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Empirical calculation method of residual flexural tensile strength $f_{R,1}$ of SFRC

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1. Introduction

Fibre reinforced concrete (FRC) is cement based composite material. Basically, the concrete has a low tensile strength, low strain capacity, and it fails in brittle manner. The addition of the fibre improves the mechanical properties of concrete, especially the post-cracking properties [1, 2].

The steel fibre reinforcement has been studied and developed intensively over past four decades, and now it is lots of different fibre in the market. The fibre can be made of different materials, such like steel, carbon, synthetic, glass and others. However, the steel fibre is the most commonly used for the structural purposes. Length, shape, and the cross section of the steel fibre are various. Numerous researches have been performed to develop better bond between the fibre and matrix of concrete [3]. Despite of that the circular cross section hooked end steel fibre is one of the most widely used because of the simplicity of manufacturing and the good bonding parameters [1, 2, 4, 5].

It is known that fibres bridge the cracks and transfer the tensile stress across the cracked sections. Therefore, the concrete becomes more ductile and durable material, cracks are restricted, and even bearing capacity of the member can be enhanced. Nevertheless, the application of the steel fibre is still limited. There are two main reasons for these limitations: first – lack of the generally accepted design method, second – complicated (experimental) determination of the post-cracking properties of steel fibre reinforced concrete (SFRC). Although it is not generally accepted design method, however there are some design proposals, and some countries already have its design standards or recommendations [1, 6-13]. Nevertheless, the second problem still remains open.

The determination method of the post-cracking parameters is prescribed by the design method, and usually it is experimental. Residual flexural tensile strength $f_{R,1}$ is the mostly applied parameter for the SLS (crack width) calculations. This parameter should be established from the experiments, and later recalculated to residual tensile stress ($\sigma_{fb} = kf_{R,1}$). Coefficient k can differ in different crack width calculation methods (0.40 – 0.45). Mean or characteristic values of $f_{R,1}$ could be used depending on the crack width calculation method. Recalculation methods are also described together with the crack width calculation methods [1, 6, 8-10, 12-14].

In order to use the discussed crack width calculation methods the tests are necessary for the determination of residual flexural tensile strength $f_{R,1}$. Method of these tests is given in EN 14651:2005+A1:2007 [14]. Nevertheless, these tests require time and other resources, therefore it would be great practical benefit if it would be possible to calculate $f_{R,1}$ without it. Even in those cases where designer is responsible only for specifying the requirements of residual tensile strength the calculation method of $f_{R,1}$ could help to analyse and choose the most economical solution.

Calculation of the residual flexural tensile strength is complicated due to the random distribution of the steel fibre in the concrete, due to a large variety of the fibre types, due to the different fibre bond and other aspects. Despite of that, it could be found in the literature some calculation proposals of SFRC post-cracking properties [1, 3, 15-19]. Most of these methods are developed using experimental results. But still, none of these methods is intended for the calculation of the residual flexural tensile strength $f_{R,1}$.

The new calculation method of the residual flexural tensile strength ($f_{R,1}$) is presented in this paper. The method was developed using 60 series of three-point bending test (446 prisms). 12 test series (132 prisms) has been tested by authors, and results of remaining series has been taken from the references [2, 15, 20, 21-34]. Only hooked end steel fibre was used for this method. The scatter of the experimental results and the relative errors of the method are discussed. The recommendation for designers and researchers are given.

2. Testing procedure and results

In order to determine $f_{R,1}$ the three-point bending tests were performed according to the regulations of EN 14651:2005+A1:2007 [14]. The scheme of three-point bending test is given in Fig. 1. Loading was performed according to a deformation control. The method allows to measure force-displacement or force-*CMOD* (crack mouth opening displacement) relations. When the displacement of the beam reaches 0.46 mm or *CMOD* reaches 0.5 mm, then the value of load is recorded and the residual flexural tensile strength $f_{R,1}$ is calculated according to Eq. (1).

10 test series have been performed in Kaunas University of Technology (KTU) and 2 test series in Norwegian University of Science and Technology (NTNU). In order to develop precise calculation method more

experimental results of three-point bending tests were taken from the references [2, 15, 20-34]. The information about the test series and its specimens is given in Table 1.

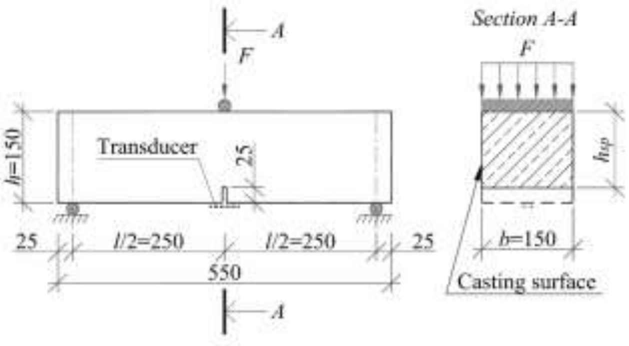

Fig. 1 Three-point bending test scheme according to EN 14651:2005+A1:2007 standard method

$$f_{R,i} = \frac{3F_{R,i}l}{2bh_{sp}^2}, \tag{1}$$

where $F_{R,i}$ is load corresponding with $CMOD = CMOD_j$ or $\delta = \delta_j$ (j = 1, 2, 3, 4), l is span length, b is width of the specimen, h_{sp} is distance between the tip of the notch and the top of the specimen.

Two types of concrete were used – traditionally vibrated and self-compacting SFRC. Traditionally vibrated SFRC was up to series No 41 (inclusive). All remaining series were made of self-compacting SFRC. Compositions of SFRC are not given in this paper because of huge number of different mixes. The main parameter which was used in further calculations is the average compressive strength of SFRC ($f_{cm,fb}$) – given in Fig. 2. The compressive strength of SFRC ($f_{cm,fb}$) was determined for every series, together with the residual flexural tensile strength ($f_{R,1}$).

Table 1

Specimens of three-point bending tests

Series No.	Reference	No. of spec.	l_{fb} / d_{fb}	l_{fb}, mm	f_y, MPa	V_{fb}, kg/m^3	Series No.	Reference	No. of spec.	l_{fb} / d_{fb}	l_{fb}, mm	f_y, MPa	V_{fb}, kg/m^3
1	KTU*	6	50	50	1200	25	31	[29]	5	64	35	1100	79
2	KTU*	12	50	50	1150	25	32	[30]	6	80	60	1050	20
3	KTU*	12	50	50	1200	30	33	[29]	5	80	60	1050	39
4	KTU*	12	50	50	1150	30	34	[30]	6	80	60	1050	40
5	KTU*	12	50	50	1150	35	35	[29]	5	80	60	1050	79
6	KTU*	12	50	50	1150	35	36	[30]	6	81	50	1270	40
7	KTU*	12	69	52	1500	15	37	[30]	6	81	50	1270	40
8	KTU*	12	69	52	1500	20	38	[30]	6	81	50	1270	40
9	KTU*	12	67	50	1150	30	39	[30]	6	81	50	1270	40
10	KTU*	12	50	30	1150	35	40	[15, 27]	8	78	35	1050	40
11	[28]	7	50	50	1100	20	41	[15, 27]	8	78	35	1050	60
12	[15, 27]	8	48	50	1000	20	42	[31]	6	63	50	1550	30
13	[15, 27]	8	48	50	1000	60	43	[22]	6	65	40	1050	30
14	[21, 22]	6	65	40	1050	30	44	[22]	6	65	40	1050	30
15	[22]	6	63	25	1700	30	45	[22]	6	65	40	1050	30
16	[15, 27]	8	67	60	1000	20	46	[22]	6	63	25	1700	30
17	[15, 27]	8	67	60	1000	20	47	[22]	6	63	25	1700	30
18	[15]	6	67	60	1000	20	48	[22]	6	63	25	1700	30
19	[22]	6	67	60	1000	30	49	[22]	6	67	60	1000	30
20	[15, 27]	8	67	60	1000	40	50	[22]	6	67	60	1000	30
21	[15, 27]	8	67	60	1000	40	51	[22]	6	67	60	1000	30
22	[15, 27]	8	67	60	1000	60	52	[32]	6	67	60	1160	39
23	[15, 27]	8	67	60	1000	60	53	[25]	5	67	60	1160	80
24	[15]	6	67	60	1000	60	54	[33]	6	67	60	1160	78
25	[34]	16	67	60	1000	75	55	[33]	6	67	60	1160	78
26	[30]	6	64	35	1100	20	56	[24, 26]**	9	64	35	1100	50
27	[20]	6	64	35	1100	30	57	[24, 26]**	4	64	35	1100	50
28	[29]	4	64	35	1100	39	58	NTNU*	9	80	60	1050	40
29	[30]	6	64	35	1100	40	59	NTNU*	9	80	60	1050	40
30	[2]	6	64	35	1100	60	60	[23]	6	80	60	1050	80

* – indicates that the tests have been performed by the authors. KTU or NTNU is the name of the university where the tests have been performed.

** – specimens were divided into two series because the mould filling procedure was different and the orientation coefficients also differed highly.

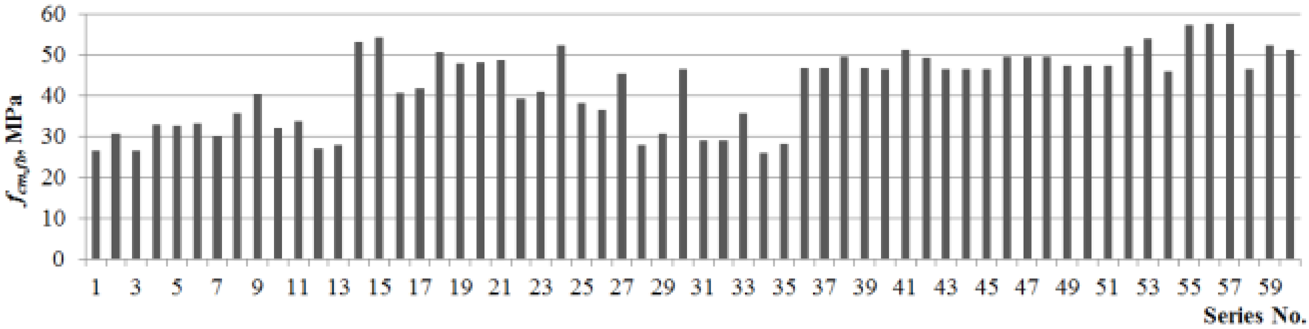

Fig. 2 Compressive strength ($f_{cm,fb}$) of SFRC series

The large scatter of the residual flexural tensile strength was obtained almost in all the series. The maximum relative error between the specimens of the same series was 71.3% (6th series). The average coefficient of variation of all series with known standard deviation (50 series) was 16.6%. While the average coefficient of variation of traditionally vibrated SFRC (31 series) was 19.2%, and 12.4% of self-compacting SFRC (19 series). Standard deviation of these coefficients was 7.65, 7.60 and 5.75, respectively. As an example stress-CMOD relation of ninth test series is given in Fig. 3. The coefficient of variation is equal to 23.43% here.

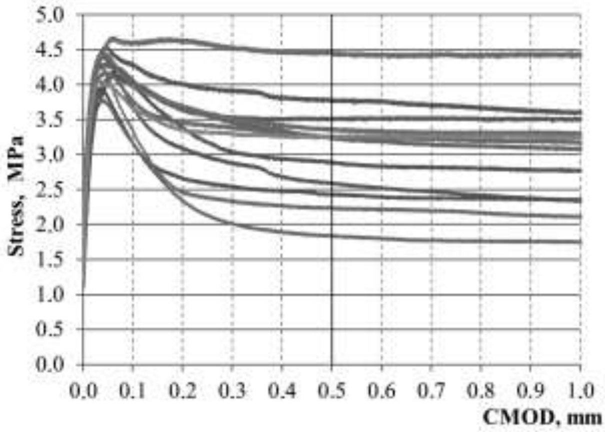

Fig. 3 Stress-CMOD relation of nineth test series

3. Analysis of relevant factors

In order to calculate the residual flexural tensile strength $f_{R,1}$ the main factors which has an influence on the post-cracking properties of SFRC should be established:

- Fibre length – l_{fb};
- Fibre diameter – d_{fb};
- Aspect ratio – l_{fb} / d_{fb};
- Fibre material properties (tensile strength) – $f_{y,fb}$;
- Fibre cross-section – shape of the section;
- Fibre shape – deformated shape of the fibre;
- Fibre content – V_{fb};
- Fibre orientation;
- Bond strength between fibre and matrix of concrete;
- Others.

The fibre length and the fibre diameter are ones of the main parameters as well as the aspect ratio of the fibre. Vandewalle determined that very short and short fibers are more effective for the narrower cracks, and the longer fibres are more effective for the larger cracks [20]. The fibre length also defines the embedment length. This is especially important for thick, short hooked end steel fibre and low strength concrete. When the embedment length is too short then the fibre(s) is pulled out of the concrete with or without the surrounding matrix. The fibre diameter defines the cross section of the fibre as well as it influences the concrete spalling in the crack surface. The aspect ratio defines the contact surface between the fibre and concrete matrix and so the stress level before the fibre de-bonding [15, 32].

The material that the fibre is made of defines such significant properties as the tensile strength of fibre ($f_{y,fb}$), modulus of elasticity (E) and others. The tensile strength ($f_{y,fb}$) defines the maximum available stress level in the fibre as well as the limit of the fibre bond [15, 16]. The modulus of elasticity (E) defines the deformations of the fibres as well as deformations of cracked SFRC members. However, while all the fibre is made of steel the modulus of elasticity (E) is approximately the same and it becomes not relevant for this research.

The different shape of the fibre defines the bond between the fibre and the concrete matrix. The circle shape of cross-section is the least effective comparing with other shapes such like rectangular, triangular or especially with the cross-section shape of the "Torex" fibre. However, the circular cross-section is the most common in practice and therefore only this cross-section shape was used in this research [3, 4].

To pull out the straight fibre which is perpendicularly embedded to concrete surface, the pulling force should exceed the shear stress-slip reaction (adhesion + friction). In order to improve the bond the deformed shape of the fibre was started to use. Lots of types of deformed steel fibre are in the market today – crimped (wavy), hooked end, with end paddles, with end buttons, etc. Nevertheless, the hooked end steel fibre is one of the most widely used. Usually, the hooked end steel fibre is produced from cold-drawn wire. To pull out such fibre, despite of the mentioned shear stress-slip reaction, the hooked end should be deformed into the straight during the pull-out. The extra pull out force depends on hooks, material properties, etc [3, 4, 15, 32]. For this research only the hooked end steel fibre was used (some fibre are given in Fig. 4).

The fibre content has the direct influence on the post-cracking properties of SFRC. This parameter is used in every proposal of calculation of the post-cracking properties [1, 3, 5, 15-18]. Nevertheless, in some proposals its influence is nonlinear. The main reason is the group effect, i.e. when the number of fibres being pulled out from the same area considerably increases the bond strength per fibre decreases [15, 35].

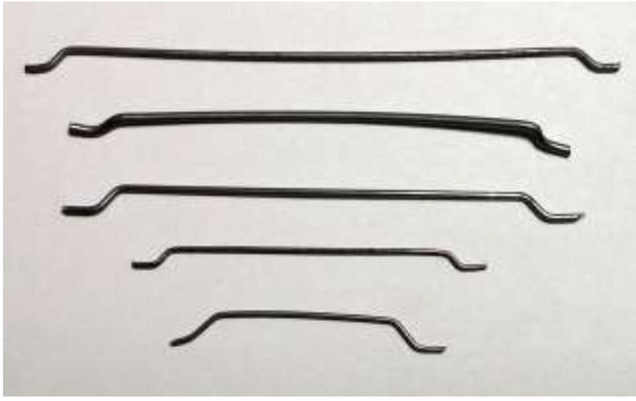

Fig. 4 Hooked end steel fibre

It is known that fibres can distribute and orientate randomly in concrete. Due to the random fibre orientation the angle between longitudinal axis of the fibre and the pull-out direction can be not equal to zero, which means that snubbing and spalling effects start to come hand in hand. The higher inclination angle the higher force is needed to pull-out the fibre due to snubbing effect. Meanwhile the higher inclination angle the lower force is needed to pull-out the fibre due to spalling of concrete in the crack. In more details these two effects are described in references [15, 35]. Also, depending on the orientation the number of fibres crossing the crack can differ significantly as well as post-cracking properties of SFRC [16, 35].

The orientation of steel fibre in SFRC can be described using the orientation factor. Theoretically, when all the fibres are orientated in one direction the orientation factor is equal to 1.0. This factor is equal to 0.637 when fibres are randomly orientated in plane, and it is equal to 0.5 when fibres are randomly orientated in space. The orientation factor can be calculated according to the Eq. (2) using the number of fibres per cross-section of SFRC [16, 19, 32]:

$$\alpha = \frac{n_{fb} A_{fb}}{A_c V_{fb}}, \tag{2}$$

where n_{fb} is a number of fibres per area of SFRC; A_{fb} is cross section area of single fibre; A_c is cross section area of SFRC; V_{fb} is fibre content (fibre volume ratio).

To evaluate the influence of the fibre orientation on the residual tensile strength a capacity factor (η_0) is used, which is calculated according to the literature [16]:

$$\left.\begin{array}{lll} \text{for} & 0 < \alpha \le 0.5: & \eta_0 = \dfrac{2}{3}\alpha; \\ \text{for} & 0.5 < \alpha \le 1.0: & \eta_0 = \dfrac{4}{3}\alpha - \dfrac{1}{3}, \end{array}\right\} \tag{3}$$

where α – the fibre orientation coefficient.

It is recommended in some calculation proposals of post-cracking properties that the fibre bond strength should be given from fibre pull-out tests. However the bond strength between the fibre and the matrix of concrete as well as resistance against the spalling can be approximately defined by strength of the concrete. The tensile strength of SFRC is used in some calculation proposals whereas the compression strength is used in another [1, 5, 15, 17, 18, 35]. While the compressive strength of SFRC ($f_{cm,fb}$) can be simply experimentally determined it was chosen to characterize the bond strength between fibre and matrix of concrete.

Other factors also can influence the post-cracking properties of the SFRC, such like local fibre distribution, local concentration of the fibres, member size, uneven properties of the concrete matrix, etc. The post-cracking properties can vary due to these factors. However, all experiments were performed according to the standard method given in EN 14651:2005+A1:2007. Such factors as member shape and its size as well as testing procedure are clearly described, and were the same for all specimens. Also, the experimental data which was taken from the references was limited. Therefore, it is assumed in this research that only factors which were described earlier are essential and should be considered in further research.

4. Analytical prediction of $f_{R,1}$

The main factors which have the influence on the residual flexural tensile strength ($f_{R,1}$) are described in previous section. These factors were combined while the most accurate calculation formula of $f_{Rm,1}$ was deduced. First of all the mentioned factors were partitioned in to three parts as it is given in Eq (4):

$$f_{Rm,1} = \beta \times \gamma \times y, \tag{4}$$

where the parameter β depends only on the compressive strength of SFRC $f_{cm,fb}$ (average value) – ($\beta \in f_{cm,fb}$), the parameter γ depends on fibre length l_{fb}, fibre diameter d_{fb}, the tensile strength of the fibre $f_{y,fb}$, and the fibre capacity factor η_0 ($\gamma \in l_{fb}, d_{fb}, f_{y,fb}, \eta_0$). The function y depends on the fibre content V_{fb} (fibre mass per cube / density of the fibre) and the fibre reinforcement efficiency factor k_{fb} – ($y \in V_{fb}, k_{fb}$).

In order to find the best relations of the discussed parameters some functions were analysed. The best relations were established when the sequent functions were used:

$$\beta = k_{c1} f_{cm,fb}{}^{n_{c1}} + k_{c2} f_{cm,fb}{}^{n_{c2}} + k_{c3}, \tag{5}$$

where k_{c1}, k_{c2}, k_{c3}, n_{c1}, and n_{c2} are coefficients which were combined while the most accurate combination was found.

$$\gamma = k \eta_0{}^{n_{\eta 1}} \left(\frac{l_{fb}}{d_{fb}}\right)^{n_{l1}} l_{fb}{}^{n_{l2}} d_{fb}{}^{n_{d1}} \left(\frac{f_{y,fb}{}^{n_{y1}}}{1000}\right)^{n_{y2}}, \tag{6}$$

where k, n_{l1}, n_{l2}, n_{d1}, n_{y1}, n_{y2}, and $n_{\eta 1}$ are coefficients which were combined while the most accurate combination was found.

The best fit function $y = f(x) = \dfrac{f_{Rm,1}}{\beta \times \gamma}$ and the coefficient of determination R^2 was established during analysis (Fig. 5).

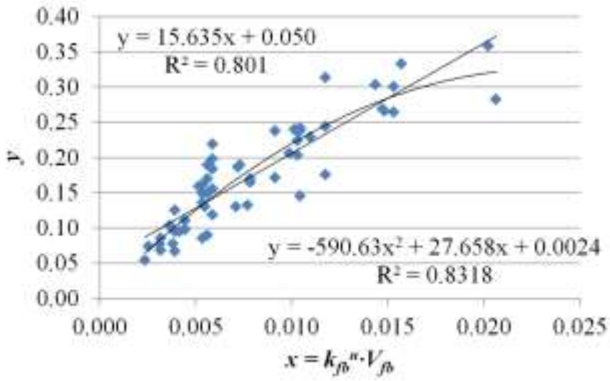

Fig. 5 Determination of function y

As it can be seen from Fig. 5 that two functions y were established and compared (linear and second order polynomial). Due to higher coefficient of determination (R^2) the second order polynomial function is used.

The only one difference between traditionally vibrated and self-compacting SFRC was assumed – the orientation factor α. For traditionally vibrated SFRC $\alpha = 0.60$ and for self-compacting SFRC $\alpha = 0.80$. The capacity factor was equal to 0.467 and 0.733, respectively. Such values of the orientation factor were chosen considering the experimental results (including the results from references) [21, 22, 24, 31, 33] and guidance from other references [13, 15].

As a result of this analysis the Eq. (7) is proposed for calculation of the residual flexural tensile strength $f_{Rm,1}$ (mean value). All experimental values of $f_{Rm,1}$ were compared with the results calculated according to the Eq. (7). The average relative error of calculated residual flexural tensile strength ($f_{Rm,1}$) is 0% due to the coefficient k_{adj}. The maximum relative error reaches 50%, and the standard deviation of the ratio between calculated and experimental results is 0.20. The comparison of calculated and experimental values of $f_{Rm,1}$ is given in Fig. 6.

$$f_{Rm,1} = k_{adj}\left(16.5 f_{cm,fb} - 0.185 f_{cm,fb}^{\ 2} - 155\right)\eta_0 \left(\frac{l_{fb}}{d_{fb}}\right)^{-\frac{1}{3}} \left(\frac{f_{y,fb}}{1000}\right)^{\frac{1}{2}} \left[27.658\left(k_{fb}^{\ 1.5} V_{fb}\right) - 590.63\left(k_{fb}^{\ 1.5} V_{fb}\right)^2 + 0.0024\right], \quad (7)$$

where the adjustment coefficient k_{adj} is equal to 0.96 and the fibre reinforcement efficiency factor – $k_{fb} = \dfrac{l_{fb}}{50 d_{fb}}$.

Although the maximum relative error is quite high, however higher then 30 % relative error was reached only for 7 test series. Considering the relative error between the specimens of the same series as well as the relative error between the comparable series (such like 5th and 6th, 37th and 39th, etc.) the accuracy of the calculation method is satisfactory.

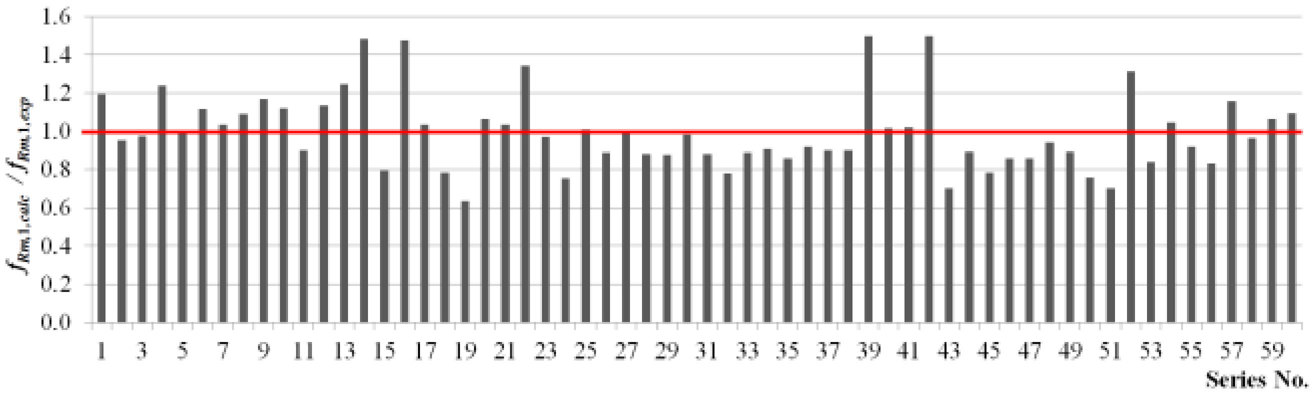

Fig. 6 The ratio between calculated and experimental residual flexural tensile strength $f_{Rm,1}$

5. Discussions

Two general orientation factors were assumed for two different types of SFRC (traditionally vibrated and self-compacting). However, despite of the clearly defined experimental program these factors (α) can vary depending on various other factors, such like vibration time, vibrator, mixer, workability of mortar, etc. The variations of the orientation factor could lead such high relative errors for some test series. Also, the other factors such like actual local fibre concentration, water cement ratio, precision and dimensions of the fibre hooks could have a significant influence on $f_{R,1,}$ but these factors were not included into the research due to lack of information.

The precision of the proposed calculation method could be revised in future research after the inclusion of more experimental results as well as the mentioned additional factors. In order to use this method directly for particular structures (beams, walls, plates, etc.) the more detailed analysis of orientation factor and its influence on $f_{R,1}$ is required as well as possible adjustments of the method. The intended application of the proposed calculation method is the same as tests results of the standard beams, which should be cast and tested according to EN 14651:2005+A1:2007 [14]. The indirect application is given in related codes, standards and recommendations [8, 10, 12, 13].

6. Conclusions

1. The calculation method of the residual flexural tensile strength ($f_{Rm,1}$) was developed using the experi-

mental results of 446 standard beams. The important parameters: fibre length, fibre diameter, tensile strength of the fibre, fibre orientation factor, fibre content, and compressive strength of SFRC was included in the research. The proposed calculation method is suitable for circular cross-section hooked end steel fibre reinforced concrete, where the mean compressive strength $f_{cm,fb}$ varies from 25 to 60 MPa and the fibre content varies from 15 to 80 kg/m^3. Method is suitable for the traditionally vibrated and self-compacting SFRC.

2. The calculated residual flexural tensile strength ($f_{Rm,1}$) could be applied in the SLS calculations (crack width calculations, etc.) according to suitable codes, standards and recommendations. The relative error of calculation method exceeded 30% in few cases. However, comparing it with the deviations between separate specimens of the same series the precision of the method is satisfactory. For the practical purposes the method could be used as a first approximation in design. The proposal and the related future research could lead to the reduction or even to elimination of the necessary tests from the design process.

References

1. **Jansson, A.** 2007. analysis and design methods for fibre reinforced concrete: a state-of-the-art report, Göteborg, Sweden: Department of Civil - and Environmental Engineering Division of Structural Engineering, Chalmers university of technology.
2. **Vandewalle, L.** 2007. Postcracking behaviour of hybrid steel fiber reinforced concrete, Proceedings of FraMCoS-6, 1367-1375.
3. **Naaman, A.E.** 2003. Engineered steel fibers with optimal properties for reinforcement of cement composites, Journal of Advanced Concrete Technology 1(3): 241-252. http://dx.doi.org/10.3151/jact.1.241.
4. **Kaklauskas, G.; Bačinskas, D.; Gribniak, V.; Jakubovskis, R.; Ulbinas, D.; Gudonis, E.; Meškėnas, A.; Timinskas, E.; Sokolov, A.** 2012. VGTU . Concrete structures reinforced by steel fibres and non-steel bars. Vilnius: Technika. 391p. (in Lithuanian).
5. **Grünewald, S.** 2004. Performance-based design of self-compacting fibre reinforced concrete, PhD thesis, Technical University of Delft.
6. **Kelpša, Š; Augonis, M.; Daukšys, M.; Augonis, A.** 2014. Analysis of crack width calculation of steel fibre and ordinary reinforced concrete flexural members, Journal of Sustainable Architecture and Civil Engineering 6(1): 50-57. http://dx.doi.org/10.5755/j01.sace.6.1.6336.
7. CNR-DT 204/2006, Guide for the design and construction of fibre-reinforced concrete structures. CNR.
8. **RILEM TC 162-TDF.** 2003. Final recommendation of RILEM TC 162-TDF: Test and design methods for steel fibre reinforced concrete sigma-epsilon-design method, Materials and Structures 36(262): 560-567. http://dx.doi.org/10.1617/14007.
9. **Jansson, A.; Löfgren, I.; Gylltoft, K.** 2010. Flexural behaviour of members with a combination of steel fibres and conventional reinforcement, Nordic Concrete Research 42(2): 155-171.
10. Model Code of Concrete Structures 2010 (Final Draft). FIB.
11. DafStb Guideline, Steel fibre reinforced concrete. German Committe for Reinforced Concrete.
12. ftSS 812310:2014 Design of fibre concrete structures, Swedish Standards Institute.
13. design guideline for structural applications of steel fibre reinforced concrete, SFRC Consorcium.
14. EN 14651:2005+A1:2007 - Test method for metallic fibre concrete. Measuring the flexural tensile strength (limit of proportionality (LOP), residual), CEN/TC 229.
15. **Dupont, D.** 2003. Modelling and experimental validation of the constitutive law (σ−ε) and cracking behaviour of steel fibre reinforced concrete, PhD thesis, Catholic University of Leuven, Department of Civil Engineering.
16. **Thorenfeldt, E.T.** 2003. Theoretical Tensile strength after cracking, fibre orientation and average stress in fibres, Workshop Proceedings from a Nordic Miniseminar: Design Rules for Steel Fibre Reinforced Concrete Structures, 43-60.
17. **Naaman, A.E.** 2003. Strain hardening and deflection hardening fiber reinforced cement composites, Proceedings of the 4th International RILEM Workshop on High Performance Fiber Reinforced Cement Composites (HPFRCC4), 95-113.
18. **Sujivorakul, C.** 2012, RILEM bookseries. High performance fiber reinforced cement composites 6. Model of hooked steel fibers reinforced concrete under tension, Dordrecht--Heidelberg--London--New York: Springer. 19-26.
19. **Kanstad, T.; Juvik, D.A.; Vatnar, A.; Mathisen, A.E.; Sandbakk, S.; Vikan, H.; Nikolaisen, E.; Døssland, Å; Leirud, N.; Overrein, G.O.** 2011. COIN report: guidlines for sizing, execution and control of fiber reinforced concrete structures.
20. **Vandewalle, L.** 2007. Mechanical properties of hybrid fiber reinforced concrete, Proceedings of the 1st International Conference on Sustainable Construction Materials and Technologies, 283-290.
21. **Vandewalle, L.; Heirman, G.** 2009. Properties of Self-compacting fibre reinforced concrete, 4th International Conference on Construction Materials: Performance, Innovations and Structural Implications (CONMAT09), 406-411.
22. **Vandewalle, L.; Heirman, G.; Van Rickstal, F.** 2008. Fibre orientation in self-compacting fibre reinforced concrete, 719-728.
23. **Olimb, A.M.** 2012. Testing of fibre reinforced concrete structures: shear capacity of beams with openings, Master thesis, Norwegian University of Science and Technology, Faculty of Engineering Science and Technology, Department of Structural Engineering.
24. **Prisco, M.; Ferrara, L.; Caverzan, A.** 2012. Self-compacting fibre reinforced concrete: is the material really isotropic? 3º Congreso Iberoamericano Sobre Hormigón Autocompactante Avances Y Oportunidades, 17-31. Available http://www.autocompacto.net/wp-content/themes/splendio/pdf/ponencias/03_ID02_MdiPrisco_def.pdf.
25. **Flakk, Ø; Tordal, K.N.** 2012. Testing of fibre reinforced concrete structures: structural behaviour in the serviceability and ultimate limit states, Master thesis,

Norwegian University of Science and Technology, Department of Structural Engineering.
26. **Ferrara, L.; Caverzan, A.; Muhaxheri, M.; Prisco, M.** 2012. Identification of tensile behaviour of SFR-SCC: Direct Vs. Indirect Tests, Eighth RILEM International Symposium on Fibre Reinforced Concrete (BEFIB 2012): Challenges and Opportunities, 209-221.
27. **Dupont, D.; Vandewalle, L.** 2003. Calculation of crack widths with the (σ –ε) method, Proceedings of International RILEM Workshop on Test and Design Methods for Steel Fibre Reinforced Concrete - Background and Experiences, 119-144.
28. **Buratti, N.; Mazzotti, C.; Savoia, M.** 2010. Experimental study on the flexural behaviour of fibre reinforced concretes strengthened with steel and macro-synthetic fibres, Proceedings of FraMCoS-7, 1286-1294.
29. **Amirineni, K.C.** 2009. Fracture properties of fiber reinforced concrete, Master thesis, The Pennsylvania State University.
30. **Álvarez, A.B.** 2013. Characterization and modelling of SFRC elements, Doctoral Thesis, Universitat Politècnica de Catalunya.
31. **Hanssen, H.E.; Hallberg, M.A.** 2013. Post-Tensioned fibre reinforced flat slab, Master Thesis, Norwegian University of Science and Technology.
32. **Sandbakk, S.** 2011. Fibre reinforced concrete: evaluation of test methods and material development, PhD Thesis, Norwegian University of Science and Technology.
33. **Rød, A.; Aspås, Ø.** 2013. Testing of fibre reinforced concrete structures: shear capacity of beams with openings, Master thesis, Norwegian University of Science and Technology.
34. **Strack, M.** 2008. Modelling of crack opening in steel fibre reinforced concrete under tension and bending, Proceedings of the 7th RILEM International Symposium on Fibre Reinforced Concrete: Design and Applications - BEFIB, 323-332.
35. **Naaman, A.E.** 2008, Engineering materials for technological needs. High-performance construction materials. Sci Appl. High Performance Fiber Reinforced Cement Composites. Singapore: World Scientific Publishing Co. Pte. Ltd. 91-153p.

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EMPIRICAL CALCULATION METHOD OF RESIDUAL FLEXURAL TENSILE STRENGTH $f_{R,1}$

S u m m a r y

An empirical calculation method is proposed in the article for the calculation of residual flexural tensile strength of steel fibre reinforced concrete. The method was developed analyzing factors which have an influence on residual tensile strength, and using the experimental results of 446 specimens. Assuming that the residual flexural tensile strength depends on three functions the relevant variables and constants were deduced, and so the optional functions were established. The proposed calculation method is suitable for the SFRC, where fibre content varies from 15 to 80kg/m^3 as well as mean compressive strength varies from 25 to 60 MPa. Only circular cross-section hooked end steel fibre is available however, traditionally vibrated as well as self-compacting concrete is suitable for this method. The residual flexural tensile strength ($f_{R,1}$) is used in various crack width calculation methods, and it should be determined experimentally according to standards. Therefore, the proposed calculation method has a great practical benefit.

Keywords: Steel fibre, SFRC, residual flexural tensile strength, $f_{R,1}$, CMOD.

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