases} C_0, & t \le t_0 \\ s_d \operatorname{cov}\left(X_0, \ldots, X_{t-1}\right) + s_d \varepsilon I_d, & t > t_0 \end{cases},$$

where s_d is the parameter dependent on the model dimensions, C_0 is the initial covariance matrix used until the moment t_0; ε is necessary to ensure the non-degeneracy of the covariance matrix.

It should be noted that algorithmic realizations were carried out by the open source software **R[6].**

Conditional posterior distributions for hierarchical Bayesian model

In Section 2.2, it was stated that, up to normalization constant, posterior probability density function of Poisson-Normal hierarchical model is as follows:

$$\pi\left(\theta_1^1, \ldots, \theta_1^N, \theta_2^1, \ldots, \theta_2^N, \mu, \sigma\right) \propto$$
$$\left(\sigma_1 \sigma_2\right)^{-(N+1)} \exp\left\{-\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i\left(t, \theta_1^i, \theta_2^i\right) + \sum_{i=1}^{N}\sum_{t=1}^{T} y_t^i \ln \lambda_i\left(t, \theta_1^i, \theta_2^i\right) - \frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(\theta_1^i - \mu_1\right)^2}{\sigma_1^2} + \frac{\left(\theta_2^i - \mu_2\right)^2}{\sigma_2^2}\right]\right\}.$$

Conditional density function of any parameter can be obtained by considering all other parameters being constants, i.e. one can drop all multiplication constants involving all parameters except those, for which one wants to obtain conditional density function. Thus, density function of parameters $\theta_1^1, \ldots, \theta_1^N, \theta_2^1, \ldots, \theta_2^N$ conditional on parameters μ, σ, y is, up to a normalization constant, as follows:

$$\theta_1^1, \ldots, \theta_1^N, \theta_2^1, \ldots, \theta_2^N \mid \mu, \sigma, y \sim$$
$$\exp\left\{-\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i\left(t, \theta_1^i, \theta_2^i\right) + \sum_{i=1}^{N}\sum_{t=1}^{T} y_t^i \ln \lambda_i\left(t, \theta_1^i, \theta_2^i\right) - \frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(\theta_1^i - \mu_1\right)^2}{\sigma_1^2} + \frac{\left(\theta_2^i - \mu_2\right)^2}{\sigma_2^2}\right]\right\},$$

Conditional density function of scale parameters σ_k has the following form:

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[5] H. Haario, E. Saksman, J. Tamminen, An adaptive Metropolis algorithm. Bernoulli 7, 223-242, 2001.

[6] R Core Team (2013). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/.

$$\pi\left(\sigma_k^2 \mid \mu_1, \mu_2, \theta_1^1, \ldots, \theta_1^N, \theta_2^1, \ldots, \theta_2^N\right) \propto \left(\sigma_k\right)^{-(N+1)} \exp\left\{-\frac{\frac{1}{2}\sum_{i=1}^{N}\left[\left(\theta_k^i - \mu_k\right)^2\right]}{\sigma_k^2}\right\}.$$

It is exactly of the form of inverse Gamma distribution with parameters $\frac{N}{2}-1$ and $\frac{1}{2}\sum_{i=1}^{N}\left(\theta_{ki}-\mu_k\right)^2$, i.e.:

$$\sigma_k^2 \mid \mu_k, \theta_{1i}, \theta_{2i} \sim Inverse\_Gamma\left(\frac{N}{2}-1, \frac{1}{2}\sum_{i=1}^{N}\left(\theta_{ki}-\mu_k\right)^2\right), k=1,2,$$

Analogously, conditional density function for location parameter μ_k is obtained:

$$\pi\left(\mu_k \mid \sigma_k, \theta_{1i}, \theta_{2i}\right) \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(\theta_k^i-\mu_k\right)^2}{\sigma_k^2}\right]\right\}.$$

Since

$$-\frac{1}{2}\sum_{i=1}^{N}\left[\frac{\left(\theta_k^i-\mu_k\right)^2}{\sigma_k^2}\right] = -\frac{1}{2\sigma_k^2}\sum_{i=1}^{N}\left[\left(\theta_k^i-\mu_k\right)^2\right] = -\frac{1}{2\sigma_k^2}\left(N\mu_k^2 - 2\mu_k\sum_{i=1}^{N}\left(\theta_k^i\right) + \sum_{i=1}^{N}\left(\theta_k^i\right)^2\right) =$$

$$= -\frac{N}{2\sigma_k^2}\left(\left(\mu_k - \frac{1}{N}\sum_{i=1}^{N}\left(\theta_k^i\right)\right)^2 + \frac{1}{N}\sum_{i=1}^{N}\left(\theta_k^i\right)^2 - \left(\frac{1}{N}\sum_{i=1}^{N}\left(\theta_k^i\right)\right)^2\right),$$

then, again, by dropping all multiplication constants independent of μ_k,

$$\pi\left(\mu_k \mid \sigma_k, \theta_{1i}, \theta_{2i}\right) \propto \exp\left\{-\frac{N}{2\sigma_k^2}\left(\mu_k - \frac{1}{N}\sum_{i=1}^{N}\left(\theta_k^i\right)\right)^2\right\},$$

which is exactly of Gaussian distribution form, up to a normalization constant,

$$\mu_k \mid \sigma_k, \theta_{1i}, \theta_{2i} \sim N\left(\frac{1}{N}\sum_{i=1}^{N}\theta_{ki}, \frac{1}{N}\sigma_k^2\right), k=1,2.$$

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